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Decay of the Pi Meson*

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It is known experimentally that, relative to the normal pion decay mode $\pi\rightarrow\mu+\nu$, the branching ratios ρ and ρ_γ for the alternate decay modes $\pi\rightarrow e+\nu$ and $\pi\rightarrow e+\nu+\gamma$ are very small (ρ, $\rho_\gamma\lesssim 5\times10^{-5}$). We investigate the question of whether these limits on the branching ratios are consistent with the idea that pion decay occurs through a universal Fermi interaction via disintegration into a virtual nucleon pair: $\pi\rightarrow$virtual nucleon pair$\rightarrow\mu$ (or e)$+\nu$. A value for ρ consistent with the experiments can be obtained if the pseudoscalar coupling constant g_P in the universal Fermi interaction is small compared to the axial vector coupling constant g_A. An estimate of ρ_γ is made assuming that the photodecay occurs through the axial vector coupling, and it is found that there is probably no disagreement with experiment on this score. However, the photodecay can also occur through the tensor interaction (forbidden for $\pi\rightarrow\mu+\nu$ and $\pi\rightarrow e+\nu$). Using the experimental value $g_A/g_T\lesssim0.02$ obtained from beta-decay experiments, we estimate $\rho_\gamma\gtrsim0.025$, a result which is inconsistent with the experimental value. It is shown that the disagreement cannot be removed by using a linear combination of all possible Fermi couplings. An upper limit on g_P for beta decay, valid even in the absence of a universal Fermi interaction, is obtained.

I. INTRODUCTION

THE observed reactions

$$n\rightarrow p+e^-+\nu, \quad (1)$$

$$p+\mu^-\rightarrow n+\nu, \quad (2)$$

$$\mu^\pm\rightarrow e^\pm+2\nu, \quad (3)$$

are usually discussed in terms of the Fermi coupling of four spinor fields

$$\mathcal{H}_{\text{int}}=\textstyle\sum_i g_i(\bar\psi_a\Gamma_i\psi_b)(\bar\psi_c\Gamma_i\psi_d)+\text{c.c.}, \quad (4)$$

where the $(\bar\psi_a\Gamma_i\psi_b)$ are the respective covariants ($i=1\cdots5$): scalar, vector, tensor, axial vector, and pseudoscalar. The relative magnitudes of the various coupling constants g_i are not yet well established for any of the above processes. However, it is known from the observed reaction rates that the dominant coupling constants have very nearly the same magnitude for all three reactions.[1] This remarkable fact suggests some sort of universality in the interaction of four Fermi particles.[2]

In its simplest form, this is expressed by the Tiomno-Wheeler triangle,[2] in which each of the pairs of spinor particles, (n,p), (e,ν), and (μ,ν), is assumed to interact with each other pair with the same combination of coupling constants. This limited form of universality makes no provision for other conceivable four-particle interactions, e.g., $\mu^-+p\rightarrow p+e^-$, $\mu^\pm\rightarrow e^\pm+e^-+e^+$, etc.; but these processes are in any case not observed.

In the case of charged pion decay, $\pi^\pm\rightarrow\mu^\pm+\nu$, it seems natural to invoke the Fermi interaction of Eq. (2), according to the scheme: $\pi\rightarrow$virtual nuclear pair$\rightarrow$ $\mu+\nu$. This process is permitted by the conservation laws for both axial vector and pseudoscalar Fermi coupling. On the basis of this interpretation of pion decay, it must then be supposed that one or both of these couplings is present in the interaction of (n,p) and (μ,ν). Because of the ambiguities associated with perturbation treatments of the pion-nucleon interaction, it has not been possible so far to obtain a reliable theoretical estimate for the absolute rate of pion decay. Nevertheless, in the important case of axial vector coupling, a perturbation calculation employing a cutoff has been made[3] and leads to fairly reasonable agreement with experiment if one takes for the Fermi coupling constant the value $g_A\approx10^{-49}$ erg cm^3. This is about equal to the magnitude of the dominant coupling constant for each of the reactions (1)-(3).[1]

