powers of the uniformly precessing part of the magnetization. Thus they modulate the dc field [and hence the natural frequency $\omega(\mathbf{k})$ of the spin wave of wave number $\mathbf{k}^{3,4}$], terms of nth order in the uniform precession giving a modulation rate n times the signal frequency ω. Thus the motion of the kth spin wave is not unlike that of a pendulum subject to a periodic vertical force: if the modulation is deep enough, instability can occur, most readily when one of the modulation frequencies equals twice the natural frequency of the pendulum. Thus the spin wave $\mathbf{k}$ is potentially unstable whenever $2\omega(\mathbf{k}) = n\omega$. The case previously discussed[5] is $n = 2$. The driving terms, being quadratic in the uniform precession, are then too small to give instability in the presence of losses, except when the dc field is approximately adjusted for resonance, $\omega = \omega(0)$. The condition $\omega(\mathbf{k}) = \omega = \omega(0)$ then gives the range of $\mathbf{k}$ for which instability may occur.[4]

The instability responsible for the subsidiary peak is $n = 1$, that is $2\omega(\mathbf{k}) = \omega$. It, too, is most readily excited at resonance $\omega = \omega(0)$; however, the modulation, now linear in the uniform precession, can be as large remote from resonance as the quadratic modulation is at resonance, and so instability will occur at comparable signal levels.

Detailed analysis now gives the following results: spin waves unstable for the least signal powers are so long that exchange effects are negligible. $\omega(\mathbf{k})$ then depends on $\mathbf{k}$ only through the angle, θ, between $\mathbf{k}$ and the magnetization direction. The dc field H_1, also involved in $\omega(\mathbf{k})$, is found in terms of θ from $2\omega(\mathbf{k}) = \omega$. Spin waves along θ will begin to grow when the signal field exceeds

$$h = \frac{\omega - \omega(0)}{4\pi\gamma M_0} \Delta H \cdot F, \qquad (1)$$

where M_0 is the saturation magnetization, γ the gyromagnetic ratio, $\omega(0)$ the resonance frequency in the field $H_1 = H_1(\theta)$, $\Delta H = 1/\gamma T_1$ the line width obtained from the relaxation time T_1,[3] and F a numerical factor depending on θ, sample shape, and $\omega/4\pi\gamma M_0$. $F = \infty$ at $\theta = 0$ and $\pi/2$, and achieves a minimum of order unity
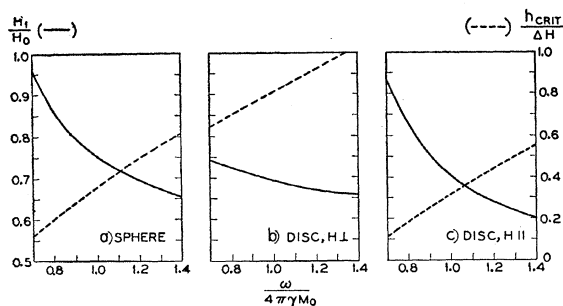

FIG. 1. Minimum critical signal field, and the location of the subsidiary peak for commonly found experimental conditions.

TABLE I. Ratio of field H_1 at subsidiary absorption to the field H_0 required for resonance at 9000 Mc/sec, and the corresponding theoretical threshold fields.

		H_1/H_0 from Theory	Experiment[a]	$\frac{h_{\rm crit}}{\Delta H}$ (theory)
MnZn ferrite disk	($H \perp$ disk)	0.72	0.72	0.732
$4\pi M_0 \sim 3880$ oe	($H \parallel$ disk)[b]	0.78	0.66	0.230
Ni-ferrite sphere $4\pi M_0 \sim 3320$ oe		0.83	0.73	0.232

[a] See reference 1.
[b] Circular polarization of the uniform precession has been assumed here as elsewhere. This is not correct for $H \parallel$ disk, but can hardly change H_1/H_0

between these two limits. h can be minimized with respect to θ, yielding the least signal field $h_{\rm crit}$ above which instability will occur, and a corresponding value of the dc field H_1, at which the subsidiary absorption will first be noticed. The results are shown in Fig. 1. If conditions are such that $2\omega(\mathbf{k}) = \omega = \omega(0)$ can be satisfied, $\omega - \omega(0)$ in (1) should be replaced by $\gamma\Delta H$, yielding a particularly small critical field.

