TABLE II. Direct and indirect values. Each indirect value is the result of a least-squares solution from which the direct measurement has been omitted and hence represents the value of the quantity which may be inferred from the totality of other measurements.

Equation number (DC55)	Direct value	Indirect value
0–1	0.00±3.00	2.45±1.53
1–1	3.50±3.78	3.19±8.66
2–1	4.00±0.45	0.76±2.83
3–1	−2.30±2.29	−1.77±1.86
4–1	11.10±1.31	12.08±2.24
5–2	13.50±1.10	12.47±2.32
6–3	−5.60±8.16	8.30±1.48

indicates to what extent the output value of a given function is determined by the indirect implications of that other data.

The direct input values and the indirect least squares values are presented in Table II. From this table we see for example that the indirect value of 0–1 (the conversion factor from x-units to milliangstroms) is somewhat more accurate than the direct value. On the other hand item 1–1 (the Siegbahn-Avogadro number, $N_s'=N\Lambda^3$) is given much more accurately by the direct data than by the indirect. That this was so has been established previously in more cumbersome ways.[4]

The directly measured value of item 2–1 (fine-structure splitting in deuterium) is more than six times as accurate, giving it almost forty times as much weight, as the indirect value. This demonstrates forcibly how important Dayhoff, Triebwasser, and Lamb's measurement of the fine-structure constant is in providing a crucial datum for the values of the atomic constants. On the other hand, the situation is reversed with respect to the h/e determinations, (6–3); in this case the indirect value is much more accurate than the direct measurement. If the direct measurement were omitted from the analysis, the weight assignable to the value of h/e would be changed only slightly and the output value would be altered by less than half the probable error. The need for further measurements of the short-wavelength limit of the continuous x-ray spectrum has of course been previously emphasized.[2,5]

[4] J. W. M. DuMond and E. R. Cohen, Phys. Rev. **94**, 1790 (1954); E. R. Cohen and J. W. M. DuMond, Phys. Rev. **98**, 1128 (1955).

[5] J. A. Bearden and J. S. Thomsen, "A Survey of Atomic Constants," The Johns Hopkins University, Baltimore, 1955 (unpublished).

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Field Dependence of Magnetoconductivity

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A theoretical calculation of the magnetic field dependence of the elements of the conductivity tensor has been performed for a crystal with a general electronic energy band structure. It was assumed that the Boltzmann equation is valid, and that an energy-dependent relaxation time exists. The results are the same as would be given by a superposition of electron gases, whose cyclotron frequencies are related harmonically. The strengths of the harmonics depend upon the energy band structure; in particular, there are certain relations among them which are required by symmetry. The diagonal elements of the conductivity tensor are found to be monotonically decreasing functions of the magnetic field strength. Extension of the calculation to alternating electric fields reveals harmonics in the cyclotron resonance.

1. INTRODUCTION

UNTIL recently, calculations of the magnetic field dependence of the Hall effect and magnetoresistivity have been confined to materials with simple ellipsoidal band structures.[1] For general band structures, the limiting cases of very weak fields[2] and very strong fields[3] have been studied. Zeiger has reported a calculation based on a particular (nonellipsoidal) model for the band structure of p-type germanium.[4] This paper consists of a calculation which applies, under restrictions which are discussed below, at all field strengths to general band structures. We show that certain new features found by Zeiger are to be expected in general.

In materials which contain more than one type of carrier, the field dependence of the Hall effect and magnetoresistivity may be used to separate the effects of the different carriers, and to obtain concentrations and mobilities for each type. Such analyses have been carried out for p-type germanium, in which the two carriers are light and heavy holes.[5,6] The separation was accomplished by fitting the experimental data to theoretical formulas derived on the basis of spherical energy surfaces. To make a similar analysis on a material with

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[1] See, for instance, H. Jones, Proc. Roy. Soc. (London) **A155**, 653 (1936); B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954); and M. Shibuya, Phys. Rev. **95**, 1385 (1954).

[2] H. Jones and C. Zener, Proc. Roy. Soc. (London) **A145**, 268 (1934).

[3] J. A. Swanson, Phys. Rev. **98**, 1534 (1955); **99**, 1799 (1955).

[4] H. J. Zeiger, Phys. Rev. **98**, 1560 (1955).

[5] Willardson, Harmon, and Beer, Phys. Rev. **96**, 1512 (1954).

