C. Decay of As^{72} to Ge^{72}

The decay of As^{72} has been studied by several investigators.[39–42] As^{72} decays primarily by positrons of end-point energies of 3.339 (19.3 percent), 2.498 (61.6 percent), 1.844 (12.1 percent), 0.669 (5.0 percent), and 0.271 (2.0 percent) Mev.[39] Gamma rays of energy 0.835 and 1.05 Mev were resolved. The $\log ft$ value (8.3) and the shape of the 3.339-Mev ground state positron branch indicated[39] that the transition was first forbidden ($\Delta J=\pm 2$, change of parity). A 2^- spin and parity was then implied for As^{72}, possibly accounted for by an $f_{5/2}$ proton and a $g_{9/2}$ neutron configuration.

We have started to investigate the decay of As^{72} using scintillation spectrometers. Although the results will be reported in more detail at a later date, it is worthwhile noting that the decay scheme is at least as complex as that of Ga^{72}, but more difficult to study because of the relatively lower intensity of the gamma rays above 0.84 Mev. From preliminary results it appears that most of levels observed in the decay of Ga^{72} are also populated in the decay of As^{72}. In addition a level at 2.90 Mev and apparently levels above 3.34 Mev are required. The 2.90-Mev level seems to participate in populating the 0.69-Mev level. At present no decisive information concerning the spins and parities of the excited levels of Ge^{72} can be obtained from the decay of As^{72}, beyond what is known from the decay of Ga^{72}.

[39] Mei, Mitchell, and Huddleston, Phys. Rev. 79, 19 (1950).
[40] P. H. Stoker and O. Ping Hok, Physica 19, 279 (1953).
[41] Mitchell, Jurney, and Ramsey, Phys. Rev. 71, 825 (1947).
[42] McCown, Woodward, and Pool, Phys. Rev. 74, 1315 (1948).

ACKNOWLEDGMENTS

We would like to thank Mr. Lloyd Chase for helping us with some of the experimental work and in particular with the analysis of the pair spectrometer results. We are grateful also to Professor M. W. Johns for permission to quote some of his unpublished results and to Dr. D. G. Ravenhall and Dr. D. H. Wilkinson for illuminating discussions.

PHYSICAL REVIEW VOLUME 101, NUMBER 1 JANUARY 1, 1956

Fission Theory and Semiempirical Mass Formula

TH. A. J. MARIS
Nobel Institute of Physics, Stockholm, Sweden
(Received July 11, 1955)

The fission theory of Bohr and Wheeler employs the semiempirical mass formula with the following constants: E_s(=surface energy)$=14A^{\frac{2}{3}}$ Mev; $x=E_{\text{Coulomb}}/2E_s=(1/47.8)(Z^2/A)$; nuclear radius=Coulomb radius$=1.47\times10^{-13}A^{\frac{1}{3}}$ cm. The experimental masses deviate systematically from the values calculated using this formula. In the present note it is shown that these differences may severely influence the results of the fission theory.

A reduction of the standard error in the mass formula from 8 to 2 mMU has been achieved by using the following constants: $E_s=17.8A^{\frac{2}{3}}$ Mev; Coulomb radius$=1.216\times10^{-13}A^{\frac{1}{3}}$ cm; $x=(1/50.1)(Z^2/A)$. The smaller x-value and Coulomb radius, in addition to possible shell effects in the two halves of the deformed nucleus, decrease the stability of a symmetric deformation of the nucleus.

SINCE the publication of the original liquid drop theory,[1] a considerable amount of new experimental data on exact masses has been obtained.[2,3] To fit these values more satisfactorily the constants in the semiempirical mass formula[4,5] have to be altered. This change affects the liquid drop theory of fission. With the revised values the critical form of the nucleus is more strongly deformed than hitherto assumed and tends more to asymmetry.

In the fission theory of Bohr and Wheeler the following constants are employed: E_s(=surface energy) $=14A^{\frac{2}{3}}$ Mev; $x=E_c/2E_s=(1/47.8)(Z^2/A)$ (E_c=Coulomb energy, Z=charge number, A=mass number); nuclear radius=Coulomb radius$=1.47\times10^{-13}A^{\frac{1}{3}}$ cm. As mentioned, the semiempirical mass formula in which these values are used,[5] results in large systematic deviations from the experimental masses.

To show the extent to which these errors may affect the results of the theory, we have plotted the energy necessary for the deformation of a U^{238}-nucleus into two touching spheres with charges proportional to their masses, as a function of the sphere masses (Fig. 1). From the liquid drop theory, with the original constants mentioned above, one finds curve I.[6] Curve II is calculated from the experimental mass values[3] of nuclei with the same mass numbers as the spheres. These masses have been corrected with the following formula

[1] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).
[2] Glass, Thompson, and Seaborg, J. Inorg. and Nucl. Chem. 1, 3 (1955).
[3] A. H. Wapstra, Isotopic Masses II, *IKO*, Amsterdam (unpublished).
[4] C. F. von Weizsäcker, Z. Physik 96, 431 (1935).
[5] E. Fermi, *Nuclear Physics* (The University of Chicago Press, Chicago, 1949).
[6] S. Frankel and N. Metropolis, Phys. Rev. 72, 914 (1947).

