additional broadening of the resonance line should occur. This broadening could not be studied in the present experiment, since it is of the same order as the experimental error in $1/T_2$. A constant value $1/T_2=4\times10^9$ sec^{-1} between 4.2°K and 293°K is consistent with the obtained results. A temperature-independent line width of the observed order of magnitude has been predicted by Kasuya[15] on the basis of a theoretical investigation of the interaction between the ferromagnetic spins and the conduction electrons.

The value of the spectroscopic splitting factor and of the anisotropy constants found at X-band agreed closely with those found at K-band. At X-band, with the dc field parallel to the easy direction of magnetization, a full resonance curve could not be observed below about -50°C because of the increasing magnitude of the crystalline anisotropy. At about -160°C, the resonance peaks had disappeared completely, since the anisotropy field shifted the resonance to higher frequencies. With the dc field parallel to the hard direction, the expected secondary peaks[16] were observed.

I am greatly indebted to Professor N. Bloembergen for his hospitality and active interest, leading to several stimulating discussions, to C. J. Hubbard for helpful suggestions concerning experimental details and to the Department of State for a Fulbright and Smith-Mundt grant.

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Hall Effect and Magnetic Properties of Armco Iron*

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The Hall effect for Armco iron is examined in detail over a large range of fields. The effect is given by a sum of the ordinary and the extraordinary effect over the range investigated, including the nonlinear region. It is shown that the extraordinary Hall constant is independent of field within the experimental error. The approach to magnetic saturation in Armco iron is examined at high fields. The Hall effect is shown to be useful for studying detailed behavior of magnetic properties.

1. INTRODUCTION

A NUMBER of experiments have demonstrated[1–6] that the Hall effect in ferromagnetic materials can be expressed as the sum of two terms, one proportional to the magnetizing field, H, and the other proportional to the magnetization M. Thus

$$E_H=(R_0H+R_14\pi M)I/t$$
$$=[R_0B+(R_1-R_0)4\pi M]I/t, \quad (1)$$

where E_H is the Hall potential difference, R_0 and R_1[7] are the ordinary and extraordinary Hall constants respectively, I is the total current, t the thickness of the sample in the direction of H, and B the magnetic induction. Here we assume the sample is a thin rectangular plate (6.0 cm×2.0 cm×0.1 cm in the case of Armco iron) so that H is uniform throughout the plate. Recently a theory for R_1 has been given by Karplus and Luttinger.[8] The constant R_0, which should yield a measure of the number of conducting particles per unit volume and their sign, has been found to agree satisfactorily with estimates obtained from simple band models for a number of ferromagnetic materials.[3,5,9]

In this paper, the previously presented experimental results for Armco iron[5] (hereafter referred to as A) are reanalyzed in greater detail. Because of the high precision of the measurements and the particular magnetic characteristics of this material, Eq. (1) can be examined quite accurately. The results show that within the accuracy of the measurements R_1 is independent of H and that Eq. (1) is obeyed over the entire range of fields used. The possibilities of employing the Hall effect to study the magnetic properties of ferromagnetic materials is discussed briefly.[10] The procedure is particularly useful for examining the detailed approach to magnetic saturation at high magnetizing fields.

2. METHOD OF MEASUREMENT

A detailed discussion of the experimental procedures for the Hall effect measurements described in this paper

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may be found in A.[5] A brief summary of the method is given in this section with a slightly different approach.

In order to obtain accurate measurements, the Hall potential is measured with a high-precision dc apparatus[5] by an incremental method. A change in Hall potential, $(\Delta E_H)_i$, is measured for a corresponding change in magnetic induction $(\Delta B)_i$ with respect to a fixed reference field, B, chosen well above magnetic saturation. The magnetic states obtained by this procedure are shown in Fig. 1, where P (or P') is the reference value of B, and successive loops from P to x_1, x_2, x_i are traced always returning to P before proceeding to the next measurement. For convenience the portion of L_1 observed is that for $B \geq 0$, although the entire section L_1 could be traversed during the measurements. After reversing between P and P', the corresponding section L_2 is investigated for the same increments of B where $B \leq 0$, and since the large hysteresis loop PP' is symmetric with respect to the origin, the corresponding points on L_1 and L_2 result in the same magnetic state for the sample. The values of M, and therefore H, obtained in this manner are thus a unique function of B (a result which is verified to high accuracy by the Hall effect results at high fields). Any other potential which does not reverse with the direction of B (resistive type effects which are large at low B) is eliminated by observing the Hall potential as a function of the direction of B.

