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Angular Correlations in Associated Production*

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Five examples of associated production have been found in $\pi^- - p$ collisions at 1.0 Bev. Three cases of $\Lambda^0+\theta^0$, one case of $\Sigma^0+\theta^0$, and one case of Σ^-+K^+ have been found. From these cases a cross section of about 1 mb is deduced. The center-of-mass angular distribution of the hyperons is compared with that found by W. B. Fowler *et al.* at 1.4 Bev. The hyperons show a strong correlation between their decay planes and their planes of production as has been found previously by Fowler *et al.* The θ^0's do not show this tendency. The possibilities of observational bias are considered. One explanation of these phenomena is that the orbital angular momentum vector and spin vector of the hyperon are either parallel or antiparallel.

From these and other considerations it seems likely that the spin of the hyperons Σ and Λ^0 is 3/2 or 5/2 or possibly as high as 7/2 and the spin of the θ^0 is probably 0 but could be 1.

I. INTRODUCTION

THE present paper is a summary of work on associated production of hyperons and heavy mesons in 1.0-Bev $\pi^- - p$ collisions. These data are compared with those obtained by the Brookhaven cloud chamber group in their study of $\pi^- - p$ interactions at 1.4 Bev.[1] Particular emphasis is placed on the angular correlation in the production processes.

II. COMPARISON OF CROSS SECTION

Scanning diffusion cloud-chamber pictures made with the Brookhaven magnet chamber, in which 200 $\pi^- - p$ interactions were found, revealed five examples of associated production. Two of these cases were reported previously.[2] If one estimates from emulsion data the number of zero-prong interactions and uses the total $\pi^- - p$ cross sections as measured by Cool, Madansky, and Piccioni,[3] a cross section of 1 mb for associated production is deduced which is to be compared with 0.9 mb at 1.4 Bev.[1] No attempt has been made to correct for geometrical loss of the neutral particles; however, because of the lower center-of-mass energy of the products the geometrical correction must be considerably less than at 1.4 Bev. The ratio of the cross sections can be found by comparing the number of Λ^0's seen emerging from the walls and bottom of the chamber at 1.4 Bev and 1.0 Bev. The Brookhaven workers found 27 Λ^0's and 150 $\pi^- - p$ interactions[1]; we find 27 Λ^0's and 197 $\pi^- - p$ interactions. The cross section is about 50 mb at 1.0 Bev, and 34 mb at 1.4 Bev. Calculations from these data give the result that the cross section for associated production goes up by about a factor of 2 between 1.0 and 1.4 Bev.

In any event the $\pi^- - p$ cross section for associated production seems to be nearly constant between 1.0 and 1.4 Bev.

III. NEW CASES OF ASSOCIATED PRODUCTION

Previously we reported two cases of associated production.[2] Case I indicated the production of a Σ^0 hyperon. Cases II, III, and IV are consistent with the production of $\Lambda^0+\theta^0$. Case V is consistent with the production of a Σ^- and K^+ meson of mass about 970 m_e. The experimental measurements and the Q values deduced are given in Table I. The momenta of the incoming pions are estimated by measuring parallel tracks and by spectrum measurements from the kinematics of the elastic collisions.

IV. ANGULAR DISTRIBUTIONS

To understand in detail the mechanisms of production, it is necessary to know what angular momentum waves participate in associated production in both the incoming pion wave and the outgoing θ^0 wave. To this end we study the angular distribution of Λ^0's and Σ's in the production process. To supplement the data on angular distribution, we use some of the data on Λ^0's coming from the walls and bottom of the chamber. Scattering inside the parent nucleus will produce some perturbation of the angular distribution. In order to eliminate as much as possible these side effects we take only cases of Λ^0's which from their angle and momentum are consistent with $\pi^-+p\rightarrow\Lambda^0+\theta^0$.

Figures 1 and 2 show the angular distributions of hyperons produced at 1.0 and 1.4 Bev, respectively. The distribution at 1.0 Bev seems somewhat flatter than that at 1.4 Bev. This is not surprising since the Λ^0 has momentum 300 Mev/c in the center-of-mass system when produced in a 1.0-Bev $\pi^- - p$ collision and 500 Mev/c when produced at 1.4 Bev. Crudely one can say that outgoing waves up to $L=3$ are involved at 1.4 Bev and up to $L=2$ are involved at 1.0 Bev. It is of course possible to push the orbital states higher since angular distributions generally only establish lower limits for L values. Also this implies that incoming pion waves at least up to $L=3$ at 1.4 Bev and $L=2$ at 1.0 Bev are

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[1] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **91**, 1287 (1953); Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **93**, 861 (1954); Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **98**, 121 (1955).