[6] Adams, Davis, and Goldberg, Phys. Rev. **99**, 625 (1955).

a more complicated band structure (or to correct the germanium analysis), it is necessary to know the general form of the Hall effect and magnetoresistivity as functions of magnetic field strength for a general band structure. A desire to carry out such a program for graphite[7] was the principle motivation of the present work.

We discuss here only the magnetoconductuctivity tensor, as the components of this tensor are simpler theoretically than the measured quantities. The magnetoresistance and Hall constant can easily be deduced from the conductivity tensor and vice versa.

2. GENERAL FORM OF THE MAGNETO-CONDUCTIVITY TENSOR

We now present the solution of the transport equation and obtain a form for the field dependence of the conductivity tensor. The Boltzmann equation in the presence of uniform electric and magnetic fields, and assuming the existence of a relaxation time, is[8]

$$(e/\hbar)[\boldsymbol{\mathcal{E}}+\mathbf{v}\times\mathbf{H}/c]\cdot\nabla_k f+(f_0-f)/\tau=0, \quad (2.1)$$

where the notation is the same as in Wilson's book. The general solution of this equation has been given by Shockley[9] and by Wilson.[10] We find it useful to present the solution here, using a different notation.

First, let us follow Wilson and write the distribution function as

$$f=f_0-\phi\partial f_0/\partial E, \quad (2.2)$$

where ϕ is a function which is proportional to the electric field strength.[11] The Boltzmann equation now becomes to first order in $\boldsymbol{\mathcal{E}}$,

$$-(e/\hbar c)\mathbf{v}\times\mathbf{H}\cdot\nabla_k\phi+\phi/\tau+e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}=0. \quad (2.3)$$

In obtaining (2.3), use has been made of the fact that $(1/\hbar)\nabla_k E=\mathbf{v}$, and that the operator $(\mathbf{v}\times\mathbf{H})\cdot\nabla_k$ applied to a function of energy alone gives zero.

The first term in (2.3) is the derivative along a path in k space (called the hodograph) which is formed by the intersection of a plane perpendicular to the magnetic field with a constant energy surface (see Fig. 1). To describe the position along the hodograph, it is convenient to introduce a new variable s, such that $\partial\mathbf{k}/\partial s=-(e/\hbar c)\mathbf{v}\times\mathbf{H}$. Thus, $s(\mathbf{k})$ represents the time at which an electron, precessing around the hodograph when there is no electric field, would be at the point $\mathbf{k}$.

[7] The field dependence of the Hall effect and magnetoresistance in graphite at low temperatures is remarkable, see G. H. Kinchin, Proc. Roy. Soc. (London) **A217**, 9 (1953).

[8] A. H. Wilson, *The Theory of Metals* (The University Press, Cambridge, 1953), second edition, p. 196.

[9] W. Shockley, Phys. Rev. **79**, 191 (1950). There is a misprint in his Eq. (4). The lower limit of the second integral should read θ instead of 0.

[10] Reference 8, p. 224.

[11] It is clearly sufficient to solve Eq. (2.1) to first order in the electric field, as such a solution gives a current proportional to the electric field (i.e., Ohm's law is obeyed).

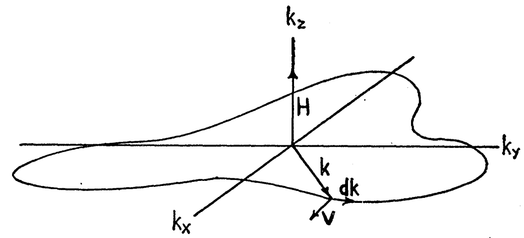

FIG. 1. Illustration of a hodograph in the k_xk_y plane. The case shown is for E increasing away from the origin, so that the representative point precesses around the hodograph in the positive direction.

In terms of the new variable, (2.3) becomes

$$\partial\phi/\partial s+\phi/\tau+e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}=0. \quad (2.4)$$

Equation (2.4) is a first-order linear differential equation in one variable and is easily solved. The general solution is

$$\phi=-\int_r^s ds' e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}(s')\exp\left[-\int_{s'}^s ds''/\tau(s'')\right], \quad (2.5)$$

where r is a constant which is determined by the boundary condition. The condition on ϕ is that it must be a single-valued function of $\mathbf{k}$; which means that it must be a periodic function of s, with a period equal to the time (T) for the particle to go completely around the hodograph once. It is obvious that $\mathbf{v}$ and τ are also periodic in s with the same period. Application of the periodicity to (2.5) yields the result that $r=-\infty$.

