collision, only the velocity of the nucleon can change from $\mathbf{v}$ to $\mathbf{v}'$. The corresponding probability for emitting m mesons with momenta $\mathbf{k}_1, \cdots \mathbf{k}_m$ due to this velocity change is

$$\mathcal{P}_m(\mathbf{k}_1\cdots\mathbf{k}_m)=|\langle\Psi^{(m)}(k_1\cdots k_m;\,\mathbf{v}')|N(\mathbf{v})\rangle|^2, \quad (A.29)$$

while the probability $\mathcal{P}_0$ for no-meson emission is given by

$$\mathcal{P}_0=|\langle N(\mathbf{v}')|N(\mathbf{v})\rangle|^2. \quad (A.30)$$

Upon using (A.27) and (A.28) we find

$$\mathcal{P}_0=\exp\left[-g^2\int f^2(\mathbf{k})d^3k\right], \quad (A.31)$$

and

$$\mathcal{P}_m(\mathbf{k}_1\cdots\mathbf{k}_m)=g^{2m}\mathcal{P}_0(m!)^{-1}\prod_{i=1}^{m} f^2(\mathbf{k}_i),$$

with

$$f(\mathbf{k})=(16\pi^3\omega)^{-\frac{1}{2}}[(\omega-\mathbf{k}\cdot\mathbf{v})^{-1}(1-v^2)^{\frac{1}{2}} - (\omega-\mathbf{k}\cdot\mathbf{v}')^{-1}(1-v'^2)^{\frac{1}{2}}].$$

It is easy to verify that the total probability is

$$\sum_{m=0}^{\infty}\int\cdots\int\mathcal{P}_m(\mathbf{k}_1\cdots\mathbf{k}_m)d^3k_1\cdots d^3k_m=1.$$

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Branching Ratio for Alternative Modes of Decay of Hyperons and θ^0 Mesons

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We study those restrictions upon the branching ratios of different decay modes of Σ^+, Λ^0, and θ^0 imposed by the invariant property of a system composed of nucleons, pions, hyperons, and heavy mesons under the Wigner time reversal. These restrictions give, in fact, upper and lower limits on the various branching ratios. Later, we use a more or less specific model of hyperons and θ mesons in order to get other possible restrictions on the branching ratios. These results are model-dependent, and subsequently provide a possible test of the model we have chosen.

I. INTRODUCTION

IT is known experimentally that the Σ^+ decays according to two different modes:

$$\Sigma^+ \begin{array}{l}\to p+\pi^0\\ \to n+\pi^+\end{array} \quad (Q\sim 116 \text{ Mev}).$$

It is likewise supposed that Λ^0 and θ^0 have an alternative decay mode besides the ordinary one, although so far there is no conclusive experimental evidence for the alternate modes:

$$\Lambda^0 \begin{array}{l}\to p+\pi^-\\ \to n+\pi^0\end{array} \quad (Q\sim 37 \text{ Mev}),$$

$$\theta^0 \begin{array}{l}\to \pi^++\pi^-\\ \to \pi^0+\pi^0\end{array} \quad (Q\sim 220 \text{ Mev}).$$

First we shall study those restrictions[1] upon the branching ratios of different decay modes of Σ^+, Λ^0, and θ^0 imposed by the invariant property of a system composed of nucleons, pions, hyperons, and heavy mesons under the Wigner time reversal. These restrictions give, in fact, upper and lower limits on the various branching ratios. Since the limitations upon the branching ratios thus obtained prove so weak (see Tables I and II), we later adopt a more or less specific model of hyperons and θ mesons. This will give us other possible restrictions on the branching ratios of various decays. These results are model-dependent, and subsequently provide a possible test of the model we choose.

II. RESTRICTIONS ON BRANCHING RATIOS OF DECAYS IMPOSED BY THE INVARIANT PROPERTY UNDER THE WIGNER TIME REVERSAL

Let us take, for example, the decay of a Λ^0 into a proton and a π^-, or a neutron and a π^0. For a given value of the spin and parity[2] of the Λ^0, the relative angular momentum l between a pion and a nucleon after the decay is fixed. We have, then, only two different final states, namely, a (p,π^-) and an (n,π^0) state with the specified j and l. Or we can use two states with definite isotopic spin values $T=\frac{3}{2}$ and $T=\frac{1}{2}$ instead of (p,π^-) and (n,π^0).

