Coupling of Nucleon Orbitals*

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The coupling of nucleon orbitals through the mediation of surface waves has been treated by the intermediate coupling procedure, considering only states with 0, 1, and 2 surfons. A single-nucleon description appropriate for $_{14}Si_{15}$ and $_{15}P_{14}$, and a three-nucleon description with $T=3/2$ appropriate for $_{14}Si_{17}$ and $_{17}Cl_{20}$ (3 proton holes) are considered. $2s_{1/2}$ and $1d_{3/2}$ orbitals are coupled in this mass region. $(2s_{1/2})^2(1d_{3/2})$, $(2s_{1/2})(1d_{3/2})^2$, and $(1d_{3/2})^3$ states are coupled in the three-nucleon systems. $M1$ transition matrix elements between $I=3/2+$ and $I=1/2+$ states are small but not l-forbidden as they are for pure $1d_{3/2}\rightarrow 2s_{1/2}$ transitions. The l-forbidden $1/2+\rightarrow 3/2+$ Gamow-Teller matrix element in the decay $_{15}P_{14}\rightarrow{}_{14}Si_{15}$ has the value 0.027 (experimental value 0.040). The correction to the $\mu_{1/2}$ Schmidt moment in $_{14}Si_{15}$ is about one-half the experimental deviation; while the correction to the $\mu_{3/2}$ moment for $_{17}Cl_{20}$ is 0.788, compared to 0.552 observed. Similar calculations for three-nucleon systems with $T=1/2$ yield results applicable to $_{15}P_{16}$ and $_{18}A_{19}$. The coupling of $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, and $3s_{1/2}$ proton orbitals in $_{51}Sb_{70}$ and $_{51}Sb_{72}$ is treated by a single-nucleon description. The electric quadrupole moments of both $_{51}Sb_{70}$ and $_{51}Sb_{72}$ are accounted for and an appreciable $M1$ matrix element is found for the "l-forbidden" transition $5/2+\rightarrow 7/2+$ in $_{51}Sb_{72}$.

1. INTRODUCTION

COUPLING between nucleon orbitals of the same parity, differing by no more than two units of angular momentum, is provided through the mediation of collective surface oscillations of a spheroidal nuclear core. The Hamiltonian for the coupled system is[1,2]

$$H=H_p+H_{so}+H_{\rm int}, \tag{1}$$

where

$$H_p=-(\hbar^2/2M)\textstyle\sum_p \nabla_p^2+\sum_p V(r_p), \tag{1a}$$

$$H_{so}=(\hbar\omega/2)\sum_{\mu=-2}^{+2}(b_\mu^* b_\mu+b_\mu b_\mu^*), \tag{1b}$$

$$H_{\rm int}=-(\hbar/2B\omega)^{1/2}DR\sum_{\mu,p}[b_\mu+(-1)^\mu b_{-\mu}^*]\times Y_{2\mu}(\theta_p,\phi_p)\delta(R-r_p). \tag{1c}$$

$V(r_p)$ is assumed to represent a rectangular potential well of depth D. The summation p is over all nucleons in unfilled shells; b and b^* are destruction and creation operators for n_μ, the occupation number of surfons having z-component of angular momentum equal to μ:

$$\begin{aligned} b^*\delta_{n'}(n)&=(n'-1)^{1/2}\delta_{n'+1}(n),\\ b\delta_{n'}(n)&=(n')^{1/2}\delta_{n'-1}(n). \end{aligned} \tag{2}$$

B is a moment of inertia parameter of the core, and $\omega=(C/B)^{1/2}$, where C is a function of Coulomb energy and surface tension measuring the rigidity of the nuclear surface.

The eigenfunctions of H_p+H_{s0} are

$$\Phi(\gamma\nu JK;II_z;TT_3)=\sum_{J_z+K_z=I_z}U(\gamma JJ_zTT_3)\times V(\nu KK_z)(JJ_zKK_z|II_z). \tag{4}$$

J, K, and I are the angular momenta of the nucleons in unfilled shells, the nuclear core, and of the coupled system, respectively. T is the total isobaric spin, γ represents the type of configuration of the nucleons is unfilled shells (i.e., three nucleons in $1d_{3/2}$ orbitals), and ν is the number of quanta of surface oscillation. $V(\nu KK_z)$ represents the state of the core. Explicit formulas for the $V(\nu KK_z)$ are given by Feenberg.[1] In the present investigation, an intermediate coupling wave function is employed. The interaction Hamiltonian couples a linear superposition of the functions given by Eq. (4):

$$\Psi(II_zTT_3)=\sum_{\zeta\nu}p_{\zeta\nu}\Phi(\gamma\nu JK;II_z;TT_3), \tag{5}$$

where $\zeta\nu$ represents the quantum numbers $\gamma\nu JK$. The amplitude coefficients $p_{\zeta\nu}$ are determined to minimize the total energy. The sum is taken over all γJK coupled by $H_{\rm int}$, but the values of ν are limited to 0, 1, and 2. If the determined amplitudes indicate rapid convergence of the wave function with increasing ν, this procedure is justified.