The problem arises, however, of understanding the apparent non-occurrence of an electron decay mode for the π-meson. Experimentally, the ratio $\rho=(\pi\rightarrow e+\nu)/(\pi\rightarrow\mu+\nu)$ of the decay rates for the two modes appears to be no larger than about 5×10^{-5}.[4] If the pair (e,ν) is assumed to be coupled to nucleons in the same way as the pair (μ,ν)—in accordance with the idea of a universal Fermi interaction—the ratio ρ can be calculated rigorously, independent of any detailed treatment of the pion-nucleon interaction. This has been done by Ruderman and Finklestein.[3] For axial vector coupling this ratio is

$$\rho_A=\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2\approx10^{-4}. \quad (5)$$

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[1] An excellent review can be found in the article by L. Michel in *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952).

[2] O. Klein, Nature **161**, 897 (1948); G. Puppi, Nuovo cimento **5**, 587 (1948); Lee, Rosenbluth, and Yang, Phys. Rev. **75**, 905 (1949); J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).

[3] M. Ruderman and R. Finklestein, Phys. Rev. **76**, 1458 (1949).

[4] S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240(A) (1955).

For pseudoscalar coupling

$$\rho_P=\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2=5.4. \tag{6}$$

In the spirit of the universal Fermi interaction, therefore, any appreciable admixture of pseudoscalar coupling must be ruled out; but the experimental limits on ρ may just barely be consistent with axial vector coupling.

For later reference, we repeat here the derivation of the above result for the important case of axial vector coupling. The decay $\pi\to(\mu$ or $e)+\nu$ can be represented schematically by the "black box" diagram of Fig. 1. The black box contains all the complicated diagrams involving virtual nucleon pairs and pions. The Fermi interaction, however, is of course treated only to first order in perturbation theory. The pion momentum P_π is the only vector on which that portion of the matrix element which describes the black-box processes may depend. The total matrix element therefore has the form (in units where $\hbar=c=1$)

$$\mathfrak{M}=g_Af_Am(\bar{\psi}_{\mu,e}\gamma_5\boldsymbol{P}_\pi\psi_\nu),\quad \boldsymbol{P}_\pi=\gamma_\lambda(P_\pi)_\lambda, \tag{7}$$

where f_A is a dimensionless numerical factor "of order unity"; m is a scalar which has the dimensions of mass and which depends on the nucleon and pion masses. We may expect that m is in fact of the order of the nucleon mass. In perturbation theory, for pseudoscalar coupling between the meson and nucleon fields, the product f_Am is given by the divergent integral

$$f_Am=\frac{4\sqrt{2}G}{(2\pi)^4i}\int d^4q\frac{M}{(q^2+M^2)(q^2+M^2-m_\pi^2-2P_\pi\cdot q)}, \tag{8}$$

where M is the nucleon mass and G is the pseudoscalar coupling constant. If a Feynman convergence factor $K^2/(q^2+K^2)$ is introduced, and the cutoff chosen to be the nucleon mass, $K=M$, then with the neglect of small quantities ($m_\pi\ll M$), the expression for f_Am reduces to

$$f_Am=\frac{G}{\sqrt{2}(2\pi)^2}M. \tag{8'}$$

The decay rate is given by the expression

$$\tau^{-1}=\frac{1}{8\pi}g_A^2f_A^2\left(\frac{m}{m_\pi}\right)^2\left(\frac{m_{\mu,e}}{m_\pi}\right)^2\left(1-\frac{m_{\mu,e}^2}{m_\pi^2}\right)^2m_\pi^5. \tag{9}$$

The ratio of Eq. (5) follows immediately. We also note that if one takes for g_A the value 10^{-49} erg cm^3, the observed $\pi\to\mu+\nu$ lifetime is obtained with the choice $f_Am=0.2M$, which appears to be a reasonable result.