The reaction matrix K and the scattering (or decay) matrix T are given by

$$K=V+V\left[\frac{1}{E-H_0}\right]K, \quad (1)$$

[1] This possibility was first suggested to the author by K. M. Watson. Also see K. M. Watson, Phys. Rev. 95, 228 (1954).

[2] This is the intrinsic parity of a Λ^0 relative to the intrinsic parity of a nucleon.

and

$$T = \bar{K} + i\bar{K}\cdot T, \tag{2}$$

where H_0 is the free Hamiltonian, V is the interaction Hamiltonian for the elementary particles, and $\bar{K}$ is K on the energy shell.

The matrix $\bar{K}$ can be written as[3]

$$\bar{K} = \Sigma_{r,r'}(r'|\bar{K}|r)\phi_{r',j}{}^{m*}\phi_{r,j}{}^{m}, \tag{3}$$

where r and r' ($r,r'=1, 2, \cdots$) correspond in turn to the various channels of H_0. Here $\phi_{r,j}{}^m$ denotes an eigenfunction of H_0 with r, j, and m.

If we use the most common representation of $\phi_{r,j}{}^m$, we have

$$W\phi_{r,j}{}^m = i^{2m}\phi_{r,j}{}^m, \tag{4}$$

where W is the Wigner time-reversal operator.[4]

The invariance of $\bar{K}$ under W,

$$W\bar{K}W^\dagger = \bar{K}, \tag{5}$$

which follows from the invariance of H_0 and V under W, assures that the $\bar{K}$ matrix is real:

$$(r'|\bar{K}|r)^* = (r'|\bar{K}|r). \tag{6}$$

The invariance of $\bar{K}$ under W, together with the Hermitian property of $\bar{K}$, imply further that $\bar{K}$ is symmetric:

$$(r'|\bar{K}|r) = (r|\bar{K}|r'). \tag{7}$$

For the decay of a Λ^0 we have only three channels. We shall denote a Λ^0 state $\phi(\Lambda^0)$ before its decay, a nucleon-pion state $\phi_j{}^l(T=\frac{3}{2})$ with $(j,\ l,\ T=\frac{3}{2})$, and a nucleon-pion state $\phi_j{}^l(T=\frac{1}{2})$ with $(j,\ l,\ T=\frac{1}{2})$, by $r=1$, 2, and 3, respectively. Then the $\bar{K}$ matrix becomes

$$\bar{K} = \begin{vmatrix} 0 & \gamma_3 & \gamma_1 \\ \gamma_3 & \tan\eta_3 & \alpha \\ \gamma_1 & \alpha & \tan\eta_1 \end{vmatrix}. \tag{8}$$

Here η_3 and η_1 are pion-nucleon scattering phase shifts in the $(j,\ l,\ T=\frac{3}{2})$ and $(j,\ l,\ T=\frac{1}{2})$ states, respectively; α is a matrix element of $\bar{K}$ for pion-nucleon scattering which changes the magnitude of the isotopic spin; γ_3 and γ_1 are matrix elements of $\bar{K}$ responsible for the Λ^0 decay into $T=\frac{3}{2}$ and $T=\frac{1}{2}$ state, respectively. From Eq. (6), all these quantities are real.

The T matrix is obtained from Eq. (2) and Eq. (8):

$$T \approx \begin{vmatrix} 0 & \gamma_3\cos\eta_3 e^{i\eta_3} & \gamma_1\cos\eta_1 e^{i\eta_1} \\ \gamma_3\cos\eta_3 e^{i\eta_3} & \tan\eta_3 & \alpha\cos\eta_1\cos\eta_3 \times e^{i(\eta_1+\eta_3)} \\ \gamma_1\cos\eta_1 e^{i\eta_1} & \alpha\cos\eta_1\cos\eta_3 \times e^{i(\eta_1+\eta_3)} & \tan\eta_1 \end{vmatrix}, \tag{9}$$

where terms of higher order in α, γ_3, and γ_1 have been neglected.