2. GENERAL RELATIONS

Matrix elements of the interaction Hamiltonian are computed by the Racah formalism[3]:

$$(\gamma\nu JK;II;TT|H_{\rm int}|\gamma'\nu'J'K';II;TT)=-\hbar\omega\left(\frac{16\pi}{5}P\right)^{1/2}W(JKJ'K';I2)(\nu K\|Z^2\|\nu'K')\times(-1)^{J-K'-I}(\gamma JT\|\textstyle\sum_p Y^2(\theta_p,\phi_p)\|\gamma'J'T), \tag{6}$$

where

$$(\nu K\|Z^2\|\nu'K')=(\nu KK|b_{K-K'}^*+(-1)^{K-K'}b_{K'-K}|\nu'K'K')f(K,K'), \tag{7}$$

$$(\gamma JT\|\textstyle\sum_p Y^2(\theta_p,\phi_p)\|\gamma'J'T)=(\gamma JJTT|\sum_p Y_{2,(J-J')}(\theta_p,\phi_p)|\gamma'J'J'TT)f(J,J'), \tag{8}$$

$$f(A,B)=\left[\frac{(A+B+3)!}{(2A)!}\frac{(A+B-2)!}{(2B)!}\right]^{1/2}. \tag{9}$$

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[1] E. Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, 1955).

[2] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

[3] G. Racah, Phys. Rev. **63**, 367 (1943).

TABLE I. Reduced matrix elements of Z^2.

ν	K	ν'	K'	$(\nu K\|\|Z^2\|\|\nu' K')$
1	2	0	0	$5^{1/2}$
2	0	1	2	$2^{1/2}$
2	2	1	2	$-(10)^{1/2}$
2	4	1	2	$(18)^{1/2}$

$W(JKJ'K;I2)$ is the Racah function, which has been tabulated by Biedenharn.[4]

The parameter P measures the strength of the coupling and is given by

$$P=\frac{5k^2\hbar}{32\pi(BC)^{1/2}}\Big/(\hbar\omega)^2, \tag{10}$$

where k is the radial factor in the matrix element of the Rainwater interaction operator, equal to about 40 Mev.[5-7]

The total energy of the system is

$$E=\int\Psi^*H\Psi=\sum_{\zeta\nu}\sum_{\zeta'\nu'}H_{\zeta\nu\zeta'\nu'}p_{\zeta\nu}p_{\zeta'\nu'}, \tag{11}$$

subject to the normalization condition:

$$\sum_{\zeta\nu}p_{\zeta\nu}^2=1. \tag{11a}$$

Letting $(\zeta\nu|H_{\rm int}|\zeta'\nu')=H_{\zeta\nu\zeta'\nu'}$, the relations $\partial E/\partial p_{\zeta\nu}=0$ yield:

$$\sum_{\zeta'\nu'}(H_{\zeta\nu\zeta'\nu'}-E\delta_{\zeta\nu\zeta'\nu'})p_{\zeta'\nu'}=0. \tag{12}$$

Since H has nonvanishing off-diagonal matrix elements only between states which differ by one surfon, and we consider only ν equal to 0, 1, and 2, it is convenient to eliminate the 0- and 2-surfon amplitudes from Eqs. (12) and obtain a form involving only the amplitudes of 1-quantum components:

$$(E-H_{\zeta 1\zeta 1})p_{\zeta 1}=\sum_{\pi 1}\Big[\sum_{\chi 0}(E-H_{\chi 0\chi 0})^{-1}H_{\zeta 1\chi 0}H_{\chi 0\pi 1}p_{\pi 1}+\sum_{\chi 2}(E-H_{\chi 2\chi 2})^{-1}H_{\zeta 1\chi 2}H_{\chi 2\pi 1}p_{\pi 1}\Big]. \tag{13}$$

Since several of the determinants to be solved are quite formidable a simple method of evaluation was developed to make certain that one has the lowest root E. One inspects Eqs. (13) to see which of the $p_{\zeta 1}$ is expected to have the greatest magnitude. One then sets all the other amplitudes of one-surfon components equal to zero, and easily finds the lowest root of the resulting cubic equation in E. One then lets the second largest $p_{\zeta 1}$ also be different from zero and solves the resulting sixth-order equation, guided by the fact that the new root must be lower than the first. This procedure is repeated until all of the amplitudes are included. Obviously, the value of E must be lower for each successive iteration. When the $p_{\zeta 1}$ were included in the order of decreasing magnitude, only one new root was found in the calculations to be lower than the root derived in the preceding iteration. Consequently, the lowest root is finally obtained.

3. COUPLING OF $2s_{\frac{1}{2}}$ AND $1d_{\frac{3}{2}}$ ORBITALS

Coupling between $2s_{1/2}$ and $1d_{3/2}$ orbitals is expected in the mass range $29\leqslant A\leqslant 39$. The energy difference

$$\Delta=E(1d_{3/2})-E(2s_{1/2})\cong 1\text{ Mev} \tag{14}$$

is neglected in the calculations since it is substantially smaller than $\hbar\omega$ in this mass range ($\hbar\omega\cong 5$ Mev).

A single-nucleon description is appropriate for $_{14}Si_{15}$ and $_{15}P_{14}$. This problem has been treated by Feenberg,[1] and will be summarized here. Reduced matrix elements of Z^2 needed in all applications involving no more than two surfons are displayed in Table I. Only two reduced matrix elements of Y^2 are needed for a one-nucleon description:

$$(2s_{1/2}\|Y^2\|1d_{3/2})=(1d_{3/2}\|Y^2\|1d_{3/2})=-(1/\pi)^{1/2}. \tag{15}$$

The amplitude coefficients for the $I=1/2$ and $I=3/2$ levels are given in Tables II and III. The value of P in this mass range is taken to be 0.5. The energy displacements of the two levels, measured from the level with zero surfons, coincide only in the approximation $\Delta=0$. Convergence is quite satisfactory, the two-surfon components amounting to only 4.1% of the total wave function.