[2] W. D. Walker, Phys. Rev. **98**, 1407 (1955).

[3] Cool, Madansky, and Piccioni, Phys. Rev. **93**, 637 (1954).

TABLE I. Description of new cases of associated production.[a]

	P_i in Mev/c	θ_K	θ_Y	$\varphi_{\pi Y}$	φ_{pY}	$\varphi_{\pi^+ K}$	$\varphi_{\pi^- K}$	P_{pY} in Mev/c	$P_{\pi Y}$ in Mev/c	P_{+K} in Mev/c	P_{-K} in Mev/c	Q in Mev	Q_K in Mev
III	1100±70	5°±2°	13°±1°	35°±2°	16.5°±1.5°	18.7°±1°	50°±2°	310±10		538±40	···	41±5	216±20
IV	1150±50	···	20°±3°	54°±10°	6°±2°	···	···	···	70±14	···	···	$22\pm{}^{15}_{10}$	220±40
V	1150±50	31.5±4°	16°±2°	70°±10°	···				···			···	182±50

[a] P_i = Incoming momentum in Mev/c;
P_{pY} = momentum of the nucleon from the decay of the hyperon in Mev/c;
$P_{\pi Y}$ = momentum of the π from the decay of the hyperon in Mev/c;
$P_{\pi K}$ = momentum of the π from the decay of the K particle in Mev/c;
Q = Q value of the hyperon in Mev;
Q_K = Q value of the K in Mev;
θ = angle of production;
φ = opening angle of the decay.

responsible for the production processes. Work on $\pi^- - p$ processes[4,5] at both of these energies indicates that incoming waves up to $L=3$ or 4 at 1.0 Bev and $L=4$ or 5 at 1.4 Bev are important in pion production processes—and could certainly be important in associated production as well.

V. ANGULAR CORRELATIONS

One of the most interesting features of the production processes observed by the Brookhaven group[1] is the apparent strong correlation between the plane of production of the hyperon and the plane formed by the decay products of the hyperon. The five cases reported here show these same features. Adding these to the Brookhaven cases, there are now 12 cases of hyperon production and decay in which the dihedral angle between planes of production and decay of the hyperon is less than 45°. Table II gives the angles between the planes of production and decay for the five cases found here for both the hyperons and the θ^0's. Figures 3 and 4 give a summation of the Brookhaven and Wisconsin data on angular correlation.

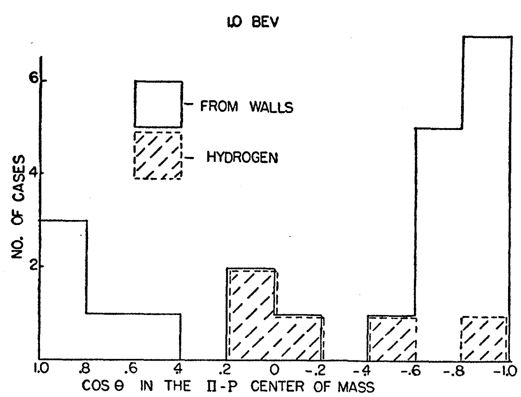

FIG. 1. This histogram shows the angular distribution in the center-of-mass system for hyperons produced at 1 Bev. The cross-hatched cases are those produced in the gas. The rest of the histogram is taken from cases of Λ^0's decaying when the point of origin is not seen. The momentum of the Λ^0 is calculated in the $\pi^- - p$ center-of-mass system and is required to be within 150 Mev/c of the 300 Mev/c expected in the process $\pi^- + p \rightarrow \Lambda^0 + \theta^0$ before it is included in the distribution.

[4] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).
[5] W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955).

The calculated curves are the distributions of dihedral angles for particles decaying with $L=2$ and $L=3$ assuming initially spins $S=5/2$ and $7/2$ and S vector oriented perpendicular to the plane of production. It should be noted that the θ^0's do not seem to show the correlation. The probability of obtaining the observed distribution for hyperons if the true distribution were uniform is about 10^{-3}.