So the wave function ψ after the decay of Λ^0 is

$$\psi \sim \gamma_3\cos\eta_3 e^{i\eta_3}\phi_j{}^m(T=\tfrac{3}{2}) + \gamma_1\cos\eta_1 e^{i\eta_1}\phi_j{}^m(T=\tfrac{1}{2}), \tag{10}$$

where

$$\phi_j{}^m(T=\tfrac{3}{2}) = (\tfrac{2}{3})^{\frac{1}{2}}\phi_j{}^m(n,\pi^0) + (\tfrac{1}{3})^{\frac{1}{2}}\phi_j{}^m(p,\pi^-), \tag{11}$$

and

$$\phi_j{}^m(T=\tfrac{1}{2}) = -(\tfrac{1}{3})^{\frac{1}{2}}\phi_j{}^m(n,\pi^0) + (\tfrac{2}{3})^{\frac{1}{2}}\phi_j{}^m(p,\pi^-).$$

The branching ratio $R(\Lambda^0)$ of alternative decays of Λ^0 is obtained from Eqs. (10) and (11):

$$R(\Lambda^0) \equiv \frac{P(\Lambda^0\to n+\pi^0)}{P(\Lambda^0\to p+\pi^-)} = \frac{2z^2+1-2\sqrt{2}z\cos(\eta_1-\eta_3)}{z^2+2+2\sqrt{2}z\cos(\eta_1-\eta_3)}, \tag{12}$$

with

$$z = \gamma_3\cos\eta_3/\gamma_1\cos\eta_1. \tag{13}$$

The mesonic phase shifts, η_3 and η_1, can be obtained from the experimental data for pion-nucleon scattering with pion energy corresponding to the Q-value of the Λ^0 decay. γ_3 and γ_1 are unknown but real. The quantity z can take, then, any real value from $-\infty$ to $+\infty$.

From Eq. (12), we find the maximum and minimum allowed values of $R(\Lambda^0)$ for given η_3 and η_1. When $|\eta_3-\eta_1|\ll 1$, which is the case for the Λ^0 decay, these bounds are

$$R_{\max} \approx (9/2)[1/\sin^2(\eta_1-\eta_3)],$$

and

$$R_{\min} \approx (2/9)\sin^2(\eta_1-\eta_3). \tag{14}$$

We list $R_{\max}$ and $R_{\min}$ in Table I for various possible values of spin and parity of Λ^0. Notice that for $\eta_1=\eta_3$, there is no limitation on $R(\Lambda^0)$.

A quite similar argument can be applied to the decay of a Σ^+ or a θ^0 particle. The branching ratio $R(\Sigma^+)$,

$$R(\Sigma^+) \equiv P(\Sigma^+\to p+\pi^0)/P(\Sigma^+\to n+\pi^+), \tag{15}$$

is given by the same expression as Eq. (12), except the mesonic phase shifts, η_3 and η_1, must be evaluated at the pion energy corresponding to the Q-value of the Σ^+ decay. The result is given in Table II.

Similarly, the branching ratio $R(\theta^0)$ of θ^0 decay can be given as follows:

TABLE I. Maximum and minimum allowed values of the branching ratio R for Λ^0 decay obtained from the invariant property under the Wigner time reversal. We have used the experimental value of the phase difference $|\eta_1-\eta_3|$ for given total and orbital angular momenta, j and l, of the pion-nucleon system after the Λ^0 decay. l is uniquely determined by the spin j and parity of Λ^0.

j	Λ^0 Parity	l	$\lvert\eta_1-\eta_3\rvert$	$R_{\max}$	$R_{\min}$
$\frac{1}{2}$	+	1	~0°	~∞	~0
$\frac{1}{2}$	−	0	~10°	~150	~0.007
$\frac{3}{2}$	+	1	~5°	~600	~0.002
...		...			