As mentioned above, the experimental upper limit on the rate of the decay mode $\pi\to e+\nu$ is only barely consistent with the idea of decay through a pure axial vector coupling. Additional difficulties arise in connection with the possibility of photon emission: $\pi\to e+\nu+\gamma$. In the first place, Ruderman[5] has suggested that the small probability for the reaction $\pi\to e+\nu$, in the case of axial vector coupling, does not obtain for the reaction $\pi\to e+\nu+\gamma$. He finds for the ratio $\rho_\gamma=(\pi\to e+\nu+\gamma)/(\pi\to\mu+\nu)$ a value of 2.4×10^{-3}, in the case of axial vector coupling. This is clearly inconsistent with the experimental results. Ruderman's calculation involves certain approximations, however, and these are re-examined in the following section. It is argued there that the ratio ρ_γ is apt to be much smaller than is indicated by Ruderman's work and that in fact there is no contradiction with the idea of a universal Fermi interaction on this score.

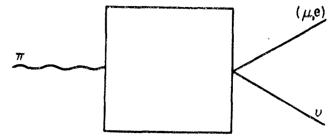

FIG. 1. Diagram for the decay $\pi^\pm\to\mu^\pm$ (or $e^\pm$)$+\nu$.

However, in Sec. III, an additional problem is raised: recent experimental beta-decay evidence indicates that the tensor coupling constant is much larger than the axial vector constant.[6,7] It is necessary then to consider the competition of the decay $\pi\to e+\nu+\gamma$ occurring through the tensor interaction[8] with the decay $\pi\to\mu+\nu$ occurring through a much weaker axial vecor interaction. It is found that the ratio of the rates for the two processes is much larger than the experimental upper limit. Despite the theoretical uncertainties involved in this estimate, it appears to represent a very serious difficulty for the idea that the pairs (e,ν) and (μ,ν) are similarly coupled to nucleons.[9,10]

In Sec. IV, the possibility of linear combinations of the various Fermi couplings is considered. It is shown there that the difficulties discussed in Sec. III cannot be overcome in this way.

The theoretical estimates which are made in the present work are based mainly on general invariance and dimensional arguments, as in the derivation leading to Eq. (7). Perturbation theory results for the pion-nucleon interaction are referred to briefly only as a confirmation of arguments having to do with the form of matrix elements.

[5] M. Ruderman, Phys. Rev. **85**, 157 (1952).
[6] R. Sherr and R. H. Miller, Phys Rev. **93**, 1076 (1954). The ratio (g_A/g_T) reported in this paper has been revised (private communication) to 0 ± 0.02.
[7] B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955).
[8] Photodecay through the scalar interaction is still forbidden; and as we shall show in Sec. IV, the contributions from vector and pseudoscalar couplings are likely to be small.
[9] After the present work was completed, R. G. Sachs called our attention to a paper by Iwata, Ogawa, Okonogi, Sakita, and Oneda, Progr. Theoret. Phys. **13**, 19 (1955) in which this same problem of photodecay via tensor coupling is discussed from the point of view of perturbation theory. These authors come to conclusions similar to ours (see Sec. III).

II. PHOTODECAY via AXIAL VECTOR COUPLING

In this section, we wish to reconsider the calculation of the branching ratio for the two processes,

$$\pi^{\pm}\rightarrow e^{\pm}+\nu+\gamma \qquad (10)$$

and

$$\pi^{\pm}\rightarrow\mu^{\pm}+\nu, \qquad (11)$$

assuming that both proceed via a universal axial vector coupling. The decay (10) occurs through the three diagrams (a), (b), and (c) of Fig. 2. The black-box portion of diagrams (a) and (b) is identical with that in Fig. 1 describing the decay (11). The black box of diagram (c) differs in that a photon is emitted by one of the virtual charged nucleons or pions. Ruderman[5] supposes that only diagram (a) contributes significantly to process (10), so that he includes only this diagram in his calculation. We shall show, however, that when all diagrams are included, a very much smaller result is obtained for the ratio ρ_γ of the rates for the two processes (10) and (11). The contribution of diagram (b) vanishes if the calculation is made in the gauge in which the scalar potential vanishes, but its presence is necessary for certain gauge invariance considerations.