The normal magnetization curve L_3 is also shown in Fig. 1 with a series of hysteresis loops (y_i) which would be obtained if the Hall potential were measured while B was reversed between the corresponding points. It has been pointed out[5] that this method is not used because of the inherent inaccuracy obtained when a small difference between two large potentials is observed, although resistive type effects are also eliminated by the reversal method. It is apparent from Fig. 1 that M as a function of B will depend on the particular method of measurement at low values of B. Thus a plot of E_H *versus* B for the incremental measurements may

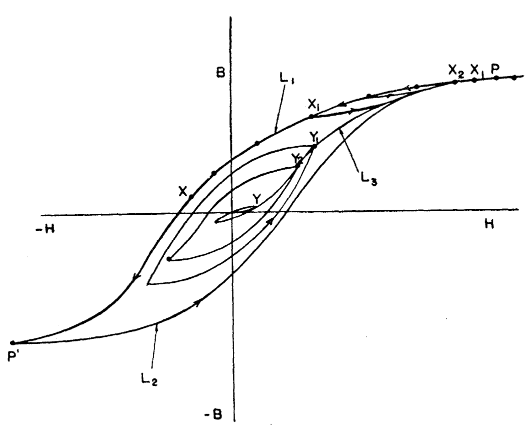

FIG. 1. B *versus* H curve illustrating the magnetic states produced in the material during the course of Hall measurements.

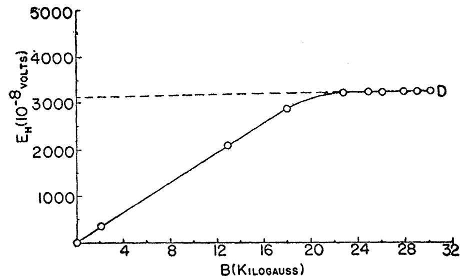

FIG. 2. Hall effect of Armco iron.

show a larger slope than the reversal measurements at low B [compare points x and y in Fig. 1 with Eq. (1)].

From Eq. (1),

$$\frac{\partial E_H}{\partial B}\frac{t}{I}=R_0+(R_1-R_0)4\pi\frac{\partial M}{\partial B}$$

$$=R_0+(R_1-R_0)\left(\frac{\mu-1}{\mu}\right), \quad (2)$$

where μ is the differential permeability. For low fields $\mu \gg 1$ and for high fields $\mu \cong 1$, so that $\partial E_H/\partial B$ is constant for these regions. The quantities R_0 and R_1 can be determined from the measurements if two values of B are known for two corresponding values of M using Eq. (1) or if two values of $\partial E_H/\partial B$ are known for two values of μ in Eq. (2). Only the high-field data and the knowledge of $4\pi M_s$ (obtained by independent methods) are necessary to determine R_0 and R_1. The additional low-field data can be used to check the value of R_1. Generally the linear portions of E_H *versus* B are examined because R_0 and R_1 are easily obtained from this data. From Eq. (2), we see that if $(R_1-R_0) \gg R_0$, the $\partial M/\partial B$ term may be magnified sufficiently to allow this term to be examined accurately. The data shows that the nonlinear Hall data follows the magnetic properties in detail.

The data for Armco iron are shown in Figs. 2 and 3 where the curves are normalized to $I=25.0$ amperes and $t=0.100$ cm, values quite close to the experimental conditions. Figure 3 illustrates the detailed behavior of E_H *versus* B at high B. The magnifying effect of R_1, discussed above, makes it possible to observe that the sample is not saturated up to 26 kilogauss. Even the point at 26 kilogauss is almost 2×10^{-9} volt below the indicated straight line while the three points at higher fields deviate by less than 1×10^{-9} volt.[11] Each point shown is the result of averaging 8 or more experimental values.