Before going any further with these considerations, it is necessary to consider the effect of scanning bias on the data. It is conceivable that the effects observed are just a result of the method of search. The probability of seeing a Λ^0 decay is probably proportional to the length of the tracks in the sensitive region of the chamber. Also the probability of seeing a decay must be proportional to the ionization of the tracks. The sensitive volume of the chamber is effectively thicker for a highly ionizing event. It is worth noting that in the cases in which Λ^0 and θ^0 were both found, the scanner never saw the θ^0 but only the Λ^0. In this investigation only four cases have been found which could be the decay of θ^0's coming out of the bottom.

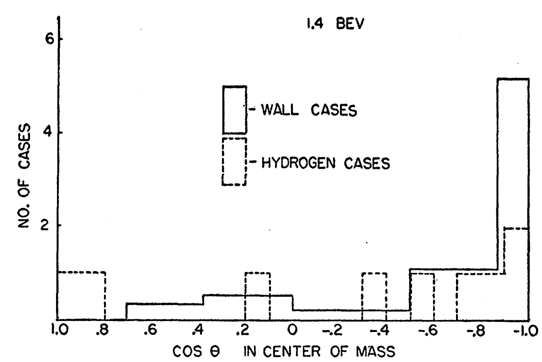

FIG. 2. The histogram shows the center-of-mass angular distribution for hyperons produced in collisions at 1.4 Bev. These data were taken from the 1955 report of Fowler *et al.*[1] The dashed histogram is for the cases of associated production in hydrogen.

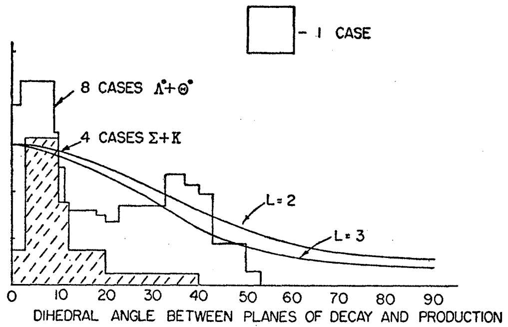

FIG. 3. This histogram shows the number of cases of hyperon production in which a given dihedral angle between planes of production and decay is found. These are the cases found in the present investigation plus those published by Fowler *et al.*[1] The curves are calculated assuming that the decays are for particles of $L=2$, $J=5/2$ and $L=3$, $J=7/2$ in which J is originally oriented perpendicular to the plane of production. Each case is represented by a rectangle whose width is proportional to the error in the measurement of the dihedral angle and whose height is inversely proportional to this error. All cases are represented by equal areas.

Some decay configurations are easier to see than others. In particular, if a Λ^0 is moving up through the sensitive region of the chamber, then it will produce more track length in the chamber if its plane of decay is perpendicular to the plane of the chamber since probably one of the tracks will traverse a considerable fraction of the sensitive region of the chamber. Actually, if the plane of decay is perpendicular, the Λ^0 will be viewed edge on and might well be missed so that an angle somewhat less than 90° is perhaps most favorable. In Fig. 5 a histogram is drawn for the number of cases of Λ^0 decay showing a given dihedral angle between the plane of the chamber and the plane of decay. There seems to be an indication of a peak around 60° or 70° and perhaps another at 0°.

If one examines the Λ^0's from the walls and bottom for correlation between planes of production and decay, essentially no correlation is found.[6] The results of this

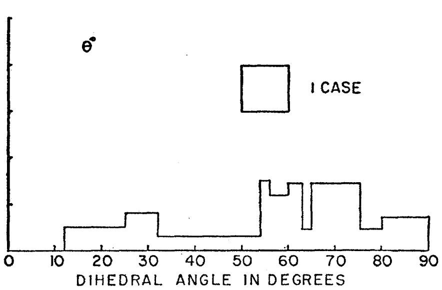

FIG. 4. This histogram shows the number of cases of θ^0 decay having a given angle between planes of decay and production. Three of the cases are from the present investigation and three from the work of Fowler *et al.*[1] The data are presented as in Fig. 3.