Magnetic moments for the $I=1/2$ state are displaced from the Schmidt $s_{1/2}$ line by an amount:

$$\begin{aligned}\mu-\mu_{1/2}&=-(p_{1\frac{3}{2}}^2-p_{2\frac{3}{2}}^2)(\mu_{1/2}+\tfrac{1}{3}\mu_{3/2}-g_c)\\&=-0.826\text{ (odd }Z)\\&=+0.656\text{ (odd }N),\end{aligned} \tag{16}$$

TABLE II. Amplitude coefficients for a single-nucleon description with $I=3/2$, $E=-0.622\hbar\omega$. The asterisk indicates that in order to obtain the proper phase of p, one should take the negative square root of p^2.

ν	K	j	p^2
0	0	3/2	0.645
1	2	3/2	0.157*
1	2	1/2	0.157*
2	0	3/2	0.015
2	2	3/2	0.013
2	2	1/2	0.013

TABLE III. Amplitude coefficients for a single nucleon description with $I=1/2$, $E=-0.622\hbar\omega$. The asteriks indicates that in order to obtain the proper phase of p, one should take the negative square root of p^2.

ν	K	j	p^2
0	0	1/2	0.645
1	2	3/2	0.314*
2	0	1/2	0.015
2	2	3/2	0.026

[4] L. C. Biedenharn, Oak Ridge National Laboratory Report ORNL-1098, 1952 (unpublished).
[5] J. Rainwater, Phys. Rev. **79**, 432 (1950).
[6] E. Feenberg and K. C. Hammack, Phys. Rev. **81**, 285 (1951).
[7] S. Gallone and C. Salvetti, Phys. Rev. **84**, 1064 (1951).

where $g_c=Z/A$ is the gyromagnetic ratio of the core and the amplitudes are labeled with νJK. The odd-N value is to be compared with an experimental deviation of 1.38 for $_{14}Si_{15}$.

$_{15}P_{14}$ has been observed[8] to decay into the first four levels of $_{14}Si_{15}$, as shown in Fig. 1. The value of the G-T matrix element for the image transition to the ground state is given by

$$\left|\int\sigma\right|^2_{\frac{1}{2}+\to\frac{1}{2}+}=3[p_{0\frac{1}{2}0}{}^2+p_{2\frac{1}{2}0}{}^2+\tfrac{1}{5}(p_{1\frac{3}{2}2}{}^2+p_{2\frac{3}{2}2}{}^2)]^2=1.57, \quad (17)$$

a substantial reduction from the value 3 obtained by assuming these levels to be pure $2s_{1/2}$ states, but still too large to fit the experimental value of about 0.2. The beta transition $s_{1/2}\to d_{3/2}$ is l-forbidden. With the mixture of orbitals provided by the surface waves one obtains the following:

$$\left|\int\sigma\right|^2_{\frac{1}{2}+\to\frac{3}{2}+}=0.027 \quad (18)$$

compared with an experimental value of 0.040. The electromagnetic transition $3/2+\to 1/2+$ in $_{14}Si_{15}$ is predicted to go very slowly by the simple shell model, the magnetic dipole transitions being l-forbidden and the electric quadrupole transition extremely slow for the odd neutron. Surface coupling provides the $M1$ matrix element:

$$\sum_M|(\tfrac{3}{2}M|\mu_z|\tfrac{1}{2}M)|^2=\frac{50}{27}\left|\int\sigma\right|^2_{\frac{1}{2}+\to\frac{3}{2}+}(\mu_{\frac{3}{2}}-\tfrac{3}{2}g_c)^2=0.016, \quad (19)$$

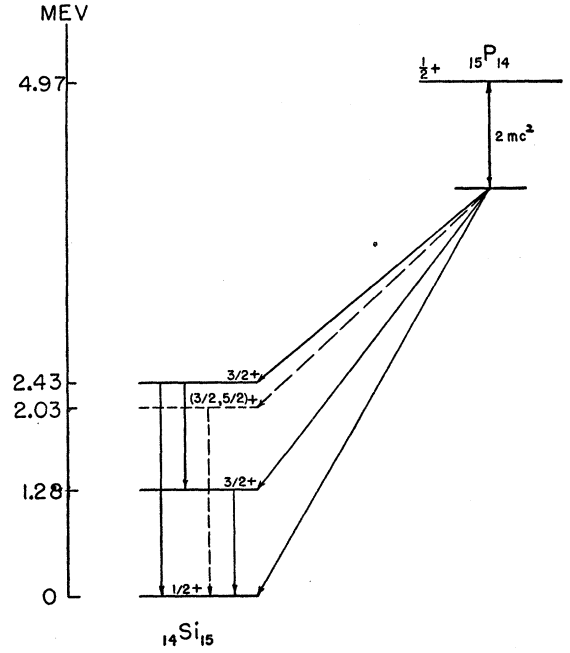

FIG. 1. Decay scheme of $_{15}P_{14}\to{}_{14}Si_{15}$.

[8] Roderick, Lonsjo, and Meyerhof, Phys. Rev. **97**, 97 (1955).

TABLE IV. Reduced matrix elements of $\Sigma_p\ Y^2(\theta_p,\phi_p)$ for three nucleons with $T=3/2$.

