Quantum-Electrodynamical Fourth-Order Corrections for Triplet Fine Structure of Helium

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(Received October 17, 1955)

The quantum-electrodynamical fourth-order corrections for the intervals of the triplet fine structure of helium are calculated by taking into account the one-electron Lamb-shift for the self-spin-orbit coupling and the effect of the anomalous magnetic moment on the mutual spin-orbit and spin-spin coupling between two electrons. The values of the radial integrals are taken from the author's previous calculation. The calculated values of the corrections are within the range of experimental errors in the case of optical measurements but they amount to observable quantities in a microwave experiment.

THE fine structure of atomic levels has recently been measured[1] by the microwave method with an accuracy of 1 Mc/sec. To such an accuracy the theoretical interpretation should take into account the quantum-electrodynamical fourth-order correction. In what follows, the corrections for the triplet fine structure of helium will be estimated. For this purpose we have only to consider J-dependent terms. The largest level shift may be the Lamb shift of the S-level type. This shift is, however, common for all the fine structure components and gives no influence on the triplet intervals.

The spin-dependent Hamiltonian of the He atom consists of three parts, namely, the self-spin-orbit coupling, the mutual spin-orbit coupling, and the spin-spin coupling, as follows[2]:

$$\begin{aligned} H_{so}{}^{s} &= 2\mu^2\mathbf{S}_1\cdot(Zr_1^{-3}\mathbf{L}_1 - r_{12}^{-3}\mathbf{L}_{12}) \\ &\quad + 2\mu^2\mathbf{S}_2\cdot(Zr_2^{-3}\mathbf{L}_2 - r_{12}^{-3}\mathbf{L}_{21}), \\ H_{so}{}^{m} &= -4\mu^2 r_{12}^{-3}(\mathbf{S}_1\cdot\mathbf{L}_{21}+\mathbf{S}_2\cdot\mathbf{L}_{12}), \\ H_{ss} &= 4\mu^2(\mathbf{S}_1\cdot\nabla_1)(\mathbf{S}_2\cdot\nabla_2)r_{12}^{-1}, \end{aligned} \tag{1}$$

where μ is the Bohr magneton. Each term of $H_{so}{}^{s}$ represents the coupling of the spin of an electron with the orbital angular momentum of the same electron through the Coulomb field of the nucleus and the other electron. Each term of $H_{so}{}^{m}$ is the magnetic energy of the spin magnetic dipole of an electron in the magnetic field produced by the angular momentum of the other electron. This is not identical with the so-called spin-other-orbit interaction. This distinction is important in the present consideration. The last part is the magnetic interaction energy of two spin dipoles.

As is well known,[3] the fourth-order fractional correction for $H_{so}{}^{s}$ is α/π. The corrections for $H_{so}{}^{m}$ and H_{ss} can be obtained by considering the anomalous magnetic moment of the electron,[3] that is, by changing $\mathbf{S}_i$ into $(\alpha/2\pi)\mathbf{S}_i$. Thus we have $(\alpha/\pi)(H_{so}{}^{s}+H_{so}{}^{m}/2+H_{ss})$ as the fourth-order correction for (1). That this consideration is correct can be confirmed by the calculation of Fulton and Martin.[4] They showed that there is another spin-dependent correction of $\mathbf{S}_1\cdot\mathbf{S}_2\delta(\mathbf{x}_{12})$ type. Such a term gives no influence on our problem because the expectation value of $\mathbf{S}_1\cdot\mathbf{S}_2$ vanishes in the singlet state and that of $\delta(\mathbf{x}_{12})$ vanishes too in the triplet state in which the orbital function of the atom is antisymmetric with respect to two electrons.

The matrix elements of $H_{so}{}^{s}$, $H_{so}{}^{m}$, and H_{ss} can easily be evaluated according to the method previously explained.[5] From these matrix elements we have the triplet intervals as follows:

$$\begin{aligned} E(^3L_{L+1})-E(^3L_L) &= (L+1)[Z\zeta^N-3\zeta^e+6(2L-1)\eta] \\ &\quad +(\alpha/\pi)(L+1)[Z\zeta^N-2\zeta^e+6(2L-1)\eta], \\ E(^3L_L)-E(^3L_{L-1}) &= L[Z\zeta^N-3\zeta^e-6(2L+3)\eta] \\ &\quad +(\alpha/\pi)L[Z\zeta^N-2\zeta^e-6(2L+3)\eta], \end{aligned} \tag{2}$$

where

$$\begin{aligned} \zeta^N &= \mu^2L^{-1}\int\int\psi_{nLL}{}^*(r_1^{-3}L_{1z}+r_2^{-3}L_{2z})\psi_{nLL}dv_1dv_2, \\ \zeta^e &= \mu^2L^{-1}\int\int\psi_{nLL}{}^*r_{12}^{-3}(L_{12z}+L_{21z})\psi_{nLL}dv_1dv_2, \\ \eta &= \mu^2[2L(2L-1)]^{-1} \\ &\quad \times\int\int|\psi_{nLL}|^2(\partial^2/\partial z_1\partial z_2)r_{12}^{-1}dv_1dv_2, \end{aligned} \tag{3}$$

TABLE I. Calculated corrections and observed intervals of He in Mc/sec.