There are several reasons for presenting the Armco iron data in preference to others available in A.[5] First, because of its large M_s, its characteristics can be studied both where $B>4\pi M_s$ and where $B<4\pi M_s$ while B is

[11] This fit is slightly better than that previously reported in A. The corrected value of R_0 is now $+2.47\times10^{-13}$ v-cm/amp-gauss.

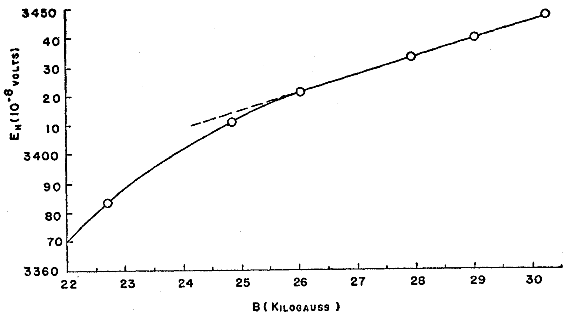

FIG. 3. High-field region of Fig. 2 with expanded scale.

moderately large[12]; second, a high thermal stability was obtained with this sample so that the ultimate sensitivity of the measuring system could be attained; and third, the value of R_1 provides a large multiplying factor. Furthermore, the sample was quite pure[13] so that a detailed comparison of its magnetic behavior could be made with independent measurements. These attributes allow a careful analysis of Eq. (1).

3. CALCULATIONS OF R_1

Two independent methods of calculating R_1 using only the low and high field data are discussed briefly in this section. It is shown that the values of R_1 obtained agree within the experimental error. The possible variation of R_1 as a function of H is considered.

The first method uses only the linear high-field data. R_0 is first obtained from Eq. (2) from the high-field slope, $\partial E_H/\partial B$, assuming $\mu=1$. Substituting $4\pi M_s = 21\,580$ gauss in Eq. (1) and extrapolating the high-field slope to $B=0$ to obtain the value of E_H at $B=0$, $R_1=25.1R_0$ is obtained. The errors involved are approximately 1% for B and 0.5% for $4\pi M_s$,[14] which results in an error of 1.5% for this determination. The second method uses Eq. (2) and the slopes at low and high field to determine R_1 and R_0 (the value of $4\pi M_s$ is not required here). In principle, the use of only $\partial E_H/\partial B$ data should yield higher accuracy because the coil calibration may be eliminated. However, the measuring procedure for this data used incremental field changes from the fixed reference point at 29 kilogauss. In order to obtain the slopes at relatively low B-values, the difference between two large $(\Delta B)_i$ is required, thus limiting the slope methods to about 1%. From Eq. (2) and the slopes at high B and low B (assuming $\mu=1$ at high B and $\mu \gg 1$ at low B) one obtains, using the low-B slope between the 13 and 18 kilogauss points, $R_1=25.4R_0$. If the line through the origin in Fig. 2 is used, $R_1=26.2R_0$, though the slope thus obtained cannot be very accurate. These independent methods of calculating R_1 show that Eq. (1) is consistent within the experimental error over the range investigated.

The methods described above may be used to estimate any variation of R_1 with H. The argument is based on the observation that in a ferromagnetic material, M is a nonlinear function of B and thus H is likewise nonlinear in B. Suppose we assume

$$R_1=R_1^0+R_1^1H+\cdots, \tag{3}$$

then

$$\partial R_1/\partial B=R_1^1(\partial H/\partial B)=R_1^1(1/\mu). \tag{4}$$

Differentiating Eq. (1) where $R_1=R_1(H)$ and using only the linear term in H,

$$\frac{\partial E_H}{\partial B}\frac{t}{I}=R_0+(R_1^0-R_0+R_1^1H)4\pi\frac{\partial M}{\partial B} +4\pi MR_1^1\left(\frac{1}{\mu}\right). \tag{5}$$