γ	J	γ'	J'	$(\gamma J\|\Sigma_p\ Y^2(\theta_p,\phi_p)\|\gamma' J')$
s^2d	3/2	sd^2	1/2	$-(1/2\pi)^{1/2}$
s^2d	3/2	sd^2	3/2	$+(2/5\pi)^{1/2}$
s^2d	3/2	sd^2	5/2	$+(21/10\pi)^{1/2}$
s^2d	3/2	s^2d	3/2	$-(1/\pi)^{1/2}$
sd^2	1/2	sd^2	5/2	$-(6/5\pi)^{1/2}$
sd^2	1/2	sd^2	3/2	$-(4/5\pi)^{1/2}$
sd^2	3/2	sd^2	3/2	0
sd^2	3/2	sd^2	5/2	0
sd^2	5/2	sd^2	5/2	0
d^3	3/2	d^3	3/2	$-(1/\pi)^{1/2}$
d^3	3/2	sd^2	1/2	$+(1/2\pi)^{1/2}$
d^3	3/2	sd^2	3/2	$+(2/5\pi)^{1/2}$
d^3	3/2	sd^2	5/2	$+(21/10\pi)^{1/2}$

which yields a mean life:

$$1/\tau_{M1}=5.37\times10^{10}\ \text{sec}^{-1}. \quad (20)$$

An appreciable $E2$ transition is provided by the distorted core. The necessary matrix elements are:

$$\begin{aligned}&A(1/2;00,12)=-(2/5)^{1/2},\ A(1/2;20,12)\\&\quad=-(4/5)^{1/2},\ A(3/2;12,22)=-(7/25)^{1/2},\\&A(3/2;12,20)=(2/25)^{1/2},\ A(3/2;22,12)\\&\quad=-(7/25)^{1/2},\ A(3/2;12,20)=(1/5)^{1/2},\end{aligned} \quad (21)$$

where

$$A(j;\nu K,\nu'K')=(\nu jK;1/2\ 1/2|b_0+b_0{}^*|\nu' jK';3/2\ 1/2). \quad (22)$$

The mean life for the $E2$ transition is then

$$1/\tau_{E2}=1.06\times10^{11}\ \text{sec}^{-1}. \quad (23)$$

Explanations of the second and third excited states of $_{14}Si_{15}$ are not lacking. The observed level order is given by the present treatment, but the level spacing derived is too large by a factor of four. Furthermore, the convergence of the wave function is poor for these excited states. This indicates that holes in the $1d_{5/2}$ shell and nuclear forces play an important role in the highly excited states, and should be included in that part of the calculation.

A three-nucleon treatment with $T=3/2$ is applicable to the three odd neutrons in $_{14}Si_{17}$ and the three proton holes in $_{17}Cl_{20}$. Here $(2s_{1/2})^2(1d_{3/2})$, $(2s_{1/2})(1d_{3/2})^2$, and $(1d_{3/2})^3$ configurations are coupled by the surface waves. The three-nucleon wave functions $U(\gamma JJ_z)$ may be expressed as a linear combination of the functions:

$$(j_1m_1,j_2m_2,j_3m_3)=(1/3!)^{1/2}|u(j_1m_1)u(j_2m_2)u(j_3m_3)|. \quad (24)$$

Explicitly, the needed functions are:

$$U(s^2d\ 3/2\ 3/2)=(1/2\ 1/2,\ 1/2\ -1/2,\ 3/2\ 3/2), \quad (25a)$$

$$U(sd^2\ 5/2\ 5/2)=(1/2\ 1/2,\ 3/2\ 3/2,\ 3/2\ 1/2), \quad (25b)$$

$$U(sd^2\ 3/2\ 3/2)=(1/5)^{1/2}[(1/2\ 1/2,\ 3/2\ 3/2,\ 3/2\ -1/2)-2(1/2\ -1/2,\ 3/2\ 3/2,\ 3/2\ 1/2)], \quad (25c)$$

TABLE V. Amplitude coefficients for three nucleons with $T=3/2$ and $I=3/2$, $E=-1.423\hbar\omega$. The asterisk indicates that in order to obtain the proper phase of p one should take the negative square root of p^2.

ν	K	γ	J	p^2
0	0	s^2d	3/2	0.228*
0	0	sd^2	3/2	0.026
0	0	d^3	3/2	0.251*
1	2	s^2d	3/2	0.061*
1	2	sd^2	1/2	0.002*
1	2	sd^2	3/2	0.033*
1	2	sd^2	5/2	0.253
1	2	d^3	3/2	0.061
2	0	s^2d	3/2	0.016*
2	0	sd^2	3/2	0.002*
2	0	d^3	3/2	0.017*
2	2	s^2d	3/2	0.013*
2	2	sd^2	1/2	0.001
2	2	sd^2	3/2	0.001*
2	2	sd^2	5/2	0.018
2	2	d^3	3/2	0.011*
2	4	sd^2	5/2	0.010

TABLE VI. Amplitude coefficients for three nucleons with $T=3/2$ and $I=1/2$, $E=-1.350\hbar\omega$. The asterisk indicates that in order to obtain the proper phase of p one should take the negative square root of p^2.