We first consider a simplified case in which the nonlocal interaction represented in the above discussion by a black box is replaced by a direct coupling. From Eq. (7), we see that as far as the decay proceeding via axial vector coupling is concerned, the nonlocal interaction is equivalent to a direct coupling

$$\mathfrak{L}_{\text{int}}=ig_Af_Am\frac{\partial\phi_\pi^*}{\partial x_\lambda}(\bar{\psi}_e\gamma_5\gamma_\lambda\psi_\nu)+\text{c.c.} \qquad (12)$$

If we now introduce the electromagnetic field, we must replace $\partial/\partial x_\lambda$ by $\partial/\partial x_\lambda-ieA_\lambda$. The term involving A_λ which arises in this way yields the analog of diagram (c) in Fig. 2. With this interaction, the matrix element for photodecay becomes

$$\mathfrak{M}=-eg_Af_Am\bar{\psi}_e\left[\boldsymbol{\varepsilon}\left(1+\frac{im_e}{p_e+k-im_e}\right)\gamma_5-\boldsymbol{\varepsilon}\gamma_5\right]\psi_\nu, \qquad (13)$$

where ϵ and k are respectively the polarization and four-momentum of the photon. In Eq. (13), the first two terms correspond to diagram (a); the last term, to diagram (c). If only the terms corresponding to diagram (a) are kept, the term proportional to the electron mass in the round brackets can be neglected, and one obtains Ruderman's result. If, however, both diagrams (a) and (c) are retained, the principal contributions from these two diagrams cancel and only the term proportional to m_e survives. Because of this, an estimate of the ratio ρ_γ for the two processes (10) and (11) based on Eq. (13) leads to a small value, of order $(1/137)\ (m_e/m_\mu)^2$.

In the presumably more realistic case of pion decay occurring through the production of virtual nucleon pairs, this proportionality to m_e no longer obtains. For this case, we first consider the perturbation-theory calculation, since this leads to a result which can be generalized. To simplify the calculation, we set $m_e=0$. It happens that all infrared-divergent terms are proportional to m_e, so these terms are being dropped (the surviving term in Eq. (13) is infrared-divergent). However, these divergences can be handled by the standard Bloch-Nordsieck method, so nothing essential to the problem in hand has been omitted. With this approximation, the sum of the matrix elements corresponding to the diagrams of Fig. 2 is

$$\mathfrak{M}=eg_Af_A'M\frac{P_\pi\cdot k}{M^2}(\bar{\psi}_e\gamma_5\boldsymbol{\varepsilon}\psi_\nu), \qquad (14)$$

where f_A' is given by the convergent integral

$$f_A'=\frac{\sqrt{2}}{(2\pi)^2}GM^2\int_0^1dx\times\int_0^{1-x}dy\frac{1-2x}{M^2-m_\pi^2x(1-x)-2P_\pi\cdot kxy} \qquad (15)$$

$$\approx\frac{\sqrt{2}}{6(2\pi)^2}G. \qquad (15')$$

This is to be compared with the expression for f_A given by Eq. (8). Although the latter is given by a divergent integral, f_A and f_A' are of the same order of magnitude if some reasonable cutoff $K\sim M$ is used to evaluate f_A.

We now show that the form (14) for the photodecay matrix element is valid in the general case, irrespective of what happens inside the black box. Indicating explicitly the argument of the dimensionless scalar f_A of Eq. (7), $f_A=f_A(P_\pi^2)$, we can write the sum of the matrix elements for the three diagrams of Fig. 2 in the form

$$\mathfrak{M}=eg_Am\bar{\psi}_e\left\{f_A(P_\pi^2)\boldsymbol{\varepsilon}\frac{1}{p_e+k-im_e}\gamma_5\boldsymbol{P}_\pi+f_A[(P_\pi-k)^2]\frac{(2P_\pi-k)\cdot\epsilon}{[(P_\pi-k)^2+m_\pi^2]}\gamma_5(\boldsymbol{P}_\pi-\boldsymbol{k})+h_1(P_\pi,k)\gamma_5\boldsymbol{\varepsilon}+h_2(P_\pi,k)\frac{P_\pi\cdot\epsilon}{m^2}\gamma_5\boldsymbol{k}+h_3(P_\pi,k)\frac{P_\pi\cdot\epsilon}{m^2}\gamma_5\boldsymbol{P}_\pi\right\}\psi_\nu. \qquad (16)$$