The time T and the angular frequency (cyclotron frequency) associated with it are given by

$$T=2\pi/\omega=(\hbar/eH)\oint dk/v_p, \quad (2.6)$$

where the integration is around the hodograph, dk is an element of arc length on the hodograph, and v_p is the component of the velocity perpendicular to the magnetic field.

We now make the assumption that the relaxation time is constant on the hodograph,[12] and make use of the periodicity in s to write $\mathbf{v}$ as a Fourier series,

$$\mathbf{v}=\sum_{m=-\infty}^{\infty}\mathbf{v}(m)\exp[im\omega s], \quad (2.7)$$

where the reality of $\mathbf{v}$ requires that $\mathbf{v}(-m)=\mathbf{v}^*(m)$. Substituting (2.7) into (2.5) and remembering that we are now assuming that τ is independent of s, we find

$$\phi=-\tau\textstyle\sum_m e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}(m)\exp[im\omega s]/(1+im\omega s). \quad (2.8)$$

We now have two tasks: to compute the current with the distribution function associated with (2.8) and to

[12] Though the integral can still be performed for a general relaxation time, the dependence of the result upon the magnetic field is not the same. Thus, success in explaining the experimental results with the simple form found in this section *may* be evidence that the relaxation time is constant on the hodograph.

find the coefficients $\mathbf{v}(m)$ which appear in (2.7). We proceed first to the calculation of the current.

The current due to a single band is given by the familiar expression

$$\mathbf{j}=[-e/(2\pi)^3]\int d^3k\mathbf{v}f=[-e/(2\pi)^3]\times\int d^3k\mathbf{v}\phi(-\partial f_0/\partial E), \quad (2.9)$$

where the integration is over the basic Brillouin zone. The total current density is given by the sum of contributions like (2.9) from each band. In expression (2.9), we may replace the integrand at any point in k space with the average over the hodograph which passes through the point. Such rewriting does not change the value of the integral as integration over the Brillouin zone includes integration around each hodograph. The current may then be written

$$\mathbf{j}=[e^2/(2\pi)^3]\int d^3k(-\partial f_0/\partial E)\tau\mathbf{M}, \quad (2.10a)$$

where

$$\mathbf{M}=-(\omega/2\pi e\tau)\oint ds\phi\mathbf{v}. \quad (2.10b)$$

The integral in (2.10b) is over one period.

Substituting expression (2.8) for ϕ into (2.10b), and using the orthogonality of the functions $\exp[im\omega s]$, we obtain

$$\mathbf{M}=\sum_m[\boldsymbol{\mathcal{E}}\cdot\mathbf{v}(m)]\mathbf{v}(-m)/(1+im\omega\tau). \quad (2.11)$$

Expression (2.11), when substituted into (2.10a), will give the current and thus allow us to calculate the conductivity.

Let us first define a tensor $\mathbf{S}$ such that $\mathbf{M}=\mathbf{S}\cdot\boldsymbol{\mathcal{E}}$. We shall choose the magnetic field to be parallel to the z axis and examine the components of $\mathbf{S}$ (each of which gives rise to a corresponding component in the conductivity tensor). The easiest component to discuss is S_{xx} which is

$$S_{xx}=\sum_{m=-\infty}^{\infty}\frac{|v_x(m)|^2}{1+im\omega\tau}=\sum_{m=1}^{\infty}\frac{2|v_x(m)|^2}{1+(m\omega\tau)^2}. \quad (2.12)$$

In writing (2.12), we have used the reality condition, and the fact that $v_x(0)=0$.[13] It is seen that S_{xx} is a positive, monotonically decreasing function of the magnetic field (all dependence upon the magnetic field strength is contained in the cyclotron frequency ω, which is linear in the magnetic field).

The expressions for the other components of $\mathbf{S}$ are given by

$$S_{xy}=\sum_{m=1}^{\infty}\left\{\frac{v_x(m)v_y(-m)+v_x(-m)v_y(m)}{1+(m\omega\tau)^2}+\frac{i[v_x(-m)v_y(m)-v_x(m)v_y(-m)]m\omega\tau}{1+(m\omega\tau)^2}\right\} \quad (2.13)$$

and

$$S_{zz}=v_z^2(0)+\sum_{m=1}^{\infty}\frac{2|v_z(m)|^2}{1+(m\omega\tau)^2}. \quad (2.14)$$

