which $g(z)$ has, respectively, no zeros in the interval $-1<z<+1$, or a single zero at $z=0$, every solution[4] of (1) must have a zero in the interval $0<z<1$. Let then $g(z)$ be a solution of the equation, such that

$$g(z_0)=g(+1)=0, \quad 0<z_0<1. \tag{2}$$

We will show that if $\lambda<\frac{1}{4}$, Eq. (2) is impossible. There is a well-known comparison theorem which states that, if there is a solution with the properties (2), then the equation

$$g''+V(z)g=0, \tag{3}$$

where

$$V(z)>\lambda v(z)>0 \tag{4}$$

has a solution such that

$$g(+1)=0; \quad g(z_1)=0, \tag{5}$$

where $z_0<z_1<+1$. We choose

$$V(z)=[4z^2(1-z^2)]^{-1}. \tag{6}$$

By means of a change of variable

$$z=(1-x)^{\frac{1}{2}}, \quad g=(1-x)^{\frac{1}{4}}xf(x), \tag{7}$$

(3) reduces to the hypergeometric equation

$$x(1-x)\frac{d^2f}{dx^2}+(2-3x)\frac{df}{dx}-\frac{15}{16}f=0. \tag{8}$$

It can be seen that the conditions (5) now reduce to

$$f(0)=\text{finite}; \quad f(x_1)=0 \tag{9}$$

where x_1 is positive and <1. Any solution of (8) which is finite at $x=0$ is a multiple of the Gauss hypergeometric function[5]

$$F(5/4,3/4,2;x).$$

This, however, is obviously positive in the whole interval $0<x<1$, and thus (9) is impossible.

[1] G. C. Wick, Phys. Rev. **96**, 1124 (1954); R. E. Cutkosky, Phys. Rev. **96**, 1135 (1954).
[2] F. L. Scarf, Phys. Rev. **100**, 912 (1955).
[3] We understand that Dr. Scarf is now in essential agreement with us; see preceding Letter by D. A. Geffen and F. L. Scarf [Phys. Rev. **101**, 1829 (1956)]. We are indebted to these authors for advance communication of their paper.
[4] Notice that eigensolutions of (1) must be either even or odd functions of z.
[5] Compare Eq. (64) of the first paper of reference 1.

Nuclear Moments of Hf^{177} and Hf^{179}†

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THE only previous estimate of the spins of the odd-neutron hafnium isotopes was made many years ago in a brief note by Rasmussen.[1] From the number of observed hyperfine structure components in the Hf I spectrum he concluded that the spins were 1/2 or at most 3/2. Various types of evidence that are incompatible with such low values have recently appeared.[2–4] A reinvestigation of the hyperfine structure using enriched isotopes therefore seemed indicated.

Under the high resolution afforded by multilayer coated Fabry-Perot etalons, both isotopes show flag patterns with as many as eight components. The over-all

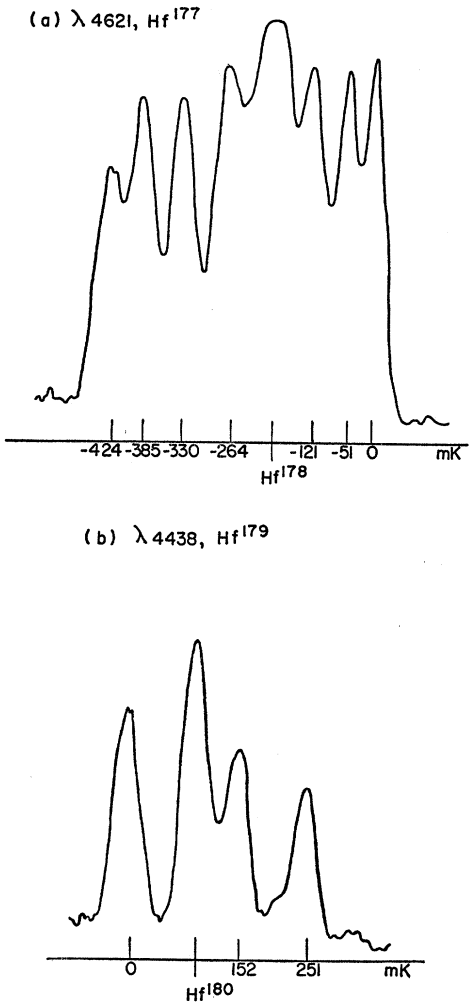

FIG. 1. Microphotometer traces of structure in Hf I lines.