Here h_1, h_2, and h_3 are dimensionless scalars, which may depend on P_π and k; and the last three terms represent the only relativistically invariant possibilities for the diagram (c) of Fig. 2 which do not vanish for both the actual calculation and the gauge invariance condition. If we now make the approximation $m_e=0$

and use conservation of momentum and the properties of the spinors ψ_e and ψ_ν to simplify Eq. (16), we obtain

$$\mathfrak{M}=eg_Am\bar{\psi}_e\bigg\{f_A(P_\pi^2)\gamma_5\boldsymbol{\varepsilon}+h_1(P_\pi,k)\gamma_5\boldsymbol{\varepsilon}+h_2(P_\pi,k)\frac{P_\pi\cdot\epsilon}{m^2}\gamma_5\boldsymbol{k}+h_3(P_\pi,k)\frac{P_\pi\cdot\epsilon}{m^2}\gamma_5\boldsymbol{k}\bigg\}\psi_\nu. \quad (17)$$

Since for the ordinary gauge $P_\pi\cdot\epsilon=0$, the actual matrix element is given by

$$\mathfrak{M}=eg_Am[f_A(P_\pi^2)+h_1(P_\pi,k)](\bar{\psi}_e\gamma_5\boldsymbol{\varepsilon}\psi_\nu). \quad (18)$$

Now in order for expression (17) to be gauge invariant, it must vanish if we replace ϵ by k. This gives us the condition

$$f_A(P_\pi^2)+h_1(P_\pi,k)=-[h_2(P_\pi,k)+h_3(P_\pi,k)]\frac{P_\pi\cdot k}{m^2}. \quad (19)$$

Thus, it appears that the matrix element for photodecay via axial vector coupling has the form indicated in the perturbation calculation

$$\mathfrak{M}=eg_Amf_A'\frac{P_\pi\cdot k}{m^2}(\bar{\psi}_e\gamma_5\boldsymbol{\varepsilon}\psi_\nu), \quad (20)$$

where

$$f_A'=f_A'(P_\pi,k)=-[h_2+h_3].$$

In the absence of any reliable theory of what happens inside the black box, we can only speculate about the factors f_A' and m. We anticipate that m is of the order of the nucleon mass, since the black box arises from the formation of nucleon-antinucleon pairs. Moreover, the results of the present calculation should reduce to those obtained with direct coupling in the limit where the nucleon mass$\rightarrow\infty$. Since we have dropped terms proportional to m_e in the present calculation, this means that the result (20) should vanish in this limit. The dimensionless factor f_A' is presumably of the same order of magnitude as f_A. These speculations are consistent with the perturbation results presented earlier.

Neglecting the possible dependence of f_A' on k, we can now calculate the ratio ρ_γ for the decay rates of processes (10) and (11). The result is

$$\rho_\gamma=\frac{1}{240\pi}\left(\frac{e^2}{4\pi}\right)\left(\frac{m_\pi}{m}\right)^4\left(\frac{m_\pi}{m_\mu}\right)^2\times\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^{-2}\left(\frac{f_A'}{f_A}\right)^2. \quad (21)$$

If we take $f_A'=f_A$ and $m=M$ this gives

$$\rho_\gamma\sim4\times10^{-8}. \quad (22)$$

Although the estimates of m and f_A'/f_A are uncertain we feel that the ratio is sufficiently small that the photodecay process considered in this section does not represent a serious problem for the idea of pion decay occurring through a universal Fermi interaction.