The expression for S_{yy} is similar to that for S_{xx}, and those for S_{xz} and S_{yz} are similar to that for S_{xy} (with the important difference that the relations among the Fourier coefficients of the velocities are different). The components of $\mathbf{S}$ obey the general symmetry requirement that $S_{ij}(\mathbf{H})=S_{ji}(-\mathbf{H})$.[14] Note that S_{zz} is the only component which approaches a finite limit as the magnetic field becomes infinite. The first term in S_{xy} remains finite when the magnetic field is zero. If such a term is nonzero, it means that the zero magnetic-field conductivity is anisotropic. The term will be zero if the hodograph possesses sufficient symmetry. The second term in S_{xy}, which is odd in the magnetic field strength, is responsible for the normal Hall effect.

To find the conductivity, we must integrate $\mathbf{S}$ over the zone, obtaining

$$\boldsymbol{\sigma}=[e^2/(2\pi)^3]\int d^3k(-\partial f_0/\partial E)\tau\mathbf{S}. \quad (2.15)$$

If one considers a case in which degenerate statistics apply, the quantity $\partial f_0/\partial E$ is appreciable only for values of E near the Fermi energy. In that case, $\boldsymbol{\sigma}$ may be expressed as an integral over the Fermi surface. In the fortunate case that $\omega\tau$ is constant over the Fermi surface, the form of $\boldsymbol{\sigma}$ would be the same as that of $\mathbf{S}$. If the variation of $\omega\tau$ over the Fermi surface were small, $\boldsymbol{\sigma}$ could be approximated by an expression of the same form, using appropriate average values of $\omega\tau$.[15] If Boltzmann statistics were applicable, the variation would probably be greater, as the conductivity will be a combination of $\mathbf{S}$'s corresponding to different energies.

3. AMPLITUDES OF THE HARMONICS

We shall begin our discussion of the Fourier coefficients using cylindrical coordinates in k-space, and then transform to the variable s. Let θ be the azimuthal angle about the z-axis ($\theta=0$ being the direction of the x-axis) and let ρ be the distance in k-space from the z-axis. A given hodograph can be described by $k_z=$constant, $\rho=\rho(\theta)$. We may write the energy in a Fourier series,

$$E=\sum_n E_n(\rho,k_z)\exp[in\theta], \quad (3.1)$$

where the reality of E requires that $E_{-n}=E_n^*$. Other restrictions on the E_n's may result from symmetry requirements. If the k_xk_z plane is a mirror plane, then E_n is real. If the symmetry operations include a p-fold rotation about the z-axis, then E_n is zero unless n is an integral multiple of p.

The components of the velocity are found by taking appropriate gradients of the energy,

[13] This result, which is proved in Appendix A, means that the orbits of electrons, in the absence of an electric field, do not drift in a direction perpendicular to the magnetic field.

[14] M. Kohler, Ann. Phys. (5), 40, 601 (1941).

[15] If the relaxation time is a function of energy alone, then the variation of $\omega\tau$ over the Fermi surface is due to that of ω.

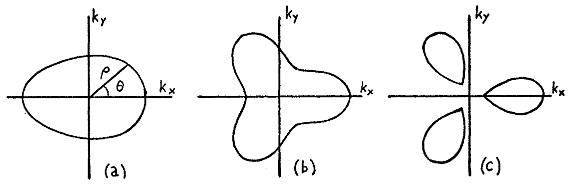

FIG. 2. Possible forms for hodographs.

$$v_x=(1/\hbar)\sum_n[E_n'\cos\theta-(n/\rho)E_n\sin\theta]\exp[in\theta], \quad (3.2a)$$

$$v_y=(1/\hbar)\sum_n[E_n'\sin\theta+(n/\rho)E_n\cos\theta]\exp[in\theta], \quad (3.2b)$$

where the prime denotes the derivative with respect to ρ. We rewrite these expressions as follows:

$$v_x=(1/\sqrt{2})(g+g^*), \quad (3.3a)$$

$$v_y=(i/\sqrt{2})(g-g^*), \quad (3.3b)$$

where

$$g=\sum_n(1/\sqrt{2}\hbar)[E_n'-(n/\rho)E_n]\exp[i(n+1)\theta]. \quad (3.3c)$$

We need to write v_x and v_y as functions of s. To do this, we must take the ρ, which appears in (3.3c) (both explicitly and as the argument of E_n) to be a periodic function of θ, and take θ to be a function of s ($\theta=\omega s$ +periodic function of s). If this were done, we would have

$$g=\sum_n B(n)\exp[i(n+1)\omega s]. \quad (3.4)$$

We shall make use of the form (3.4), but not attempt to calculate the B's here. It can be shown, however, by tracing through the steps outlined in the foregoing, that if certain of the E_n's are zero by symmetry, then the corresponding $B(n)$'s are also zero.