[3] See, e.g., M. Gell-Mann and K. M. Watson, Ann. Rev. Nuc. Sci. 4, 267 (1955).
[4] E. P. Wigner, Gott. Nachr. (1927).

(a) If the spin of the θ^0 is even,

$$R(\theta^0)\equiv\frac{P(\theta^0\rightarrow 2\pi^0)}{P(\theta^0\rightarrow\pi^++\pi^-)}=\frac{2x^2+1+2\sqrt{2}x\cos(\delta_0-\delta_2)}{x^2+2-2\sqrt{2}x\cos(\delta_0-\delta_2)},\qquad(16)$$

with

$$x=\beta_2\cos\delta_2/\beta_0\cos\delta_0.\qquad(17)$$

Here β_2 and β_0 are matrix elements of $\bar{K}$ corresponding to θ_0 decay into two pion states with isotopic spin $T=2$ and $T=0$, respectively. δ_2 and δ_0 are, respectively, the phase shifts of pion-pion scattering in $T=2$ and $T=0$ states, evaluated at a pion energy corresponding to the Q-value of θ^0 decay.

(b) If the spin of θ^0 is odd,

$$R(\theta^0)=0.\qquad(18)$$

There has been some question as to whether the θ meson and the τ meson merely denote different decay modes of the same particle. If this is true, the θ and τ must be a vector mesons $(1,-)$ or have spin higher than one. Dalitz's analysis[5] of the energy distribution of π^- in τ^+ decay renders it highly improbable that the spin and parity of the τ meson be $(1,-)$.

The experimental determination of $R(\theta^0)$ gives us another way to answer this question. If we find even a single case of θ^0 decay into two π^0, the possibility of the θ^0 meson being $(1,-)$ can be ruled out.[6] On the other hand, if $R(\theta^0)$ is shown to be smaller than $R_{\min}$ obtained from Eq. (16) for δ_0 and δ_2 of the S-wave pion-pion scattering, the θ^0 could not be a scalar meson $(0,+)$, and so is probably $(1,-)$.

Although there are no direct experimental data available for δ_0 and δ_2, there are indications[7] of a strong pion-pion scattering at pion energy $\sim$110 Mev. (In the center-of-mass system, 110 Mev corresponds to the Q-value of θ^0 decay.) However, for tentative values of $|\delta_0-\delta_2|$, we have evaluated $R_{\min}$ as well as $R_{\max}$ in Table III.

TABLE II. Maximum and minimum allowed values of the branching ratio R for Σ^+ decay obtained from the invariant property under the Wigner time reversal. We have used the experimental value of the phase difference $|\eta_1-\eta_3|$ for given total and orbital angular momenta, j and l, of the pion-nucleon system after the Σ^+ decay. l is uniquely determined by the spin j and parity of Σ^+.

j	Σ^+ Parity	l	$\|\eta_1-\eta_3\|$	$R_{\max}$	$R_{\min}$
$\frac{1}{2}$	+	1	$\sim5°$	~600	~0.002
$\frac{1}{2}$	−	0	$\sim25°$	~25	~0.04
$\frac{3}{2}$	+	1	$\sim25°$	~25	~0.04
...			...		

TABLE III. Maximum and minimum values of the branching ratio R for θ^0 decay, obtained from the property of invariance under the Wigner time reversal. Tentative values of the phase difference $|\delta_0-\delta_2|$ for pion-pion scattering are used in computing R.

$\|\delta_0-\delta_2\|$	$R_{\max}$	$R_{\min}$
0°	∞	0
30°	16.0	0.06
60°	3.8	0.26
90°	2.0	0.50

III. MODEL-DEPENDENT RESTRICTIONS ON BRANCHING RATIOS

Since the invariance property of the system under the Wigner time reversal alone does not give us sufficient information about branching ratios of these unstable particles, we shall adopt various models of unstable particles to obtain more detailed information.