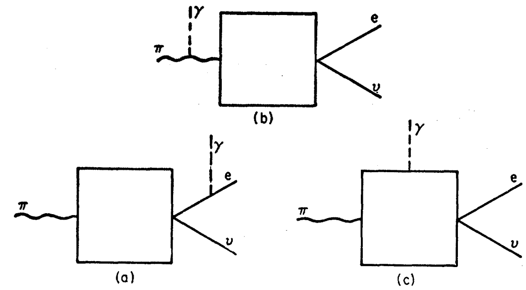

FIG. 2. Diagrams for the decay $\pi^\pm\rightarrow e^\pm+\nu+\gamma$.

III. PHOTODECAY via TENSOR COUPLING

Recent experimental evidence indicates that the scalar and tensor coupling constants in ordinary beta decay are larger than the vector and axial vector constants.[6,7,10] In particular, Sherr and Miller report a lower limit of ~50 for the ratio g_T/g_A.[6] If the same couplings are assumed for (μ,ν) there arises in the first place the problem of understanding the absolute rate for $\pi\rightarrow\mu+\nu$ decay on the basis of an axial vector coupling constant no larger than about 10^{-51} erg cm^3. Despite the ambiguities associated with the theoretical predictions, this reduction by a factor of $(50)^2$ in the value of g_A^2 assumed in the calculations of Ruderman and Finklestein[3] may be difficult to reconcile with the observed $\pi\rightarrow\mu+\nu$ lifetime. In addition, we must now consider the competing decay mode $\pi\rightarrow e+\nu+\gamma$ occurring through the much larger tensor coupling.

This process can occur only through photon emission by one of the virtual pions or nucleons, as indicated schematically in diagram (c) of Fig. 2. In this case, the portion of the matrix element describing the black box processes can be constructed from the vectors P_π, k, and ϵ. For reasons of gauge invariance, the form of the matrix element must be

$$\mathfrak{M}=ieg_Tf_T'(\bar{\psi}_e\gamma_5\boldsymbol{\varepsilon}\boldsymbol{k}\psi_\nu), \quad (23)$$

where f_T' is a dimensionless scalar quantity which is a function of the nucleon and pion masses and which may also depend on the scalar product $P_\pi\cdot k$. We neglect the possible dependence on the latter quantity. In comparing Eqs. (7) and (23), we shall assume that the numerical factors f_A and f_T' are essentially equal and that m is of the order of the nucleon mass.

The decay rate for $\pi\rightarrow e+\nu+\gamma$ is given by the expression

$$\tau^{-1}=\frac{1}{96}\frac{1}{(2\pi)^2}\left(\frac{e^2}{4\pi}\right)g_T^2f_T'^2m_\pi^5. \quad (24)$$

The ratio $(\pi\rightarrow e+\nu+\gamma)/(\pi\rightarrow\mu+\nu)$ of the rates of the

[10] H. M. Mahmoud and E. J. Konopinski, Phys. Rev. 88, 1266 (1952).

two decay modes is

$$\rho_\gamma = 0.067(1/137)(g_T f_T'/g_A f_A)^2 (m_\pi/m)^2. \quad (25)$$

If we now take $f_T' = f_A$, m = nucleon mass, and $g_T/g_A \gtrsim 50$, the ratio becomes

$$\rho_\gamma \gtrsim 0.025, \quad (26)$$

a result which is inconsistent with the experimental upper limit.

IV. LINEAR COMBINATIONS OF COUPLINGS

Up to this point, we have considered only single Fermi couplings (pure axial vector, pure tensor, etc.). It is necessary now to see what happens when a general linear combination of couplings is taken into account. In order to include in our discussion the possibility that there is no universal Fermi interaction, we shall, when appropriate, distinguish by superscripts (e) and (μ) the Fermi coupling constants connecting (n,p) with (e,ν) and (μ,ν), respectively.