In terms of the B's, the desired Fourier coefficients of the velocities are

$$v_x(m)=(1/\sqrt{2})[B(m-1)+B^*(-m-1)], \quad (3.5a)$$

$$v_y(m)=(i/\sqrt{2})[-B(m-1)+B^*(-m-1)]. \quad (3.5b)$$

Using these expressions, we may work out the combinations of velocity components which appear in (2.13),

$$v_x(m)v_y(-m)+v_x(-m)v_y(m) = -i[B(m-1)B(-m-1) - B^*(m-1)B^*(-m-1)], \quad (3.6a)$$

$$i[v_x(m)v_y(-m)-v_x(-m)v_y(m)] = -[|B(m-1)|^2-|B(-m-1)|^2]. \quad (3.6b)$$

We now consider several special cases. First, if the k_xk_z plane is a mirror plane [see Fig. 2(a)] then all the B's are real and (3.6a) vanishes. This result is in agreement with the expectation that if the k_xk_z plane is a mirror plane, then the xy part of the zero-magnetic-field conductivity should be diagonal in the k_xk_y coordinates. For S_{xy} in this case, we have

$$S_{xy}=\sum_{m=1}^{\infty}\frac{m\omega\tau[B^2(-m-1)-B^2(m-1)]}{1+(m\omega\tau)^2}. \quad (3.7)$$

Second, if the hodograph is simply connected and is invariant under threefold rotation about the z-axis [as in Fig. 2(b)], we have

$$S_{xy}=-\frac{\omega\tau B^2(0)}{1+(\omega\tau)}+\frac{2\omega\tau B^2(-3)}{1+(2\omega\tau)^2}-\frac{4\omega\tau B^2(3)}{1+(4\omega\tau)^2}+\cdots. \quad (3.8a)$$

In this same case, S_{xx} can be written

$$S_{xx}=\frac{B^2(0)}{1+(\omega\tau)^2}+\frac{B^2(-3)}{1+(2\omega\tau)^2}+\frac{B^2(3)}{1+(4\omega\tau)^2}+\cdots. \quad (3.8b)$$

Note that the third-order harmonics are missing in both expressions, and that the mth order term in S_{xy} is equal to the mth order term in S_{xx} times $\pm m\omega\tau$ (signs alternating).[16]

We next consider another possible band structure with threefold symmetry; one composed of three hodographs, each with mirror symmetry, arranged as shown in Fig. 2(c). We may, in this case, take **S** to be the average of the contributions from each hodograph. We then find

$$S_{xy}=\frac{\omega\tau[B^2(-2)-B^2(0)]}{1+(\omega\tau)^2}+\frac{2\omega\tau[B^2(-3)-B^2(1)]}{1+(2\omega\tau)^2}+\frac{3\omega\tau[B^2(-4)-B^2(2)]}{1+(3\omega\tau)^2}+\cdots \quad (3.9a)$$

and

$$S_{xx}=\frac{B^2(-2)+B^2(0)}{1+(\omega\tau)^2}+\frac{B^2(-3)+B^2(1)}{1+(2\omega\tau)}+\frac{B^2(-4)+B^2(2)}{1+(3\omega\tau)^2}+\cdots. \quad (3.9b)$$

Note that all the harmonics are now present. The mth term in S_{xy} is equal to $m\omega\tau$ times a quantity which may be positive or negative, but the magnitude of which is less than, or equal to, the mth term in S_{xx}.

If, in the case just discussed, there is the additional symmetry of a twofold axis through the center of each hodograph and parallel to the z-axis, then the B's with odd indices vanish. Thus, the even harmonics in S_{xx} and S_{xy} would vanish.

For a hodograph with fourfold symmetry [analogous to the one in Fig. 2(b)], only B's with indices which are

[16] From Eq. (2.6), it is seen that ω is positive if the energy increases going away from the center of the hodograph, and is negative if the energy decreases. Thus, the leading term in (3.8a) is negative for electrons, and is positive for holes, in agreement with the traditional result.

multiples of four will be nonzero. It follows that only odd harmonics appear in S_{xx} and S_{xy} and that the mth term in S_{xy} is given by $\pm m\omega\tau$ times the mth term in S_{xx}. If four hodographs having mirror symmetry are arranged in a cross [analogous to Fig. 2(c)], the resultant expression is the same as (3.9). The foregoing discussion applies when the magnetic field is parallel to a fourfold [100] axis. There are also threefold axes [111] in cubic crystals, and the discussion of threefold symmetry already given applies. We shall not discuss other symmetries here.