The right side of Eq. (5) becomes $(R_1^0+R_1^1H_c)$ at low B where $\mu \gg 1$ and $H\sim H_c$ (the coercive force) assuming $4\pi MR_1/\mu$ is small. At high B, $\mu=1$, and the right side of Eq. (2) is given by $R_0+4\pi M_sR_1^1$. Extrapolating the linear high-field data to $B=0$ where $E_H=E_H^0$, one obtains

$$E_H^0t/I=(R_1^0-R_0+R_1^1H_c)4\pi M_s. \tag{6}$$

The three quantities R_0, R_1, and R_1^1 can be calculated using Eqs. (5) and (6) in conjunction with the Hall data, knowing $4\pi M_s$, H_c and μ at two points. The maximum possible value of R_1^1 using the experimental error as a limit is $R_1^1 \leq R_1^0\times10^{-6}$ oersted^{-1} which is certainly negligible. For the remainder of this paper it will be assumed that R_1 is independent of H. This is in agreement with these experimental results and with the model used by Karplus and Luttinger[8] to calculate R_1. Their model employs a large spin-orbit interaction which one expects would be insensitive to most fields produced in the laboratory.

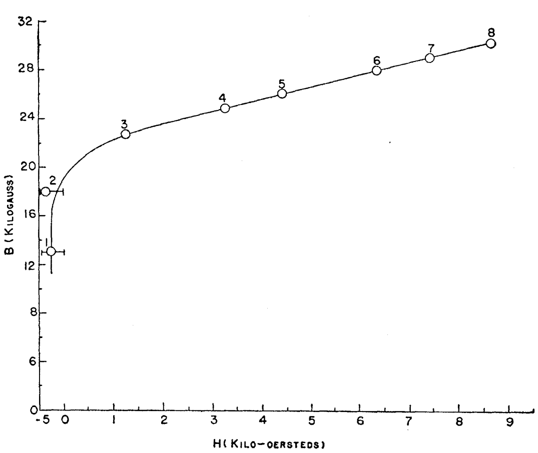

FIG. 4. B *versus* H for Armco iron calculated using the Hall effect data.

[12] High values of B were employed because R_0 was of particular interest and could be experimentally observed only at values of B well above magnetic saturation.

[13] A typical analysis for Armco iron is given in A.

[14] William R. Bitler of this laboratory measured the M_s of the Armco iron used here. He found $4\pi M_s=21\,580$ gauss within 0.5% from $H=5$ to 17 kilooersteds.

4. $B-H$ CURVE OF ARMCO IRON

The previous discussion has considered only the low and high B-regions where $\partial E_H/\partial B$ is generally constant. Now consider the nonlinear region of the E_H *versus* B-curve. Using Eqs. (1) or (2) and the values of R_0 and R_1 obtained earlier, $4\pi M$ or $4\pi\partial M/\partial B$ can be calculated from the data. Since $B=H+4\pi M$, B as a function of H can be calculated. The results are plotted in Fig. 4. No correction for the demagnetizing field is required here because B was measured directly. The retentivity, B_r, and coercivity, H_c, are 16 000 gauss and 240(±300) oersteds, respectively. These values are somewhat higher than the values of B_r (6 to 14 kilogauss) and H_c (1 oersted) listed by Bozorth for iron.[15] However, this agrees with the observation that both B_r and H_c increase with $H_{\max}$ (here $H_{\max}=9$ kilooersteds). The value of H_c obtained here is open to question because this quantity could be obscured by the 1% error of the incremental B measurements. The data shows that μ is large in the region $B=13$ to 18 kilogauss thus satisfying the approximations made in the earlier sections. The B-H curve presented here is not entirely unambiguous; i.e., if $\partial M/\partial B$ should be a constant rather than zero at the highest fields used, an undetectable systematic error would be left in the determination of R_0. Such an error would appear in the B-H curve or other magnetic properties using the above methods. Fortunately errors in the determination of R_0 do not affect the equality of the R_1's determined by the two independent methods. Furthermore, although a constant value for $\partial M/\partial B$ has been predicted[16] theoretically, its value is too small to produce observable errors in the experimental values of R_0. Detailed quantitative agreement with other methods used in obtaining these high-field magnetic data are discussed in the next section. In this way it is demonstrated that Eq. (1) describes the Hall effect over the entire range of fields investigated as indicated in Sec. 3.