Nonphoton Decay

The most general matrix element for $\pi \to e+\nu$ and $\pi \to \mu+\nu$ decay may involve a combination of axial vector and pseudoscalar couplings. On the basis of the same kinds of arguments used earlier, we can write the matrix element in the form

$$\mathfrak{M} = \bar{\psi}_{e,\mu}\gamma_5 \{g_A f_A m \boldsymbol{P}_\pi - i g_P f_P m^2\}\psi_\nu. \quad (27)$$

As before, we anticipate that $f_A \approx f_P$ and $m \approx$ nucleon mass. The ratio $\rho = (\pi \to e+\nu)/(\pi \to \mu+\nu)$ is then found to be

$$\rho = \left(\frac{m_e g_A{}^e f_A + m g_P{}^e f_P}{m_\mu g_A{}^\mu f_A + m g_P{}^\mu f_P}\right)^2 \left(\frac{m_\pi{}^2 - m_e{}^2}{m_\pi{}^2 - m_\mu{}^2}\right)^2. \quad (28)$$

In the limits where either $g_A \to 0$ or $g_P \to 0$, this reduces to the previous results.

As we have already seen, if one assumes a universal axial vector coupling ($g_P = 0$), the ratio ρ has the value 10^{-4}, which is actually somewhat larger than the experimental upper limit 5×10^{-5}. In principle, this difficulty could be avoided, in the framework of a universal Fermi interaction, by assuming that the two terms in the numerator of Eq. (28) almost cancel:

$$m_e g_A f_A \approx -m g_P f_P. \quad (29)$$

There is no way to rule out this possibility on the basis of our present knowledge of the beta-decay coupling constants. But, as we shall discuss later, this choice of coupling constants cannot be expected to account for the apparent nonoccurrence of a decay mode $\pi \to e+\nu+\gamma$.

In any case, even if we drop the idea of a universal Fermi interaction, the experimental limit on ρ implies that

$$\left|\frac{m_e}{m} g_A{}^e f_A + g_P{}^e f_P\right| \lesssim (3\times10^{-3}) \left|\frac{m_\mu}{m} g_A{}^\mu f_A + g_P{}^\mu f_P\right|. \quad (30)$$

If we assume that all of the numerical factors f have the same magnitude, this can be written

$$|g_P{}^e| \lesssim (3\times10^{-3}) \left\{\frac{m_\mu}{m}|g_A{}^\mu| + |g_P{}^\mu|\right\} + \frac{m_e}{m}|g_A{}^e|. \quad (31)$$

However, from the observed rates for reactions (1)–(3), we know that the dominant coupling constants have about the same magnitude—which in turn is about the magnitude of the tensor and scalar coupling constants, $g_T{}^e$ and $g_P{}^e$, in ordinary beta decay. We can therefore conclude that

$$g_P{}^e \lesssim 3\times10^{-3} g_T{}^e. \quad (32)$$

We have neglected the last term in Eq. (31) since we have assumed that $m \approx$ nucleon mass and since we know that $g_A{}^e \ll g_T{}^e$. The inequality of Eq. (32) is independent of any assumption of a universal Fermi interaction, though of course it rests on the assumption that pion becay occurs in the way usually pictured, through virtual nucleon pairs.

Photon Decay

As pointed out above, the nonoccurrence of the $\pi \to e+\nu$ decay mode does not in itself rule out the idea of a universal Fermi interaction. The assumption that pion decay occurs through pure axial vector coupling leads to the result that the probability for this decay mode is indeed small, though it is slightly larger than the present experimental limit. But even if the experimental limit should be lowered, one could in principle invoke a small amount of pseudoscalar coupling, such as to produce the cancellation of Eq. (29).

We must therefore consider whether, with a suitable linear combination of Fermi couplings, one can also understand the non-occurrence of a $\pi \to e+\nu+\gamma$ decay mode. We have already seen that a universal interaction involving tensor and axial couplings leads to a relative probability $\gtrsim 0.025$ for this decay mode. This is at least 500 times larger than the experimental upper limit.