widths are greater for Hf^{177} and the intensities are degraded in opposite directions, so that the magnetic moments must have opposite signs. Using a classification of the lines made by Meggers[5] we find that the observed structures can only be interpreted by assigning $I=7/2$ to Hf^{177} and $I=9/2$ to Hf^{179}. Figure 1 shows two patterns, (a) for λ 4620.87 ($a^3F_4-z^5F_5^0$) of Hf^{177} and (b) for λ 4438.03 ($a^3P_0-z^3S_1^0$) of Hf^{179}. In the former case, all eight components are resolved, thanks to the influence of a considerable quadrupole moment

which distorts the pattern in such a way as to render the spacing nearly uniform. In the latter, the intensities and separations are approximately those expected for $I=9/2$, but because of disturbing lines and the intermediate coupling they could not be relied upon for a quantitative result. Instead, we have used a method of finding which value of I gives the best fit to the second-degree hfs equation[6] for the resolved components in each of two lines of high J value (4621 and 5453 A). The result is unambiguous, and is confirmed by the fact that only for $I=9/2$ does the ratio of the quadratic coefficients for the two isotopes have the same value for the two lines. This ratio is equal to the ratio of the quadrupole moments of the two isotopes. Its mean value was found to be $Q(177)/Q(179)=0.99 \pm 0.02$.

From the coefficients of the linear term in the above solutions, the ratio of the magnetic moments is found to be $\mu(177)/\mu(179)=-1.276\pm 0.008$. The difference in sign is not surprising in view of the different spins. Calculations of the magnitudes of the magnetic and quadrupole moments are under way, and the results will be reported in the complete description of this work to be published elsewhere.

We wish to acknowledge the assistance rendered by Dr. W. F. Meggers in making available his term analysis in advance of publication. We are also indebted to Dr. G. M. Temmer and Dr. C. D. Jeffries for helpful discussions, and to the latter for the loan of the separated isotopes, which were obtained from the Stable Isotopes Division of the Oak Ridge National Laboratory.

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[5] W. F. Meggers (private communication).
[6] H. Kopfermann, *Kernmomente* (Akademische Verlagsgesellschaft M. B. H., Leipzig, 1940), p. 64, Eq. (12.4).

Optical Model of Nucleus with Absorbing Surface*

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PREVIOUS optical-model calculations using square wells or almost square wells[1] have reproduced qualitatively the angular distribution of elastically scattered 14-Mev neutrons. Attempts to fit the 14-Mev data with tailed wells which fit the 17-Mev proton data so well[2] failed except for the very light elements.[3] For the heavy elements the cross sections in the region ~90° (lab angle) dropped off considerably. Other potential wells were then tried in order to bring the cross sections into better agreement with the experimental data.

Taking the Pauli principle into account, one is inclined to think of nonelastic events as taking place at the surface of the nucleus, at least for low-energy nuclear events. For this reason a Gaussian-type well was used for the imaginary part of the potential and the usual tailed well for the real part. These wells are indicated in Fig. 1. The real part is given approximately by

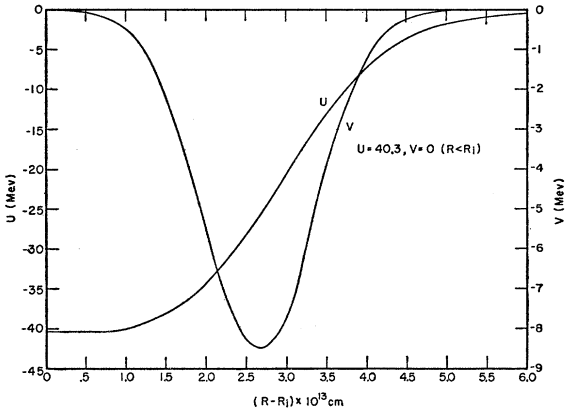

FIG. 1. Potential wells for the theoretical calculations.

$$U=U_0/[1+\exp(R-R_0)/a],$$

where $U_0=40.3$ Mev, $R_0=(1.2A^{\frac{1}{3}}+0.64)\times 10^{-13}$ cm, and $a=0.6\times 10^{-13}$ cm, and the imaginary part by

$$V=V_0\exp[-(R-R_0)^2/b^2],$$

where $V_0=8$ Mev, $b=0.978$, and R_0 is as defined above.

An exact phase-shift analysis using these wells was carried out for 14.6-Mev neutrons incident on Mg, Ca, Cd, and Bi, these being the only elements for which experimental data[4] were available over a range of 140°. The results obtained are extremely sensitive to the changes in U_0, R_0, b, and the position of the center of the Gaussian. The results of these calculations appear in Fig. 2. The total cross sections for elastic and nonelastic scattering appear in Table I.

TABLE I. Theoretical cross sections for elastic and nonelastic scattering of 14.6-Mev neutrons (in barns).

	Mg	Ca	Cd	Bi
σ_{el}	0.81	0.88	3.1	3.4
σ_{nonel}	0.99	1.23	1.9	2.3

It is possible to improve this fit somewhat by making the Gaussian wider as one goes to lighter elements. In the case of magnesium, a better fit is actually obtained by using the same shape for the imaginary part of the potential as for the real part. The width of the Gaussian