5. APPROACH TO MAGNETIC SATURATION

The approach to saturation in ferromagnetic materials has been studied for several decades. At high fields, the experimental data may be fitted by

$$M=M_s\left(1-\frac{a}{H}-\frac{b}{H^2}+\cdots\right)+K_0H. \qquad (7)$$

The term K_0H presumably is caused by the increase in spontaneous magnetization due to the alignment of elementary spins. The term b/H^2 is attributed to the magnetic crystal forces and thus involves the anisotropy constants while the a/H term (magnetic hardness) has been explained by Néel[17] as due to nonmagnetic cavities or inclusions which are present in the material. The range of these Hall measurements extended to sufficiently high fields that this equation could be examined in detail. Czerlinski[18] has examined iron up to $H\sim 2600$ oersteds and found satisfactory agreement by using only the b and K_0 term. Steinhaus, Kussman, and Schoen[19] (hereafter referred to as SKS) examined several iron samples up to $H\sim 8000$ oersteds and found that the term in a was also necessary at high fields.

The differential Hall effect data for Armco iron is tabulated in Table I where the increments are relative to the reference point 7 (at $H=8.89\times10^3$ oersteds and $B=29.01\times10^3$ gauss), and the corresponding B *vs* H curve is shown in Fig. 4. The column $4\pi M$ in Table I is calculated from the incremental form of Eq. (2) and $R_1=25.1R_0$. Values of H *versus* $4\pi M$ using the reference point data ($B=29.01$ kilogauss when $4\pi M_s$ $=21\,580$ gauss) are given in Table II. The $4\pi M$ *versus* H data of SKS[19] are compared with the results calculated from the Hall data in Fig. 5. Note that the two scales of $4\pi M$ are slightly displaced because $4\pi M_s$ of SKS is smaller than the value for Armco iron. Figure 6 shows the same data replotted with $4\pi M/H$ *versus* $4\pi M$. The slopes of these lines, which are directly related to the a's of the two different sets of data, agree within experimental error. Only the high-field data of SKS are reproduced here (i.e., $M>99\%\ M_s$). The linearity of their data shows the following: (1) the b/H^2 term is absent in this field range although at lower fields this term is prominent; (2) the K_0H term is negligible in this field range since otherwise the straight line would curve upward. The last point of SKS, which falls below the straight line by 3 times their experimental error (2 gauss), indicates additional complications. The more accurate Hall effect measurements suggest a sharp

TABLE I. Incremental Hall effect data for Armco iron.

Increment	ΔE_H (10^{-8} v)	ΔB (10^{-3} gauss)	$4\pi\Delta M$ (gauss)	ΔH (10^3 oersteds)
(8–7)	−7.38	−1.23	1.5	−1.23
(7–6)	6.79	1.07	1.1	1.07
(7–5)	18.58	2.98	1.3	2.97
(7–4)	28.60	4.16	19.2	4.14
(7–3)	56.24	6.30	116	6.17
(7–2)	554.4	11.04	3260	7.78
(7–1)	1355	16.08	8420	7.66

TABLE II. Magnetic data derived from Table I.

H (oersteds)	$4\pi M$ (gauss)	$4\pi M/H$ (gauss/oersted)
8660	21 581.5	2.49
7430	21 580.0	2.91
6360	21 578.9	3.40
4450	21 578.7	4.85
3270	21 560.8	6.59
1240	21 464	17.24

[15] R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., New York, 1951), see Table 4, p. 502 and Figs. 3–8, p. 59. The Armco iron sample was machined from rolled stock and annealed at 800°C for two hours, then cooled slowly.

[16] A discussion of this effect is given by T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).

[17] L. Néel, J. phys. radium (8) 9, 184 (1948).

[18] E. Czerlinski, Ann. Physik (5) 13, 80 (1932).

[19] Steinhaus, Kussman, and Schoen, Physik. Z. 38, 777 (1937).