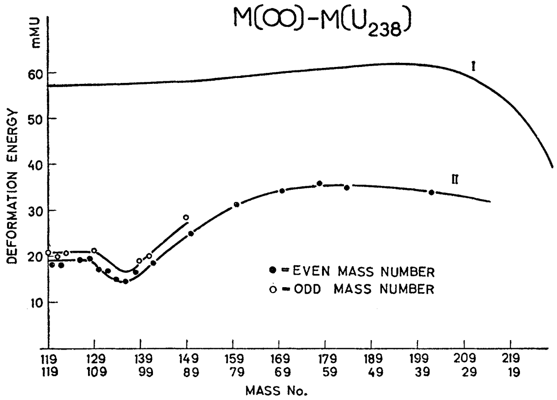

FIG. 1. Energy required to deform the U^{238} nucleus into two touching spheres. Curve I is calculated from fission theory with the usual constants; curve II is calculated from experimental masses.

that is a direct consequence of the Bohr-Wheeler form of the mass equation[1]:

$$M_1(Z_1,A)=M_2(Z_2,A)-\tfrac{1}{2}B_A(Z_2-Z_A)^2+\tfrac{1}{2}B_A(Z_1-Z_A)^2,$$

in which M_1, Z_1, A are the calculated mass, charge, and mass number of the sphere, and M_2, Z_2, A are the experimental mass, charge, and mass number of a nucleus, isobaric to the sphere. From the above-mentioned constants of the mass equation, it follows[2] that

$$B_A=0.166A^{-1}+0.001254A^{-\frac{1}{3}}$$

and

$$Z_A=A/(1.981+0.015A^{\frac{2}{3}}).$$

The same mutual Coulomb energy,

$$Z_1(92-Z_1)e^2/\{1.47\times10^{-13}[A_1^{\frac{1}{3}}+(238-A_1)^{\frac{1}{3}}]\},$$

of the two spheres as used for curve I has been added and the experimental mass of uranium[2] subtracted. (A similar function has been considered by Fong[7] in another connection).

The extrapolation from experimental data used in curve II is clearly much smaller than the one employed in curve I. Accordingly, the former should give better values for the deformation energy. The large difference between the two curves therefore casts some doubt on the quantitative results of the fission theory. The fact that curve I lies about 30 Mev higher than curve II can be understood by comparing the experimental masses with those calculated from the semiempirical mass formula.[8] The former are 5 Mev larger than the latter in the region of uranium, and about 10 Mev smaller for the medium nuclei.

[7] P. Fong, Phys. Rev. **89**, 332 (1953).
[8] Charles Noel Martin, *Nuclear Tables* (Gauthier-Villars, Paris, 1954).

A reduction of the standard error in the mass formula from 8 to 2 mMU has been achieved by using the following constants[9,10]: $E_s=17.8A^{\frac{2}{3}}$ Mev and a Coulomb radius of $1.216\times10^{-13}A^{\frac{1}{3}}$ cm, the last value being in accordance with other experimental results.[11]

From these values one finds, for U^{238}, $x=0.71$ and $E_s=683$ Mev compared to $x=0.74$ and $E_s=537$ Mev in the usual theory. As is well known, a smaller x-value corresponds to a stronger deformation and an increased tendency towards asymmetry of the critical form. In addition, a stronger "bottlenecking" of the critical form favors shell effects in both halves, which might increase the asymmetry tendency.[12,13] Furthermore, both experimental (Coulomb radius) and theoretical investigations[14,15] indicate that the proton surface in the nucleus is located "inside" of the neutron surface at a distance of roughly $0.1\times10^{-13}A^{\frac{1}{3}}$ cm. In the distorted nucleus, this last effect should be influenced by polarization. The resulting critical forms might be considerably different from those calculated for the original theory.[1,6] As a first, although unreliable, estimate, considering the effects of smaller x-values and Coulomb radii as additive, and neglecting polarization, the radius of the bottleneck of the neutron distribution would be smaller by about 15% and that of the protons less by 20–25% than hitherto assumed. These values follow from the calculated critical forms for the different x-values[6] and from an assumed location of the proton surface at a distance of $0.1\times10^{-13}A^{\frac{1}{3}}$ cm inside of the nuclear surface.

The minimum in curve II is an effect of shell structure of the nuclei whose masses were used in the calculation.[16] One should be careful in interpreting this minimum quantitatively because the real nucleus will never have the form of two touching spheres and the location of the minimum depends also on the individual numbers of protons and neutrons which are different for our hypothetical spheres and the actual nuclei from which we deduced their masses. On the other hand, as mentioned, the increase in deformation of the critical form would tend to favor shell effects in both halves of the distorted nucleus.

I am greatly indebted to the director of the Nobel Institute of Physics, Professor Manne Siegbahn, for the hospitality I have received at this institute and to the Swedish Academy of Sciences for a grant. I particularly wish to thank Dr. Stanley G. Thompson and Dr. W. J. Swiatecki for several helpful discussions and suggestions.

[9] A. Green and N. Engler, Phys. Rev. **91**, 40 (1953).
[10] A. Green, Phys. Rev. **95**, 1006 (1954).
[11] F. Bitter and H. Feshbach, Phys. Rev. **92**, 837 (1953).
[12] M. G. Mayer, Phys. Rev. **74**, 235 (1948).
[13] L. Meitner, Nature **165**, 561 (1950).
[14] M. H. Johnson and E. Teller, Phys. Rev. **93**, 357 (1954).
[15] K. Wildermuth, Z. Naturforsch. **9a**, 1047 (1954).
[16] A. H. Wapstra, Physica **18**, 83 (1952).