(A) Gell-Mann—Nishijima Model[8]

According to Gell-Mann and Nishijima, the isotopic spins of Λ^0, Σ, and θ are 0, 1, and $\frac{1}{2}$, respectively. If we assume the selection rule $\Delta T=\frac{1}{2}$ for the decay of these particles,[9] we obtain

$$\gamma_3=0 \text{ for } \Lambda^0 \text{ decay},\qquad(19)$$

and

$$\beta_2=0 \text{ for } \theta^0 \text{ decay}.\qquad(20)$$

There is no additional information for Σ^+ decay obtained from this assumption, because the $\Delta T=\frac{1}{2}$ rule still allows decay both into the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ final states.

From Eqs. (12), (13), (16), (17), (19), and (20), the $R(\Lambda^0)$ and $R(\theta^0)$ are given by

$$R(\Lambda^0)=\tfrac{1}{2},\qquad(21)$$

and

$$R(\theta^0)=\begin{cases}\frac{1}{2} \text{ for an even spin of } \theta^0,\\ 0 \text{ for an odd spin of } \theta^0.\end{cases}\qquad(22)$$

Of course, these ratios are well within the limits of $R(\Lambda^0)$ and $R(\theta^0)$ given in Tables I and III.

(B) Compound Model of Unstable Particles[10]

As an example of this model, we shall take the Goldhaber model.[11] In his model, the Λ^0 and Σ particles

[5] R. H. Dalitz (private communication). Also see R. H. Dalitz, Phys. Rev. **94**, 1046 (1954).

[6] Should the alternate θ^0 decay into $2\pi^0$ and γ rays be appreciable, this conclusion is not certain.

[7] F. J. Dyson, Phys. Rev. **99**, 1037 (1955); G. Takeda, Phys. Rev. **100**, 440 (1955).

[8] M. Gell-Mann, Phys. Rev. **92**, 833 (1953); T. Nakano and K. Mishijima, Progr. Theoret. Phys. **10**, 581 (1953).

[9] The consequences of this selection rule have been studied by A. Pais and R. H. Dalitz (private communication), and by R. Gatto (to be published).

[10] G. Wentzel has independently discussed the consequences of various compound models of unstable particles [Phys. Rev. **101**, 505 (1956). We are indebted to Professor Y. Nambu for this information.

[11] M. G. Goldhaber, Phys. Rev. **101**, 433 (1956).

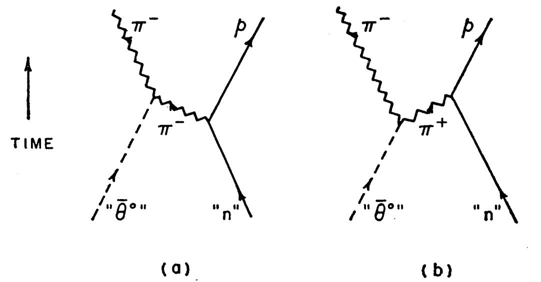

FIG. 1. Feynman diagrams of the Λ^0 decay into a proton and a π^-. (a): A Feynman diagram of the stimulated decay of "$\bar{\theta}^0$" inside Λ^0. (b): A Feynman diagram of the natural decay of "$\bar{\theta}^0$" inside a Λ^0.

are bound states of a $\bar{\theta}$ to a nucleon with total isotopic spin 0 and 1, respectively.

For example, a Λ^0 wave function can be written as

$$\psi_j{}^m(\Lambda^0)=(1/\sqrt{2})\{\psi_j{}^m(\bar{\theta}^0,n)-\psi_j{}^m(\bar{\theta}^-,p)\}, \quad (23)$$

where $\psi_j{}^m(\bar{\theta}^0,n)$ and $\psi_j{}^m(\bar{\theta}^-,p)$ are bound states of the $\bar{\theta}^0$ or $\bar{\theta}^-$ to the neutron or proton, respectively.

According to this model, it is quite natural to assume that Λ^0 or Σ decay is due to $\bar{\theta}$ decay within the bound state. However, $\bar{\theta}^-$ decay can be virtually ignored, as $\bar{\theta}^0$ decay is some hundred times faster.[12] This circumstance enables us to give certain relations among $R(\theta^0)$,[13] $R(\Lambda^0)$, and $R(\Sigma^+)$.