ν	K	γ	J	p^2
0	0	sd^2	1/2	0.500
1	2	s^2d	3/2	0.081
1	2	sd^2	3/2	0.120*
1	2	sd^2	5/2	0.180
1	2	d^3	3/2	0.042*
2	0	sd^2	1/2	0.031
2	2	s^2d	3/2	0.004
2	2	sd^2	3/2	0.001
2	2	sd^2	5/2	0.001*
2	2	d^3	3/2	0.037

$$U(sd^2\ 1/2\ 1/2)=(1/2)^{1/2}[(1/2\ 1/2,\ 3/2\ 1/2,\ 3/2\ -1/2)-(1/2\ 1/2,\ 3/2\ 3/2,\ 3/2\ -3/2)], \quad (25d)$$

$$U(d^3\ 3/2\ 3/2)=(3/2\ 3/2,\ 3/2\ 1/2,\ 3/2\ -1/2). \quad (25e)$$

Reduced matrix elements of $\sum_p Y^2(\theta_p,\phi_p)$ for these functions are given in Table IV.

Amplitude coefficients for the $I=3/2$ and $I=1/2$ states are given in Tables V and VI. The signs of these coefficients are given for a three-particle system. To obtain the coefficients for three holes one need only reverse the sign of the one-surfon components. The energy displacements indicate a ground state spin equal to 3/2, with a low first excited state of spin 1/2. This corresponds to the experimental level order in both $_{14}Si_{17}$ and $_{17}Cl_{20}$. Magnetic moments of the ground states are displaced from the Schmidt lines by

$$\mu-\mu_{3/2}=0.788 \text{ for } {}_{17}Cl_{20}$$
$$=-0.497 \text{ for } {}_{14}Si_{17}. \quad (26)$$

Matrix elements needed for this calculation are given in Table VII. The experimental deviation in $_{17}Cl_{20}$ is 0.552, quite large compared to the Schmidt value $\mu_{3/2}=0.124$ for an odd proton. The magnetic moment of $_{14}Si_{17}$ has not been measured, but the deviation is in the expected direction compared with other $d_{3/2}$ neutron states where the present description is not complete (for $_{16}S_{17}$ the deviation is about -0.6). The electric quadrupole moment of $_{14}Si_{17}$ in the ground state is given entirely by the surface oscillations:

$$(Q/R^2)_{s.o.}=-0.921. \quad (27)$$

The matrix elements needed to compute Eq. (27) have been tabulated by Feenberg.[9] If the three odd nucleons are protons, one has the small correction

$$(Q/R^2)_p=-0.062. \quad (28)$$

The derived moments are too large compared with the experimental values

$$(Q/R^2)_{\text{exp}}=-0.26 \text{ for } {}_{17}Cl_{20}, \quad (29)$$

$$(Q/R^2)_{\text{exp}}=-0.36 \text{ for } {}_{16}S_{17}. \quad (30)$$

The electromagnetic transition $1/2+\rightarrow 3/2+$ may go by $M1$ or $E2$ radiation. The $M1$ transition is l-forbidden in the simple shell-model interpretation $[(2s_{1/2})\times(1d_{3/2})^2\rightarrow(2s_{1/2})^2(1d_{3/2})]$, but the mixture of nucleon orbitals provided by the surface waves yield the nonvanishing matrix elements:

$$\sum_M|(3/2\ M|\mu_z|1/2\ M)|^2=0.028 \text{ for } {}_{14}Si_{17}, \quad (31)$$
$$=0.190 \text{ for } {}_{17}Cl_{20}. \quad (32)$$

TABLE VII. Matrix elements of μ_z for $T=3/2$, $I=3/2$.

γ	J	K	J'	$(\gamma\nu JK;\ 3/2\ 3/2;\ 3/2\ 3/2\|\mu_z\|\gamma\nu J'K;\ 3/2\ 3/2;\ 3/2\ 3/2)$
s^2d	3/2	0	3/2	$\mu_{3/2}$
sd^2	3/2	0	3/2	$(3/5)(-\mu_{1/2}+2\mu_{3/2})$
d^3	3/2	0	3/2	$\mu_{3/2}$
s^2d	3/2	2	3/2	$(1/5)(\mu_{3/2}+6g_c)$
sd^2	1/2	2	1/2	$(3/5)(-1/2\mu_{1/2}+3g_c)$
sd^2	3/2	2	3/2	$(3/25)(-\mu_{1/2}+\mu_{3/2}+10g_c)$
sd^2	5/2	2	5/2	$(13/25)[\mu_{1/2}+(4/3)\mu_{3/2}+(5/13)g_c]$
d^3	3/2	2	3/2	$(1/5)(\mu_{5/32}+6g_c)$
sd^2	5/2	4	5/2	$(-3/5)[\mu_{1/2}+(4/3)\mu_{3/2}-5g_c]$
sd^2	3/2	2	5/2	$(4/25)(21)^{1/2}[\mu_{1/2}-(1/3)\mu_{3/2}]$
sd^2	1/2	2	3/2	0
sd^2	1/2	2	5/2	0

TABLE VIII. Nonvanishing matrix elements of μ_z needed for the $M1$ transition $I=1/2+\rightarrow I=3/2+$ (three nucleons with $T=3/2$).