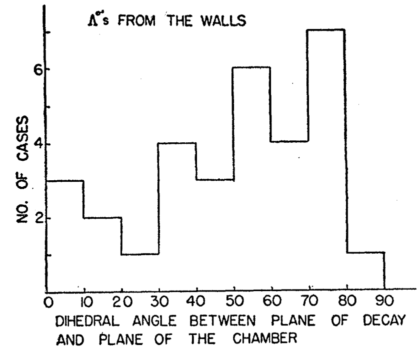

FIG. 5. These data are taken from examples of Λ^0 decay in which a point of origin is not seen. The histogram gives the number of cases *versus* the dihedral angle between the plane of the chamber and the plane of decay.

search are shown in Fig. 6. Whatever the biases are, they are rather similar for Λ^0's produced in the walls and bottom and for Λ^0's produced in the gas itself. Thus it seems that observational bias alone will not produce the correlation.

We have also looked at only Λ^0's which are consistent kinematically with having been produced in the reaction $\pi^-+p\rightarrow\Lambda^0+\theta^0$, i.e., showing the right momentum for a given angle of production. There are nine such cases, and the average angle between planes of production and decay is 35° as compared to 51° for the rest of the sample. Three of the nine cases have dihedral angles greater than 45°.

The cases of the decays of the Σ^- are rather different as far as possible biases are concerned. The main bias here would make it more likely to pick up cases in which the π^- goes at large angles relative to the direction of flight. The azimuth dependence of detection should be dependent only on the particular orientation of the Σ in chamber. The fact that the Σ's also show a strong correlation is independent evidence for the correlations of the hyperons.

If the hyperons tend to decay in the plane of production, this means that the spin of the hyperon must be oriented nearly normal to this plane. If this is the case, it means that also the spin is nearly normal to the direction of motion of the hyperon. Therefore the products of

TABLE II. Data on angular correlations.[a]

Case	Particle	η
I	Λ^0	$8°\pm10°$
	θ^0	$72°\pm18°$
II	Λ^0	$38°\pm15°$
	θ^0	$40°\pm15°$
III	Λ^0	$35°\pm15°$
	θ^0	$80°\pm20°$
IV	Λ^0	$2°\pm^{7°}_{2°}$
V	Σ^-	$5°\pm^{5°}_{2°}$

[a] η = dihedral angle between planes of decay and production.

[6] See *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1955) for a summary of the work on correlations of planes of decay and production for Λ^0's coming from interactions in complex nuclei. There are reports by Sorrels, Rossi, and Reynolds.

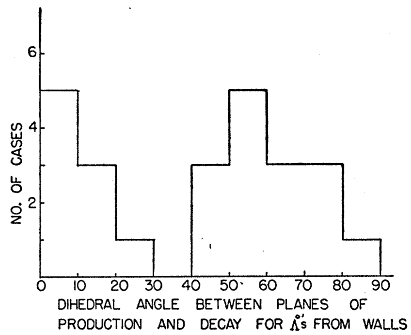

FIG. 6. This histogram shows the number of cases of Λ^0 decay in which the planes of production and decay have a given angle between them. Most of the Λ^0's seen come from the walls and bottom of the chamber. There is a pool of alcohol on the bottom so that some of the Λ^0's from the bottom plus a few from out of the illuminated region will actually have been formed in π^--proton collisions.

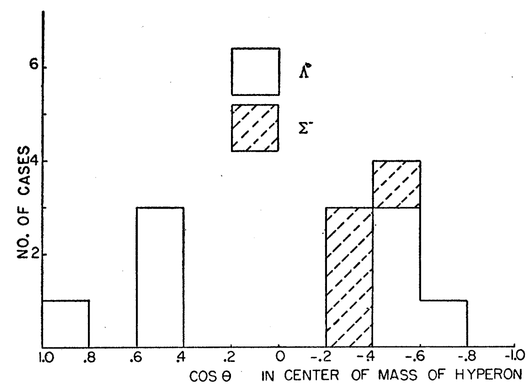

FIG. 7. This histogram shows the angular distribution in the rest system of the hyperon of the π's from the decays of hyperons formed in $\pi^- - p$ collisions. Equal intervals on the abscissa represent angular intervals containing equal solid angle. The angle 0° is in the direction of flight of the hyperon.