	1s2p Obs. interval[a]	1s2p Correction	1s3p Obs. interval	1s3p Correction
$^3P_1-{}^3P_0$	29620±30	52	7940±600[a]	12
$^3P_2-{}^3P_1$	2290±20	−21	658±1[b]	−4
	1s3d		1s4d	
$^3D_2-{}^3D_1$	1334±105	2	...	0.8
$^3D_3-{}^3D_2$	112±60	−1	...	−0.5

[a] See reference 7.
[b] See reference 1.

[1] T. H. Maiman and W. E. Lamb, Phys. Rev. 98, 1194(A) (1955).
[2] G. Araki and S. Huzinaga, Progr. Theoret. Phys. 6, 673 (1951).
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and ψ_{nLM} denotes the triplet orbital function. The values of the corrections calculated by adopting the previously evaluated approximate values of these parameters[6] are shown in Table I. They may be a little too small because the calculated intervals were smaller than observed ones, but they can indicate an order of magnitude. The observed values[1,7] of the intervals are shown in the same table. The corrections are in the range of experimental errors in the case of optical measurements while the microwave experiment will permit observation of the corrections. The previous calculation of the intervals[6] is too rough to test the correction terms even with the accurate data. We should have vastly improved orbital functions for the He excited states in order to deduce the fourth-order correction from the accurate observations.

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PHYSICAL REVIEW VOLUME 101, NUMBER 4 FEBRUARY 15, 1956

Classical Maxwell Theory with Finite-Particle Sources

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(Received June 2, 1955)

A Lorentz-invariant finite-particle model is introduced into the Maxwell theory by extending the space from space-time to all (time-like) space-time spheres. The properties of the model are examined in the classical theory as a preliminary to the quantized case. The space-time sphere radius λ is the parameter of finiteness; it has the effect of smearing point particles into bell-shaped bounded distributions which go over into the δ-function point-particle distributions in the limit $\lambda=0$. The smeared particles give rise to fields in which the Coulomb infinity no longer exists. It is shown that the finite-particle 4-current has various indispensable formal properties: that charge is conserved; and that, in interaction with its field, momentum and energy are conserved, the integrals representing the electromagnetic self-energy and self-force being convergent for $\lambda\neq 0$. This replacement of point by finite particles results in corrections to calculations which are probably negligible where the classical theory is valid, but which might be appreciable in the quantum domain at distances comparable to λ.

1. INTRODUCTION

CLASSICAL field theories suffer from infinities due to the use of a point model of the particle sources of the fields.[1] These same infinities carry over, multiplied in number and variety, into quantum field theories,[2] (which suffer as well from other infinities of a strictly quantum-mechanical nature). What is needed to eliminate this type of infinity is a finite-particle model. Moreover, it is not unreasonable to suppose that a particle model which eliminated this kind of infinity from a classical theory would do the same in a quantized theory built from it by the correspondence principle, especially if the finite-particle model were a kinematical (i.e., geometrical) element of the theory, independent of whether the classical or quantum interpretation of the fields were used. Accordingly, the study of a finite-particle model in the classical theory should serve as a useful preliminary to its eventual introduction into the quantized theory. That is the spirit in which a finite-particle model in the classical Maxwell theory is examined in this paper.

The next question is, what sort of a model shall it be? The finite-sphere model runs into group-theoretical troubles.[3] Moreover, the idea that an elementary particle has a definite volume and boundary in 3-dimensional space seems to be interpreting the phrase "finite particle" in too literal and naive a sense. Another method of avoiding the infinities is the admixture of unphysical elements like advanced fields,[4] which, besides defying causality, leads to unphysical behavior.[5] Yet, undoubtedly, elementary particles are finite in some sense. One might demand of a finite-particle model, discarding some of the prejudices carried over from macroscopic intuition, at least the following: that there be a parameter of finiteness λ which acts analytically as a cutoff in formerly infinite expressions; that the model defined by λ be meaningful against the groups employed, i.e., (at least) Lorentz-covariant; and finally, the demand of simplicity, that λ have a natural connection with, or meaning relative to, spacetime, that it remain not forever an *ad hoc* and geometrically inexplicable element in the theory. One could add to these the stronger demand that λ admit interpretation in some end formulas as the linear dimension of a finite

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[5] For yet other attempts, see M. Born and L. Infeld, Proc. Roy Soc. (London) **142**, 410 (1934); **144**, 425 (1934); **147**, 522 (1934); **150**, 141 (1935); R. Feynman, Phys. Rev. **74**, 939 (1948).