It is to be emphasized that if the hodograph is elliptical, all B's are zero except $B(0)$. In such a case, there would be no harmonics, in agreement with previous work.[1]

4. FREQUENCY DEPENDENCE OF THE MAGNETOCONDUCTIVITY

The results obtained in Sec. 2 can easily be generalized to apply when the electric field is alternating in time. To do so, it is only necessary to replace $1/\tau$ by $i\omega_a+1/\tau$, where ω_a is the angular frequency of the applied electric field. Note that the substitution must be made before the quantity τ is factored out in the definition of **S**. The expression obtained for **S** is

$$\mathbf{S}=\sum_m \mathbf{v}(m)\mathbf{v}(-m)/[1+i(m\omega+\omega_a)\tau]. \quad (4.1)$$

Since the above expression depends upon the Fourier coefficients of the velocities in the same way as the static field **S**, the results of Sec. 3 still apply.

As is well known, the real part of **S** gives the conductivity and the imaginary part gives the dielectric constant. If $|m\omega\tau|$ is of the order of one, or greater, **S** will have pronounced maxima in the neighborhood of $\omega_a=m\omega$. Such behavior is cyclotron resonance, with the harmonics predicted by Zeiger.[4] A recent calculation by Luttinger and Goodman[17] of the cyclotron resonance absorption in p-type germanium also reveals the harmonics. The third harmonic in p-type germanium has been observed experimentally by Dexter.[18] The third harmonic disappears when the magnetic field is in the [111] direction, in agreement with the conclusions of Sec. 3.

5. CONCLUSIONS

In this paper we have studied the field dependence of the magnetoconductivity for general band structures. In doing so, we have made several assumptions. It would be important to study the effects of relaxing these assumptions. Perhaps the most serious is the use of the Boltzmann equation, which neglects orbital quantization in the presence of the magnetic field.[19] It is known that the orbital quantization gives rise to the de Haas-van Alphen type oscillations in the conductivity as a function of magnetic field.

If the experimental data could be accurately represented by the formulas derived here (or their necessary generalizations), not only could the concentrations and mobilities of the carriers be determined, but information about the band structure would be gained. For example, the relative magnitudes of the harmonics could be used to fix the parameters in a theoretical expression for the electron energy as a function of wave number (such a program was suggested by Shockley in reference 9).

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APPENDIX A

We wish to show that the Fourier component $v_x(0)$ vanishes. The component is given by

$$v_x(0)=(\omega/2\pi)\oint ds v_x. \quad (A.1)$$

Let dk be the change in arc length (in k space) going with ds. Then

$$v_x(0)=(\omega\hbar c/2\pi e)\oint dk\mathbf{v}_p\cdot\mathbf{i}/v_p, \quad (A.2)$$

where $\mathbf{i}$ is a unit vector in the x direction and $\mathbf{v}_p$ is the component of the velocity perpendicular to the magnetic field $[v_p=(v_x^2+v_y^2)^{\frac{1}{2}}]$. Now

$$dk\mathbf{v}_p/v_p=\mathbf{dk}\times\mathbf{H}/H, \quad (A.3)$$

where $\mathbf{dk}$ is the vector change in $\mathbf{k}$, going along the hodograph. Thus, it follows that

$$v_x(0)=(\omega\hbar c/2\pi eH)\left(\mathbf{H}\times\oint\mathbf{dk}\right)\cdot\mathbf{i}. \quad (A.4)$$

But since $\oint\mathbf{dk}=0$, we have proven that $v_x(0)$ is zero. Obviously, the same proof holds for $v_y(0)$.

[17] J. M. Luttinger and R. R. Goodman, Phys. Rev. **100**, 673 (1955).

[18] R. N. Dexter, Phys. Rev. **98**, 1560 (1955).

[19] The effect of orbital quantization on the transport properties has been studied by S. Titeica [Ann. Phys. **22**, 120 (1935)] and by B. Davydov and I. Pomeranchuk [J. Phys. (U.S.S.R.) **2**, 147 (1940)].