γ	J	J'	$(\gamma\nu J2;\ 3/2\ 1/2;\ 3/2\ 3/2\|\mu_z\|\gamma\nu J'2;\ 1/2\ 1/2;\ 3/2\ 3/2)$
s^2d	3/2	3/2	$(2/5)[(5/3)\mu_{3/2}+(5/2)g_c)]$
sd^2	3/2	3/2	$(2/5)[(2\mu_{3/2}-\mu_{1/2}+(5/2)g_c)]$
d^3	3/2	3/2	$(2/5)[(5/3)\mu_{3/2}+(5/2)g_c]$
sd^2	5/2	5/2	$(56/225)^{1/2}[(-4/3)\mu_{3/2}-\mu_{1/2}+(5/2)g_c]$
sd^2	3/2	5/2	$+(216/13125)^{1/2}[(1/3)\mu_{3/2}-\mu_{1/2})]$
sd^2	5/2	3/2	$-(50/252)^{1/2}[(1/3)\mu_{3/2}-\mu_{1/2})]$

[9] E. Feenberg, reference 1. Note: The signs of two of the matrix elements in this treatment must be corrected. On p. 158, Eq. (IX. 40) should read: $(\cdots n_\mu\cdots|Q_c/R^2|\cdots n_\mu\cdots)\cong-(3Z/7\pi)\hbar/(BC)^{1/2}n_\mu(\mu^2-2)\cong-0.20n_\mu(\mu^2-2)$, and on p. 159, Eq. (IX. 41), one should have:

$$(00000|Q_c/R^2|01010)\cong-(3Z/7\pi)\hbar/(BC)^{1/2}.$$

Matrix elements needed in this calculation are displayed in Table VIII. The mean lives are[2]:

$$1/\tau_{M1}=3.645\times10^{12}W^3 \text{ sec}^{-1} \text{ for } {}_{17}Cl_{20}, \quad (33)$$

$$1/\tau_{M1}=5.373\times10^{11}W^3 \text{ sec}^{-1} \text{ for } {}_{14}Si_{17}, \quad (34)$$

$$1/\tau_{E2}=6.614\times10^{12}W^5 \text{ sec}^{-1} \text{ for } {}_{17}Cl_{20}, \quad (35)$$

$$1/\tau_{E2}=6.594\times10^{12}W^5 \text{ sec}^{-1} \text{ for } {}_{14}Si_{17}, \quad (36)$$

where W is the energy of the emitted γ ray in Mev. The relative phase of the matrix elements for the $M1$ and $E2$ transitions is positive for both ${}_{14}Si_{17}$ and ${}_{17}Cl_{20}$.

Next, we consider three nucleons with isobaric spin 1/2, applicable to ${}_{15}P_{16}$ and ${}_{18}A_{19}$. The needed three-nucleon wave functions are $U(s^3\ 1/2\ 1/2)$, $U(s^2d\ 5/2\ 5/2)$, $U(s^2d\ 3/2\ 3/2)$ (twice), $U(s^2d\ 1/2\ 1/2)$, $U(sd^2\ 7/2\ 7/2)$, $U(sd^2\ 5/2\ 5/2)$ (twice), $U(sd^2\ 3/2\ 3/2)$ (twice), $U(sd^2\ 1/2\ 1/2)$ (twice), $U(d^3\ 7/2\ 7/2)$, $U(d^3\ 5/2\ 5/2)$, $U(d^3\ 3/2\ 3/2)$, and $U(d^3\ 1/2\ 1/2)$. Several configurations have two independent functions with a definite J, J_z and γ. A total of thirty components are coupled by the surface waves. The resulting energy displacement is quite excessive ($E=-4.558\hbar\omega\cong22.8$ Mev), and convergence of the zero (11.6%), one (52.3%), and two (36.1%) quantum components is poor. The intermediate coupling procedure is therefore found to be invalid for this example. This failure may be due in part to taking the energy difference of the $1d_{3/2}$ and $2s_{1/2}$ orbitals equal to zero. It is also possible that the parameter P (here taken as 0.5) has been overestimated. The introduction of a positive excitation energy of one or two Mev for the $1d_{3/2}$ orbital does not appear to be sufficient to remedy the matter, and it appears that a reduction in P is required. Furthermore, nuclear forces, which are neglected here, may play an especially important role.

TABLE IX. Reduced matrix elements of $Y^2(\theta,\phi)$ for $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, and $3s_{1/2}$ orbitals.

γj	$\gamma' j'$	$(\gamma j\|\|Y^2(\theta,\phi)\|\|\gamma j')$
$s_{1/2}$	$s_{1/2}$	0
$s_{1/2}$	$d_{3/2}$	$-(1/\pi)^{1/2}$
$s_{1/2}$	$d_{5/2}$	$(3/2\pi)^{1/2}$
$s_{1/2}$	$g_{7/2}$	0
$d_{3/2}$	$d_{3/2}$	$-(1/\pi)^{1/2}$
$d_{3/2}$	$d_{5/2}$	$-(3/7\pi)^{1/2}$
$d_{3/2}$	$g_{7/2}$	$(18/7\pi)^{1/2}$
$d_{5/2}$	$d_{5/2}$	$-(12/7\pi)^{1/2}$
$d_{5/2}$	$g_{7/2}$	$-(2/7\pi)^{1/2}$
$g_{7/2}$	$g_{7/2}$	$-(50/21\pi)^{1/2}$

TABLE X. Amplitude coefficients, energy displacements, magnetic moments, and electric quadrupole moments for a single-nucleon description with $I=5/2$. Q/R^2 and $\mu-\mu_{5/2}$ may be compared with the experimental values for ${}_{51}Sb_{70}$: $\mu-\mu_{5/2}=1.434$ and $Q/R^2=-2.5$. The asterisk indicates that in order to obtain the proper phase of p, one should take the negative square root of p^2.