decay of the hyperon should go preferentially in the forward and backward directions in its rest frame.[7] The forward and backward directions are in this case parallel and antiparallel to the direction of flight of the hyperon in the laboratory system of reference. The angular distributions of the π's from Λ^0's and Σ^-'s are shown in Fig. 7. It can be seen that the angles close to 90° in the center-of-mass system are not preferred. It is puzzling that more cases close to 0° and 180° have not been found; indeed the evidence on correlation and this observation seem almost mutually exclusive. Again it is necessary to consider the possibilities of scanning bias. In the cases of Σ^-, all four decays occur with the π^- going backward in the center-of-mass system of the Σ^-. These cases are considerably easier to see than cases in which the π^- goes forward, so that one suspects that bias effects could explain the peculiarity of the Σ^-'s. In the case of the Λ^0's, there seems to be reasonable symmetry fore and aft, but many cases close to 0° and 180° are lacking if our hypothesis is correct. Such cases should not be very difficult to see. In either case a a heavily ionizing particle will be produced. If the π goes almost directly backwards in the center-of-mass system, the decay may not have the usual V appearance since the π will be quite slow and at a large angle, but it should be quite visible. In Fig. 8 we have plotted the angular distribution of the π^-'s from Λ^0's coming from the bottom of the chamber. These Λ^0's do not show any great correlation with the plane of production, so that we expect the spins to be oriented in a random fashion. Within the statistics these cases do show a slightly anisotropic distribution of the secondaries, but there is no hole in the distribution fore and aft. It thus seems likely that the cases in which the π^- mesons go very nearly forward or backward are not missed.

We are left with three alternatives in explaining the lack of consistency between the center-of-mass data and the correlation data. They are: (1) The spin is not perpendicular to the plane of production. (2) Errors in measurement may occur which tend to smear the center-of-mass data. (3) There may be statistical fluctuations which would give too strong a correlation and too weak a peaking in the center-of-mass system. The last alternative seems the best in that the cases in which the decay is nearly forward and back are the cases which give the large dihedral angles. It is precisely these cases that are missing from the angular distribution data.[8]

Figure 9 shows the angular distribution of the π's in the center-of-mass system of the θ^0. As expected, it seems isotropic.

From these data, one can make rudimentary deductions concerning the spins of the hyperons.

The first conclusion is that the hyperons seem to have their spins oriented nearly perpendicular to the plane of production. The θ's do not seem to show the effect. Thus probably the spin of the hyperons is predominant in determining the angular distribution of the reaction. It seems likely that the spin of the θ^0 is considerably less. For the sake of argument, we assume the spin of the θ^0 to be zero.

A simple model that qualitatively accounts for the data would be as follows. Associated production takes place in rather high angular momentum $\pi^- - p$ collisions, say with $J=5/2$ to $9/2$. It seems very likely from the analysis of the $\pi^- - p$ interaction data that inelastic

[7] This fact is obvious from classical considerations. Calculations have been made by L. Wolfenstein, Phys. Rev. **94**, 786 (1954), and have been particularly stressed by Treiman, Reynolds, Hodson, Phys. Rev. **97**, 244 (1955).

[8] It is possible that experimental errors in the angles and momenta of the products have pushed the distribution forward. It would be expected that errors might randomize a peaked distribution. The nine cases of Λ^0's coming from the walls which are consistent with having been produced in a $\pi^- - p$ collision do show a fairly strong peak for the π's in the forward direction. Except for the fore and aft asymmetry, this is consistent with a $\frac{3}{2}$ spin perpendicular to the trajectory. This result of asymmetry is consistent with the Brookhaven observations[1] in which the π's showed a tendency to go forward in the center-of-mass system of the Λ^0. Some of this might be a result of observational bias but perhaps not all.