In Table I the theoretical locations of the subsidiary peak are compared with the observations of Bloembergen and Wang. Agreement is better than 20% throughout. The corresponding theoretical values of $h_{\rm crit}$ are also stated.

The state of the medium at signal levels above critical has also been investigated and will be discussed in a future publication.

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[4] Clogston, Suhl, Walker, and Anderson, Phys. Rev. 101, 903 (1956).
[5] P. W. Anderson and H. Suhl, Phys. Rev. 100, 1789 (1955).

Anomalous V^0-Events as Three-Body Decays*

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"ANOMALOUS" V^0-particle events (decay processes which do not fit either of the schemes $\Lambda^0 \to p + \pi^- + 37$ Mev or $\theta^0 \to \pi^+ + \pi^- + 212$ Mev) have by now been observed in many cloud-chamber experiments. In terms of other known K-meson decay schemes, four possible interpretations suggest
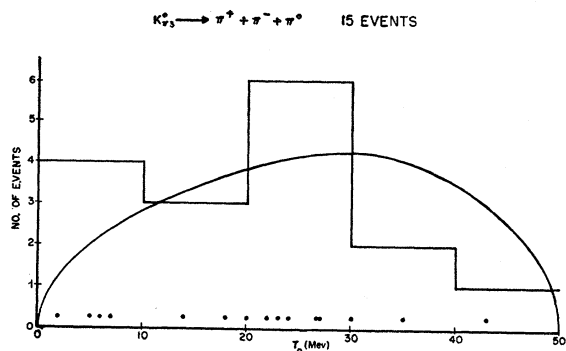
FIG. 1. "Phase space" energy distribution of the neutral pion in the $K_{\pi 3}{}^0 \rightarrow \pi^+ + \pi^- + \pi^0$ decay scheme as compared with experimental data.

FIG. 2. "Phase space" energy distribution of the neutrino in the $K_{\mu 3}{}^0 \rightarrow \mu^\pm + \pi^\mp + \nu$ decay scheme as compared with experimental data.

themselves:

$$\text{(a)} \quad K_{\pi 3}{}^0 \rightarrow \pi^+ + \pi^- + \pi^0 + 80 \text{ Mev,}$$

$$\text{(b)} \quad K_{\pi 2}{}^0 \rightarrow \pi^+ + \pi^- + \gamma + 212 \text{ Mev,}$$

$$\text{(c)} \quad K_{\mu 3}{}^0 \rightarrow \mu^\pm + \pi^\mp + \nu + 249 \text{ Mev,}$$

$$\text{(d)} \quad K_{e3}{}^0 \rightarrow e^\pm + \pi^\mp + \nu + 354 \text{ Mev.}$$

The indicated Q-values are based on the assumption that all the K mesons have mass 494 Mev. Schemes (c) and (d) are suggested as neutral counterparts of $K_{\mu 3}{}^\pm$ and $K_{e3}{}^\pm$. Scheme (b) has been suggested by Gell-Mann and Pais.[1]

From a knowledge of the laboratory momenta and included angle of the charged decay products, it is possible to calculate the kinetic energy T_0 of the neutral decay product in the rest system of the unstable particle. One finds

$$T_0 = [(M - m_0)^2 - (m_1 + m_2 + Q')^2]/2M,$$

where M, m_0, m_1, and m_2 are, respectively, the masses of the primary particle, the neutral decay product, and the charged decay products; Q' is the Q-value calculated in the usual way from the measurements on the charged secondaries on the assumption of two-body decay.

We have extracted from published data[2] some 27 "anomalous" V^0-events which are consistent with at least one of the above decay schemes. The identities of the charged secondaries are not experimentally established for any of these events; but if decay schemes (a) or (b) are assumed, the energy of the neutral decay product can be calculated without ambiguity. If scheme (c) is assumed, two values of T_0 are obtained—depending on which charged secondary is assumed to be the μ meson. The two values do not in general differ appreciably, however, and the selection has been a random one in each case. The same general procedure was used with scheme (d).