	P		1/2	1	1/2	1	1/2	1
	$e/\hbar\omega$		∞	∞	1	1	0	0
	$E/\hbar\omega$		−0.438	−0.773	−0.751	−1.389	−0.980	−1.714
ν	j	K			p^2			
0	5/2	0	0.721	0.606	0.592	0.438	0.439	0.318
1	5/2	2	0.234*	0.313*	0.176*	0.182*	0.112*	0.109*
1	7/2	2	0.027	0.030	0.027	0.031	0.022	0.025
1	3/2	2	0	0	0.038*	0.065*	0.089*	0.113*
1	1/2	2	0	0	0.084	0.131	0.198	0.233
2	5/2	0	0.003	0.019	0.019	0.032	0.021	0.030
2	5/2	2	0.002	0.005	0.001	0.004	0.006*	0.010*
2	7/2	2	0.002	0.005	0.008	0.014	0.010	0.014
2	3/2	2	0	0	0.000	0.000	0.000	0.001*
2	5/2	4	0.009	0.020	0.001*	0.004*	0.008*	0.060
2	7/2	4	0.001	0.002	0.001*	0.004*	0.008*	0.009*
2	3/2	4	0	0	0.008	0.021	0.035	0.055
2	1/2	2	0	0	0.012	0.020	0.017	0.023
	$\mu-\mu_{5/2}$		−0.463	−0.647	−0.472	−0.711	−0.758	−0.766
	Q/R^2		−2.681	−3.039	−2.603	−2.763	−2.355	−2.394

TABLE XI. Amplitude coefficients, energy displacements, magnetic moments, and electric quadrupole moments for a single-nucleon description with $I=7/2$. Q/R^2 and $\mu-\mu_{7/2}$ may be compared with the experimental values for ${}_{51}Sb_{72}$: $\mu-\mu_{7/2}=+0.831$ and $Q/R^2=-3.2$. The asterisk indicates that in order to obtain the proper phase of p, one should take the negative square root of p^2.

	P		1/2	1	1/2	1	1/2	1
	$e/\hbar\omega$		∞	∞	1	1	0	0
	$E/\hbar\omega$		−0.482	−0.871	−0.707	−1.282	−0.903	−1.569
ν	j	K			p^2			
0	7/2	0	0.666	0.532	0.628	0.494	0.495	0.371
1	7/2	2	0.245*	0.310*	0.189*	0.207*	0.127*	0.126*
1	3/2	2	0	0	0.098	0.161	0.264	0.331
1	5/2	2	0.046*	0.073*	0.024*	0.023*	0.008*	0.004*
2	7/2	0	0.010	0.019	0.017	0.036	0.019	0.028
2	7/2	2	0.008	0.016	0.000	0.002*	0.007*	0.014*
2	5/2	2	0.005	0.010	0.006	0.010	0.006	0.008
2	3/2	2	0	0	0.005	0.009	0.010	0.015
2	7/2	4	0.019	0.038	0.026	0.044	0.026	0.037
2	5/2	4	0.001	0.003	0.000	0.001*	0.002*	0.004*
2	3/2	4	0	0	0.005*	0.013*	0.019*	0.031*
2	1/2	4	0	0	0.002*	0.006*	0.017*	0.030*
	$\mu-\mu_{7/2}$		0.139	0.358	−0.111	−0.163	−0.165	−0.209
	Q/R^2		−3.429	−3.785	−3.028	−3.016	−2.290	−2.139

Although the intermediate coupling procedure fails here, the wave function derived yields physical properties of ${}_{15}P_{16}$ which are in reasonable agreement with experiment. Deviations from the Schmidt $s_{1/2}$ moments are:

$$\mu-\mu_{1/2}=-1.490 \quad (\text{odd } Z) \quad (37)$$

$$=1.164 \quad (\text{odd } N). \quad (38)$$

The experimental deviation for ${}_{15}P_{16}$ is -1.8, while the theoretical shift for odd N may be compared with deviations observed in heavier nuclei in which the odd neutron is assigned to a $3s_{1/2}$ orbital (coupling $2d_{3/2}$ orbitals):

$$\begin{aligned}\mu-\mu_{1/2}&=1.14 \quad \text{for } {}_{54}Xe_{65}\\ &=1.18-1.03 \quad \text{for } {}_{52}Te_{61,63}\\ &=1.26 \quad \text{for } {}_{48}Cd_{63}\\ &=1.05-0.87 \quad \text{for } {}_{50}Sn_{65,69}.\end{aligned} \quad (39)$$

The Gamow-Teller (G-T) matrix element for the image transition ${}_{16}S_{15}\rightarrow{}_{15}P_{16}$ is given by[10]

$$\left|\int\boldsymbol{\sigma}\right|^2=(2/T)\sum_{m'}|(I,m',T,\pm T|S_n-S_p|I,m,T,\pm T)|^2$$

$$=12|[1/2\ 1/2|(S_n-S_p)_z|1/2\ 1/2]|^2=0.032, \quad (40)$$

a substantial reduction from the value of 3 obtained

[10] M. Bolsterli and E. Feenberg, Phys. Rev. **97**, 736 (1955).