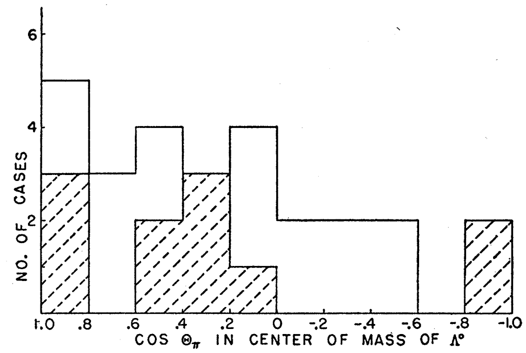

FIG. 8. Figures 7 and 8 are the same except that Fig. 8 shows the angular distribution for π's coming from Λ^0's from the walls instead of from $\pi^- - p$ collisions in the gas. The distribution is similar to that found by Fowler *et al.*[1] in that these cases show more π's going forward than backward. The dashed histogram is for cases of Λ^0 which are consistent with $\pi^- - p$ production.

collisions of these angular momenta occur. If one supposed that s-wave collisions gave rise to the particles it would be impossible to get the correlations observed. If the orbital angular momentum in the $\Lambda^0 - \theta^0$ system were zero, it would be impossible to have the correlation between planes of decay and production. In one-Bev collisions, the Λ^0 and θ^0 can come out with about two units of orbital angular momentum. The orbital angular momentum and the spin of the hyperon must be oriented parallel or antiparallel. The direction of the orbital angular momentum defines the plane of production providing it is perpendicular to the direction of the incoming beam. (**J** very nearly is.) If L, the orbital angular momentum, and S, the spin of the hyperon are parallel to each other and to **J**, then it is difficult to get correlations as strong as those observed. For example; with $J=7/2$ and $L=2$, $S=3/2$. Then in this case the correlation between plane of production and decay would be somewhat weaker than the one for the curve drawn for $L=2$ in Fig. 3. Such a possibility cannot be ruled out because of the meager statistics, especially in conjunction with the hyperon center-of-mass angular distribution. If **L** and **S** are antiparallel, then for $J=7/2$ and $L=2$, S could be as high as 11/2 which seems too high for other reasons. Any higher spin than 7/2 would be difficult to understand because of the data concerning the capture of K^-.

It is known that $K^- + p \rightarrow \Sigma^{\mp,0} + \pi^{\pm,0}$ or $\Lambda^0 + \pi^0$.[9,10] The cross section for these processes is apparently large even at low energies.[10,11] The θ^0 seems to have considerably smaller spin than the hyperons, according to the data on angular correlations. If we suppose that θ^0 and K^- have the same spin, than it is difficult to see where a large amount of angular momentum could come from in the process $K^- + p \rightarrow \Sigma^{\pm,0} + \pi^{\mp,0}$. The K^- can come in via a p state down to low energies, and the π can go out via a p state or possibly a d state. Thus it would be difficult to

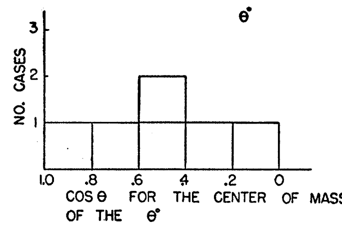

FIG. 9. This histogram shows the rest-system angular distribution of the π's coming from the decay of θ^0's. We have not attempted to separate the distribution according to the charge of the π because some of the data are taken from the work of Fowler *et al.*[1] in which there was no magnetic field.

see how the spin of the Σ could be more than perhaps 7/2 if the spin of the K^- is zero.[12] If the spin of the K's were 1, then the spin of the Σ's could be boosted by one unit of angular momentum. For the reaction $K^- + p \rightarrow \pi^0 + \Lambda^0$, it is possible to get the π^0 out in a d state a little more easily because of the extra 80-Mev kinetic energy.

The lack of correlation of the decay plane of the θ^0, in conjunction with the correlation of the Λ^0, would indicate that if the θ^0 has a spin it is on the average at right angles to that of the Λ^0. This in turn would mean that the spin of the Λ^0 could not be aligned as well with the orbital angular momentum. The fact that the Λ^0 spin and orbital angular momentum are probably closely aligned indicates that the spin of the θ^0 is probably zero or at most 1.

The apparent strong correlation between orbital angular momentum and spin of the Λ^0 or hyperon might indicate a strong spin-orbit coupling between the K particles and hyperons.