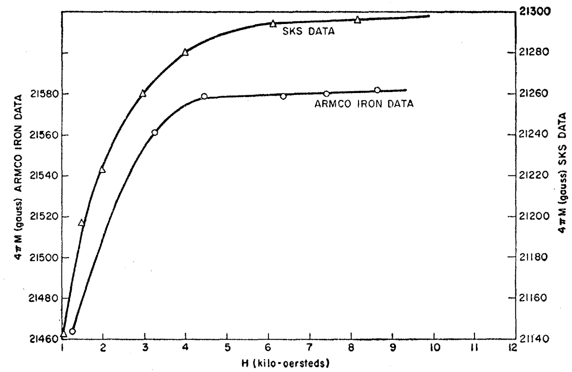

FIG. 5. Comparison of $4\pi M$ *versus* H data of Steinhaus, Kussman, and Schoen[19] (obtained by classical methods) with the results calculated from the Hall data. The different scales of $4\pi M$ in Fig. 5 and Fig. 6 are caused by slightly different values of $4\pi M_s$.

break in the high-field data which follows a steeper straight line. Apparently no other data with sufficient accuracy and field range are available to examine this effect. A possible mechanism for the additional high-field line in Fig. 6 may be based on small domains of 10^3 atoms or less as described by Stoner.[20] This effect would vary linearly with temperature. A second possibility might be the manifestation of the earlier mentioned Néel mechanism for the a/H term which varies as $1/H^2$ at very high fields. Further measurements will have to be made before this behavior is understood.

The general result obtained from the magnetic data is that, within the accuracy of measurements, Eq. (1) describes the Hall effect over the nonlinear E_H *versus* B region as indicated earlier.

6. COMMENTS ON THE APPLICATION OF THE HALL EFFECT

The results of this investigation have shown that Eq. (1) is obeyed to high accuracy and that R_1 is independent of H so that the Hall effect follows the magnetic properties in detail. In this section, the possibilities of using the inverse process of studying the magnetic properties by means of Hall effect is considered briefly.

The procedure involves a careful examination of the Hall data over a large range of fields. Equation (1), in conjunction with the low and high B-data, is used to evaluate R_0, R_1, and $4\pi M_s$. The regions where e_H is not linear with B are then examined by the methods described in earlier sections. From this the B-H curve can be obtained and the various magnetic quantities determined. The approach to magnetic saturation can be examined accurately because the K_0H term in Eq. (7) is eliminated.

The accuracy of the method depends on Eq. (1) where R_0 and R_1 are assumed independent of H. The sensitivity depends directly on the magnifying effect of R_1 which thus should be large. The value of R_1 is difficult to predict although it usually increases rapidly with temperature in pure materials. The dc potential measurements can be made to high accuracy. Since the potential depends on the current, this quantity should be made as large as possible consistent with thermal stability. Using the incremental method described earlier for Armco iron, the value $\Delta M/M_s = 2\times 10^{-5}$ could be observed for the given experimental conditions. A rather elaborate experimental setup is required for these measurements; however, when Hall measurements are to be made, the additional magnetic data are available with few additional measurements and some

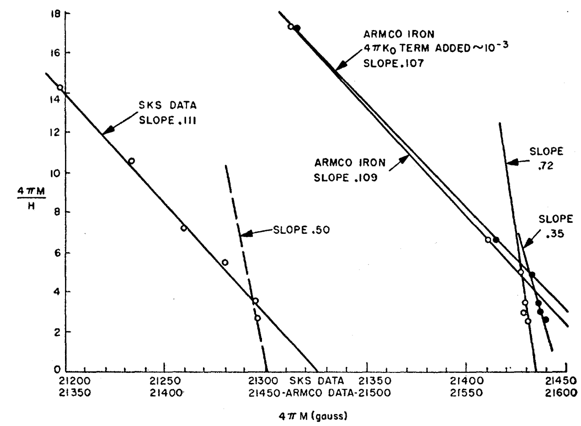

FIG. 6. $4\pi M/H$ *versus* $4\pi M$ at high magnetic fields obtained from the results of Fig. 5.

calculations. To date, the nonlinear E_H *versus* B-data have not been used to examine the magnetic properties of the material studied.

7. SUMMARY

The Hall effect has been examined in detail over a large range of fields for Armco iron. From these data, it was demonstrated that Eq. (1) is valid within the accuracy of the measurements over the entire range investigated. It was shown that Hall data frequently can be used to measure magnetic properties of ferromagnetic materials to high precision.

ACKNOWLEDGMENTS

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[20] E. C. Stoner, Trans. Roy. Soc. (London) A225, 165 (1936).