In order to find these relations, we must first separate the effects due to the final state interactions between two pions or a pion and a nucleon upon the decay matrices of θ^0 and hyperons.

If these final state interactions[14] occur in a region larger than a region where a θ^0 or hyperon decay occurs, we have a factor,

$$(ka)^{-2l}\sin(ka+\eta)e^{i\eta}, \quad (24)$$

in the corresponding matrix element of T. Here a is a characteristic radius of region of the decay interaction of $\Lambda^0(a_\Lambda)$, Σ (a_Σ), or θ^0 (a_θ); k is the momentum of each secondary in the rest system of a hyperon or θ^0; and η is a phase shift for the scattering of two secondaries.

We shall explicitly introduce this factor [Eq. (24)] into Eq. (13) and Eq. (17):

$$z=\frac{\sin(ka_\Lambda+\eta_3)}{\sin(ka_\Lambda+\eta_1)}\cdot\frac{\gamma_3'}{\gamma_1'} \quad (13')$$

and

$$x=\frac{\sin(ka_\theta+\delta_2)}{\sin(ka_\theta+\delta_0)}\cdot\frac{\beta_2'}{\beta_0'}. \quad (17')$$

Here γ_3'/γ_1' and β_2'/β_0' are considered to be free from the effect of the final state interaction.[15] For the Σ^+ decay, we can obtain $R(\Sigma^+)$ from Eqs. (12) and (13′), replacing a_Λ by a_Σ.

There are two different ways a bound $\bar{\theta}^0$("$\bar{\theta}^0$") can decay inside a Λ_0: a "$\bar{\theta}^0$" decays into one pion in the pion field of the neutron—a stimulated decay of "$\bar{\theta}^0$" —or a "$\bar{\theta}^0$" decays into two pions following by an absoprtion of one of these by the neutron—a natural decay of "$\bar{\theta}^0$."

Except for the factor due to the final state interaction, the decay amplitude for $(\Lambda^0\to p+\pi^-)$ arising from the stimulated decay of "$\bar{\theta}^0$" is proportional to a product of the following factors: the amplitude of the $(\bar{\theta}^0,n)$ state in $\Lambda^0(=1/\sqrt{2})$, the amplitude of the π^- field around the neutron $[=(\frac{2}{3})^{\frac{1}{2}}C$, where $|C|^2$ is the probability of the presence of pion field around a nucleon], and the decay amplitude $\beta'(\theta^0\to\pi^++\pi^-)$, which is proportional to the amplitude for the transition $(\bar{\theta}^0+\pi^-\to\pi^-)$. (See Fig. 1.) It is easily shown that the decay amplitude for $(\Lambda^0\to p+\pi^-)$ from the natural decay of "$\bar{\theta}^0$" is also proportional to this same factor:

$$\gamma'(\Lambda^0\to p+\pi^-)\propto(\tfrac{1}{2})^{\frac{1}{2}}(\tfrac{2}{3})^{\frac{1}{2}}C\beta'(\theta^0\to\pi^++\pi^-). \quad (25)$$

In a similar way (see Fig. 2), we have[16]

$$\gamma'(\Lambda^0\to n+\pi^0)\propto(\tfrac{1}{2})^{\frac{1}{2}}[-(\tfrac{1}{3})^{\frac{1}{2}}C]\sqrt{2}\beta'(\theta^0\to 2\pi^0), \quad (26)$$

where the proportionality constant is that of Eq. (25).