To sum up the data, we can say the following: The odds against a spin of $\frac{1}{2}$ for the hyperons are greater than 1000 to 1. The data on angular correlations indicate a spin of 5/2 or more On the other hand, the data on the distribution of π's in the rest system of the Λ^0 indicate that a spin of $\frac{3}{2}$ must be considered a possibility. These data are not independent, and consequently it seems possible to interpret the results in terms of a spin of $\frac{3}{2}$ for the Λ^0. The spins of the Σ's are probably the same or differ at most by one unit from that of the Λ^0. This follows from the correlation data and the fact that in K^- capture Σ production competes very favorably with Λ^0 production. If the π from the capture process had to come out in a state of higher orbital angular momentum in the case of the Σ than the Λ^0, then one would expect Λ^0 production to predominate. The fact that the rates of production of Σ's and Λ^0's in the capture of K^-'s are comparable in spite of the additional 80 Mev available in the case of Λ^0 production indicates actually that the π's probably come out in an s or p state relative to the hyperon. It is of course possible to circumvent these arguments by having several kinds of K^-'s and special π-hyperon forces. The data on the correlations and rest-system angular distribution indicate a spin of zero for the θ^0.

[9] H. De Staebler, Phys. Rev. **95**, 1110 (1954).
[10] J. Hornbostel and E. O. Salant, Phys. Rev. **98**, 218 (1955).
[11] Fry, Schneps, Snow, and Swami, Phys. Rev. **100**, 1448 (1955).

[12] S. B. Treiman has pointed out that the spin of the Σ's and Λ^0 can be deduced from the angular correlations observed in the capture of K^-'s in hydrogen with the subsequent decay of the hyperons. We wish to thank Dr. Treiman for a preprint of this work.

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Lateral Structure of Extensive Air Showers

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The problem presented by air showers which have particle densities ~500 per sq meter around the shower axis and which have no multiple peaks with separations of more than a meter has been analyzed previously by Hazen *et al.* using Fermi's model of meson production. The object of this paper is to show that, contrary to the conclusion of Hazen *et al.*, Fermi's model cannot generate such showers. On the other hand, we show that a modification by Bhabha, to the phenomenological models of meson production, which predicts greater angular concentration of the mesons produced in nucleon-nucleon collisions, can generate the showers of the type referred to above for a suitable choice of the parameter ϵ, which in this theory is the ratio of field mass to proper mass.

INTRODUCTION

FROM a large number of experiments,[1–5] performed during recent years with counter systems, ionization chambers, and cloud chambers, it has been established that the distribution of ionizing particles in a shower agrees closely with Molière's distribution at all places except within a distance of one meter from the axis of the shower, where there is a certain irregularity in structure. Moreover, no evidence was found for a multiplicity of cores[6] separated by distances varying from one meter to 200 meters. A cloud-chamber study of the cores of showers revealed that small showers have particle densities of the order of 500 particles per square meter around the shower axis. We shall refer, hereafter, to such showers as "showers of minimum size." If we assume that the extensive air showers are produced by the γ rays resulting from the decay of π^0 mesons created in nucleon-nucleon interactions, considerations of energy balance in the shower demand, as we shall see later, that the primary protons giving rise to showers of minimum size must have energies a few times 10^{14} ev. This fact is of considerable importance, since any theory which can predict the possibility of the generation of showers of minimum size by primaries of energy 10^{14} ev will make rather stringent demands on the model of meson production we are going to assume, particularly concerning the angular concentration of the mesons generated in a nucleon-nucleon collision.

For a theoretical approach to the problem of multiple cores, or their absence in actual showers, one needs a knowledge of the fluctuations in the number of particles which strike a detector when placed at a given distance from the axis of a shower of given energy. It would not, however, be fruitful to formulate the problem on these general lines, since the stochastic problem in the usual one-dimensional cascade theory has not been solved so far in a manner which leads to easy numerical computations. Hence we investigate the problem under the approximation that the γ rays arising from the decay of energetic π^0 mesons, created in the initial collision of a primary proton, determine essentially the core structure of an extensive air shower. Further, we consider only π^0 mesons of energy $>10^{13}$ ev since they alone will be relevant to the production of multiple cores. Whenever we speak of energetic particles we mean, hereafter, particles with energy $>10^{13}$ ev.

On these simplified lines, the problem presented by showers of minimum size has already been investigated by Hazen *et al.*,[7] employing Fermi's[8] model of meson production. Now, in order to generate a shower having

[1] Cocconi, Tongiorgi, and Greisen, Phys. Rev. 76, 1020 (1949).
[2] R. W. Williams, Phys. Rev. 74, 1689 (1948).
[3] J. M. Blatt, Phys. Rev. 75, 1584 (1949).
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