In the general case, the photodecay may involve all the Fermi couplings but the scalar. On the basis of the same kinds of dimensional and invariance arguments used earlier, we can write the matrix element in the form

$$\mathfrak{M} = e\bar{\psi}_e\gamma_5 \left\{\frac{1}{m} g_V f_V' [\boldsymbol{k}\boldsymbol{\varepsilon}\boldsymbol{P}_\pi + (P_\pi\cdot k)\boldsymbol{\varepsilon}] + i g_T f_T' \boldsymbol{\varepsilon}\boldsymbol{k} + g_A f_A' \frac{P_\pi\cdot k}{m}\boldsymbol{\varepsilon} - i g_P f_P \frac{m^2}{p_e\cdot k}\boldsymbol{\varepsilon}\boldsymbol{P}_\pi\right\}\psi_\nu. \quad (33)$$

As before, we have supplied factors m with the dimensions of mass whenever required; and we again assume that the numerical factors f all have approximately the same magnitude.

We now make use of our knowledge of the relative magnitudes of the beta-decay coupling constants; namely, $g_T \gg g_A$, g_V. Also, if the non-occurrence of the $\pi \rightarrow e+\nu$ decay mode is to be understood in the framework of a universal Fermi interaction it must be true that $g_P \lesssim (m_e/m) g_A \ll g_T$. When these results are taken into account it appears that the dominant term in the matrix element indeed comes from the tensor coupling and there is little likelihood of any cancellation effects. Cancellation between the tensor and axial vector or tensor and vector terms could arise only if the quantity m, which we have assumed to be of the order of the nucleon mass, were actually very much smaller (by a factor $\gtrsim 300$). Interference from the pseudoscalar term on the other hand would require that m be very large ($\gtrsim 100$ nucleon masses).

Barring such accidental cancellations, we conclude that the nonoccurrence of a $\pi \rightarrow e+\nu+\gamma$ decay mode constitutes a very serious difficulty with the idea of a universal Fermi interaction—at least if the customary picture of pion decay occurring through virtual nucleons is to be maintained.

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Tritium Production by High-Energy Protons*†

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The cross sections for tritium production in various substances by 450-Mev and 2.05-Bev protons have been measured. The nitrogen and oxygen cross sections—25±4 mb and 30±4 mb, respectively, at 2.05 Bev—lead to a world-wide average tritium production rate by cosmic rays of $0.14_{-0.03}^{+0.10}$ tritons/cm² sec, in good agreement with the observed value. The iron cross section—53±8 mb at 2.05 Bev—suggests a rate of production of 8.8×10^{-4} triton/g sec near the surface of a large meteorite in outer space. For the small Mt. Ayliff iron meteorite an age of 1.4×10^9 years was calculated from the observed He^3 content. Finally, the results for tritium production have been compared to those for other light particles.

INTRODUCTION AND SUMMARY

SEVERAL years ago it was postulated that tritium was being generated constantly by means of nuclear reactions. The production of tritium and its decay product, He^3, was first attributed to the reaction, $N^{14}(n,t)C^{12}$, induced by cosmic-ray neutrons.[1,2] More recently, extensive observations have been carried out on the natural abundance of tritium, and the world-wide average production rate is now believed to be 0.14 (±30%) tritons/cm² sec.[3,4] In order to gain a clearer picture of the origin of natural tritium we decided to investigate the production by the primary cosmic rays themselves. Since the flux of cosmic-ray primaries was insufficient for our measurements, we substituted protons accelerated in the Chicago Synchrocyclotron and the Brookhaven Cosmotron. From the observed proton cross sections in nitrogen and oxygen, the primary cosmic ray flux, and the tritium contribution from cosmic ray neutrons, we estimate the world-wide average tritium production rate to be $0.14_{-0.03}^{+0.10}$ tritons/cm² sec.

As the work progressed we felt that it would be interesting to measure the tritium production cross sections of other nuclei in addition. Iron was particularly interesting because of the question of the tritium and He^3 contents of iron meteorites.[5–10] It was necessary

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