These γ's are related to γ_3' and γ_1' as follows:

$$\gamma_3'=(\tfrac{1}{3})^{\frac{1}{2}}\gamma'(\Lambda^0\to p+\pi^-)+(\tfrac{2}{3})^{\frac{1}{2}}\gamma'(\Lambda^0\to n+\pi^0), \quad (27)$$

and

$$\gamma_1'=(\tfrac{2}{3})^{\frac{1}{2}}\gamma'(\Lambda^0\to p+\pi^-)-(\tfrac{1}{3})^{\frac{1}{2}}\gamma'(\Lambda^0\to n+\pi^0).$$

Then, from Eqs. (25), (26), and (27), the ratio $\beta'(\theta^0\to 2\pi^0)/\beta'(\theta^0\to\pi^++\pi^-)$ determines the ratio γ_3'/γ_1', which in turn gives us the values of z [Eq. (13′)] and $R(\Lambda^0)$ [Eq. (12)]. This ratio $\beta'(\theta^0\to 2\pi^0)/\beta'(\theta^0\to\pi^++\pi^-)$ is zero if the θ^0 spin is odd. If the θ^0 spin is even, this ratio is related to β_2'/β_0' through the following equations:

$$\beta_2'=(\tfrac{2}{3})^{\frac{1}{2}}\beta'(\theta^0\to 2\pi^0)-(\tfrac{1}{3})^{\frac{1}{2}}\beta'(\theta^0\to\pi^++\pi^-),$$

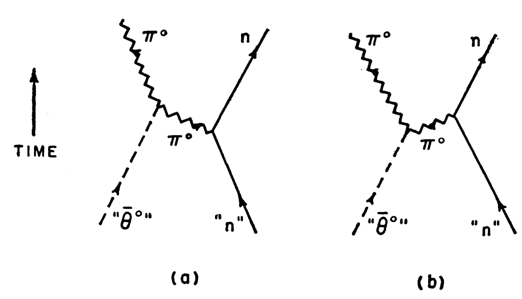

FIG. 2. Feynman diagrams of the Λ^0 decay into a neutron and a π^0. (a): A Feynman diagram of the stimulated decay of "$\bar{\theta}^0$" inside a Λ^0. (b): A Feynman diagram of the natural decay of "$\bar{\theta}^0$" inside a Λ^0.

[12] Proceedings of the Pisa Conference, June, 1955 (unpublished).

[13] The lifetime and branching ratio of $\bar{\theta}^0$ decay into two pions are the same with these of θ^0 decay. See M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[14] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[15] In an actual situation, a may be smaller but yet comparable with the range of the final state interaction. So it is better to consider a as a parameter rather than as a real physical quantity.

[16] The factor $\sqrt{2}$ before $\beta'(\theta^0\to 2\pi^0)$ is merely related to the definition of the decay amplitude for $\theta^0\to 2\pi^0$.

and (28)

$$\beta_0' = (\tfrac{1}{3})^{\frac{1}{2}}\beta'(\theta^0 \to 2\pi^0) + (\tfrac{2}{3})^{\frac{1}{2}}\beta'(\theta^0 \to \pi^+ + \pi^-).$$

Since β_2'/β_0' determines x [Eq. (17')] and $R(\theta^0)$, the $R(\Lambda^0)$ and $R(\theta^0)$ are related to each other through the common quantity $\beta'(\theta^0 \to 2\pi^0)/\beta'(\theta^0 \to \pi^+ + \pi^-)$. After combining Eqs. (12), (13'), (16), (17'), and (25)–(28), we obtain the following results.

Case (a).—The spin of θ^0 is even.

$$R(\theta^0) = \frac{2x^2 + 1 + 2\sqrt{2}x\cos(\delta_0 - \delta_2)}{x^2 + 2 - 2\sqrt{2}x\cos(\delta_0 - \delta_2)}, \tag{29}$$

with

$$x = A\sin(ka_\theta + \delta_2)/\sin(ka_\theta + \delta_0); \tag{30}$$

$$R(\Lambda_0) = \frac{2z^2 + 1 - 2\sqrt{2}z\cos(\eta_1 - \eta_3)}{z^2 + 2 + 2\sqrt{2}z\cos(\eta_1 - \eta_3)}, \tag{31}$$

with

$$z = -A\sin(ka_\Lambda + \eta_3)/\sin(ka_\Lambda + \eta_1); \tag{32}$$

and

$$R(\Sigma^+) = \frac{2y^2 + 1 - 2\sqrt{2}y\cos(\eta_1 - \eta_3)}{y^2 + 2 + 2\sqrt{2}y\cos(\eta_1 - \eta_3)}, \tag{33}$$