Of these 27 events 15 are kinematically consistent with scheme (a). In Fig. 1 the distribution in kinetic energy of the neutral particle for these events [all analyzed on the basis of scheme (a) for this purpose]
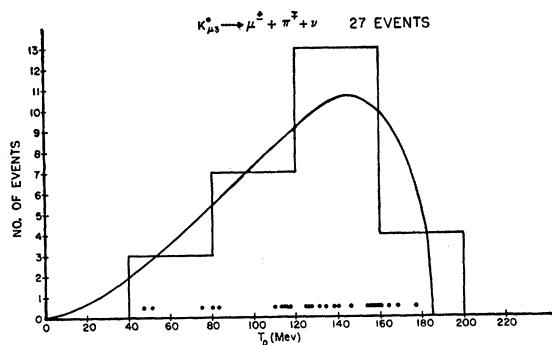
is compared with the phase space distribution. The latter is particularly relevant for scheme (a) if we assume it to be the neutral counterpart of the charged τ-meson decay, since the charged τ meson is now believed to have zero spin[3]; because of the fairly low Q-value one would expect the lowest orbital angular momentum state to dominate the distribution. The fit of the data to the predicted distribution suggests that not all the 15 events may be explained by scheme (a).

In Figs. 2 and 3, respectively, we plot the distributions obtained from all 27 events, analyzed in turn on the basis of schemes (b) and (c) together with the predicted phase space distributions. We have also compared the data with scheme (d) with essentially the same result as for scheme (c), except that the curve extends to higher energies. In this case there is more serious bias with respect to distinguishing these events from the ordinary θ^0 decays. We therefore do not plot these data.

There is of course some bias against seeing events in which Q' is very small, since in these cases the charged secondaries will often appear with a narrow included angle in the laboratory system. Events with very high Q'-values would be experimentally indistinguish-
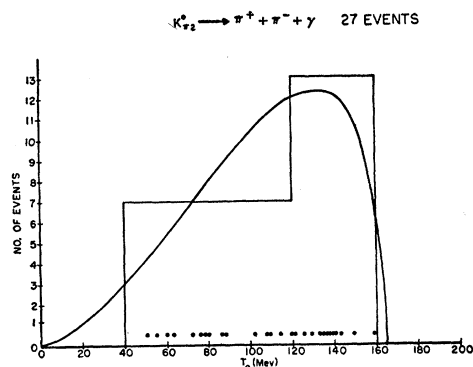

FIG. 3. "Phase space" energy distribution of the γ ray in the $K_{\pi 2}{}^0 \rightarrow \pi^+ + \pi^- + \gamma$ decay scheme as compared with experimental data.

able from ordinary θ^0 decay. However, we can think of no serious bias which operates against observation of those events where Q' has intermediate values (the major portion of the phase space distribution). All the theoretical curves are normalized to the area under the experimental curves.

It can be seen that the experimental distributions are in reasonable agreement with the phase space distributions, for either of the two schemes (b) and (c). We tentatively suggest that "anomalous" V^0-events can be largely explained in terms of these schemes along with an admixture of scheme (d). The latter could account for the occasional events involving large Q'-values > 212 Mev.

It would of course be very useful to determine the lifetime of "anomalous" V^0-particles, since a comparison here with the normal θ^0-particle lifetime would serve to clarify the question of how many distinct K-particles are required to explain all the presently known decay processes. At the present time the data are too meager for a reliable lifetime determination.

We wish to thank Professor Thompson and Professor Deutschmann for advance copies of their papers.

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[2] R. W. Thompson, in *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, to be published), Vol. III, Chap. 5, has included a list of most of the published anomalous V-particles. In addition, data have been taken from Arnold, Martin, and Wyld, Phys. Rev. **100**, 1545 (1955), and one event from Deutschmann, Cresti, Greening, Guerriaro, Loris, and Zago (to be published).
[3] Jay Orear (private communication); Roy Haddock, Phys. Rev. **100**, 1803 (A) (1955).