TABLE XII. Matrix elements of μ_z for a single-nucleon description with $I=5/2$.

j	j'	K	$(\nu jK;\ 5/2\ 5/2\|\mu_z\|\nu j'K;\ 5/2\ 5/2)$
5/2	5/2	0	$\mu_{5/2}$
5/2	5/2	2	$(23/35)\mu_{5/2}+(6/7)g_c$
7/2	7/2	2	$(27/49)\mu_{7/2}-(1/7)g_c$
3/2	3/2	2	$(23/21)\mu_{3/2}+(11/7)g_c$
1/2	1/2	2	$\mu_{1/2}+2g_c$
5/2	5/2	4	$(1/7)(-\mu_{5/2}+20g_c)$
3/2	3/2	4	$(1/7)(-5\mu_{3/2}+25g_c)$
7/2	7/2	4	$(9/49)\mu_{7/2}+(13/11)g_c$
5/2	3/2	4	$(31/130)(\sqrt{2})[(2/5)\mu_{5/2}-(5/3)\mu_{3/2}]$
5/2	3/2	2	$(8/21)[(2/5)\mu_{5/2}-(5/3)\mu_{3/2}]$

TABLE XIII. Matrix elements of μ_z for a single nucleon-description with $I=7/2$.

j	j'	K	$(\nu jK;\ 7/2\ 7/2\|\mu_z\|\nu j'K;\ 7/2\ 7/2)$
7/2	7/2	0	$\mu_{7/2}$
7/2	7/2	2	$(1/3)[(51/21)\mu_{7/2}+2g_c]$
5/2	5/2	2	$(1/9)[(37/5)\mu_{5/2}+13g_c]$
3/2	3/2	2	$\mu_{3/2}+2g_c$
7/2	7/2	4	$(1/9)[(23/21)\mu_{7/2}+20g_c]$
5/2	5/2	4	$(1/5)\mu_{5/2}+3g_c$
3/2	3/2	4	$(1/9)[(-1/3)\mu_{3/2}+32g_c]$
1/2	1/2	4	$(1/9)(7\mu_{1/2}+28g_c)$
5/2	3/2	2	$(4/15)[(2/5\mu_{5/2})-(5/3)\mu_{3/2}]$
5/2	3/2	4	$(232/225)(1/11)^{1/2}[(2/5)\mu_{5/2}-(5/3)\mu_{3/2}]$

if one assumes the odd nucleons to be in pure $2s_{1/2}$ orbitals, and in qualitative agreement with the experimental value of about 0.18.

4. COUPLING OF $1g_{7/2}$, $2d_{\frac{5}{2}}$, $2d_{\frac{3}{2}}$, AND $3s_{\frac{1}{2}}$ ORBITALS

Close competition between $1g_{7/2}$ and $2d_{5/2}$ proton orbitals is expected in $_{51}Sb_{70}$ and $_{51}Sb_{72}$. $I=5/2+$ for $_{51}Sb_{70}$ while $_{51}Sb_{72}$ has $I=7/2+$. The reason for this difference is not evident. Identical discrepancies occur for the isotope pairs $_{53}I_{74,76}$ and $_{55}Cs_{76,78}$. $2d_{3/2}$ and $3s_{1/2}$ proton orbitals are also assumed to have equal energies. The energy difference between the two pairs is designated by e:

$$e=E(2d_{3/2},\ 3s_{1/2})-E(1g_{7/2},\ 2d_{5/2}). \tag{41}$$

Reduced matrix elements of Y^2 for these single nucleon orbitals are given in Table IX.

A single-nucleon description is not strictly applicable to $_{51}Sb_{70-72}$ because of the 20–22 neutrons in unfilled shells. A complete analysis is hardly feasible here, and a single-nucleon treatment will be employed to give an indication of the importance of the surface waves for these nuclei. Tables X and XI give the amplitude coefficients, energy displacements, magnetic moments, and electric quadrupole moments of the $I=5/2+$ and $I=7/2+$ states for various values of P and e. Matrix elements of μ_z needed for the magnetic moments are given in Tables XII and XIII. In this region, $\hbar\omega$ is about 2 Mev. The sign of the difference between the energy displacements for the 5/2+ and 7/2+ states is a function of e, and hence no insight into the question of which state should be the ground state is provided by the calculation.

In both nuclei, the quadrupole moment is accounted for by the surface waves. For the 5/2+ state, the deviation of the magnetic moment from the Schmidt line is consistently in the correct direction and equal to about 1/2 of the observed shift. The deviation $\mu-\mu_{7/2}$ is small but in the correct direction if one couples only $1g_{7/2}$ and $2d_{5/2}$ orbitals, but the addition of $2d_{3/2}$ and $3s_{1/2}$ orbitals gives a small shift in the wrong direction.

The first excited state of $_{51}Sb_{72}$ has spin 5/2+ with an excitation energy of 153 kev. An investigation of the electromagnetic transition $5/2+\rightarrow 7/2+$ is therefore of interest. The magnetic dipole transition $2d_{5/2}\rightarrow 1g_{7/2}$ is l-forbidden, but a substantial $M1$ matrix element is provided by the coupled system along with a strongly reinforced $E2$ transition. The mean lives for these transitions are given in Table XIV.

TABLE XIV. Reciprocal mean lifes for the electromagnetic transition $5/2+\rightarrow 7/2+$ (applicable to $_{51}Sb_{72}$) given in sec^{-1}.

P	1/2	1	1/2	1	1/2	1
$e/\hbar\omega$	∞	∞	1	1	0	0
$1/\tau_{M1}$	7.00×10^5	1.50×10^7	4.53×10^8	6.34×10^8	5.36×10^8	4.92×10^8
$1/\tau_{E2}$	9.56×10^8	9.37×10^8	7.65×10^8	6.40×10^8	5.78×10^8	4.62×10^8

A major deficiency of the procedure applied here is that nuclear forces have been neglected. In particular, the tensor force couples nucleon orbitals of the same parity which differ by no more than two units of angular momentum. Hence the tensor force to some degree may produce effects similar to those of the surface oscillations. A detailed treatment involving nuclear forces and collective oscillations may therefore yield better agreement with experiment than the present description.

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