with

$$y = -A\sin(ka_\Sigma + \eta_3)/\sin(ka_\Sigma + \eta_1). \tag{34}$$

All these ratios are a function of A, where

$$A = -\gamma_3'/\gamma_1' = \beta_2'/\beta_0'. \tag{35}$$

Therefore, if we know one of these branching ratios $R(\theta^0)$, $R(\Lambda^0)$, and $R(\Sigma^+)$, and various phase shifts, η_3, η_1, δ_2, and δ_0, then we can determine A, which in turn gives the other two branching ratios.

In an extreme case, where $ka \gg \eta$, δ, and $|\eta_1 - \eta_3|$, $|\delta_0 - \delta_2| \ll 1$ for all the decays, we obtain the following relation:

$$R(\theta^0) = R(\Lambda^0) = R(\Sigma^+). \tag{36}$$

Case (b).—The spin of θ^0 is odd.

In this case, we have no adjustable parameter since

$$\beta'(\theta^0 \to 2\pi^0)/\beta'(\theta^0 \to \pi^+ + \pi^-) = 0, \tag{37}$$

and $R(\theta^0)$, $R(\Lambda^0)$, and $R(\Sigma^+)$ are given in terms of the various phase shifts:

$$R(\theta^0) = 0; \tag{38}$$

$$R(\Lambda^0) = \frac{2z^2 + 1 - 2\sqrt{2}z\cos(\eta_1 - \eta_3)}{z^2 + 2 + 2\sqrt{2}z\cos(\eta_1 - \eta_3)}, \tag{39}$$

with

$$z = (1/\sqrt{2})\sin(ka_\Lambda + \eta_3)/\sin(ka_\Lambda + \eta_1); \tag{40}$$

and

$$R(\Sigma^+) = \frac{2y^2 + 1 - 2\sqrt{2}y\cos(\eta_1 - \eta_3)}{y^2 + 2 + 2\sqrt{2}y\cos(\eta_1 - \eta_3)}, \tag{41}$$

with

$$y = (1/\sqrt{2})\sin(ka_\Sigma + \eta_3)/\sin(ka_\Sigma + \eta_1). \tag{42}$$

In an extreme case, $ka \gg \eta$, and $|\eta_1 - \eta_3| \ll 1$, we obtain

$$R(\theta^0) = R(\Lambda^0) = R(\Sigma^+) = 0. \tag{43}$$

So far we have not mentioned Σ^- decay. In this model, the lifetime of Σ^- decay is longer than that of Σ^+ decay, because the Σ^- contains only "$\bar{\theta}^-$" but not "$\bar{\theta}^0$." On the other hand, if there is an interaction which exchanges the charge of the $\bar{\theta}$ particle in the pion field of the nucleon, the ratio of the lifetimes will be of the order of g^2. Here g^2 is the square of the coupling constant of the charge-exchange $\bar{\theta} - \pi$ interaction.

IV. CONCLUSIONS

In the early part of this paper, we have studied how the Wigner time reversal principle gives restrictions on the branching ratios of hyperon and θ^0 decay. Tables I, II, and III give the maximum and minimum allowed values of these branching ratios evaluated for various possible spins and parities of the hyperons and θ^0 mesons. The experimental determination of $R(\theta^0)$ seems to be very important in deciding the spin of θ^0 without referring to a specific model.

In the latter part of this paper, we have adopted some specific models to find model-dependent restrictions on various branching ratios. In model (*A*), $R(\Lambda^0)$ and $R(\theta^0)$ are given by Eqs. (21) and (22), respectively, which will be subject to direct experimental verification. In model (*B*), the inter-relation among $R(\theta^0)$, $R(\Lambda^0)$, and $R(\Sigma^+)$ was deduced. Although we did not discuss this in detail because of the lack of experimental data on the phase shifts, δ_0 and δ_2, for the pion-pion scattering, this kind of relation will give us a check on any compound model of the unstable particles.