the transition is 1% as intense as the 2.9-Mev transition would have been observed.

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Radiation Widths of Nuclear Energy Levels*

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The highly excited energy levels formed by capture of slow neutrons can be studied by means of the resonances in neutron cross sections as functions of energy. In the present work the radiation widths of levels in heavy nuclei have been measured by means of total cross section curves obtained with the Brookhaven fast chopper. The "shape," "area," and "interference" methods of analyzing the neutron transmission data are described. The radiation widths obtained, together with results of similar measurements, show that radiation widths of levels in the same nuclide are nearly constant, the observed variations from level to level being of the order of the experimental error. The radiation widths decrease slowly with atomic weight, except for discontinuities at nuclear shells; these discontinuities can be satisfactorily correlated with variations in excitation energy and level spacing at the shells. The variation of radiation width with excitation energy and level spacing is consistent with theoretical calculations for electric dipole transitions; the absolute theoretical widths are too large by an order of magnitude, however.

I. INTRODUCTION

THE purpose of this investigation is to study the dependence of the radiation widths of slow-neutron resonances on atomic weight, excitation energy, level spacing, and other nuclear properties. Such information is needed to assist the advance of theoretical understanding of electromagnetic radiation phenomena in nuclei. For many years arguments have been offered to show that nuclear electric dipole radiation is improbable[1,2]; in fact, for a model which views the nucleus as a liquid drop in which the motions of the neutrons and protons are strongly correlated, no electric dipole radiation is possible. These arguments, however, do not affect magnetic radiation or higher electric multipole radiation. Theoretical estimates[3] of radiation widths of slow-neutron resonances, based on a modified independent-particle model, are a factor of 300 larger than the experimentally observed values. On the other hand, for those cases where it has been possible to compare the emission probability of competing radiations of different multipole orders, it has been found[4] that the relative probabilities are in good agreement with the predictions of the independent-particle model.

The interpretations of several recent experiments also depend on a knowledge of Γ_γ as a function of atomic weight and excitation energy. Hughes *et al.*[5] have measured the capture cross section of many elements for a spectrum of unmoderated fission neutrons. It has been shown by Bethe[6] that these cross sections are proportional to Γ_γ/D, where D is the average spacing of neutron resonances. In inferring D from these capture cross sections, Hughes *et al.* assumed a monotonic dependence of Γ_γ on atomic weight A, as given by Heidmann and Bethe.[7] If this dependence of Γ_γ on A is not monotonic, but instead shows structure, it may modify the conclusions drawn concerning the dependence of nuclear level density on atomic weight, particularly in the region of magic numbers. Another example of an experiment where information about Γ_γ is needed is found in the analysis of neutron resonances by area methods, where it is often necessary to know the radiation widths in order to obtain the other parameters of the levels. The recent work of Carter *et al.*[8] on the dependence on atomic weight of the ratio of the average reduced neutron width to the average level spacing is such a case.

At the time this study of radiation widths was begun, all the information available on Γ_γ had been summarized by Teichmann,[9] Blatt and Weisskopf,[10] Heidmann and Bethe,[7] and Feld.[11] The radiation widths of only a

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[1] M. Delbrück and G. Gamow, Z. Physik 72, 492 (1931).
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[3] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. 12.
[4] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. 93, 1260 (1954).
[5] Hughes, Garth, and Levin, Phys. Rev. 91, 6 (1953).
[6] H. A. Bethe, Phys. Rev. 57, 1125 (1940).
[7] J. Heidmann and H. A. Bethe, Phys. Rev. 84, 274 (1951).
[8] Carter, Harvey, Hughes, and Pilcher, Phys. Rev. 96, 113 (1954).
[9] T. Teichmann, Ph.D. dissertation, Princeton, 1949 (unpublished).
[10] J. M. Blatt and V. F. Weisskopf, reference 3, p. 474.
[11] B. T. Feld, Atomic Energy Commission Report NYO-3078, 1953 (unpublished).

dozen resonances had been measured with accuracies better than 50 percent, and of these only one was in an isotope of atomic weight greater than 180. In order to accumulate more data it was decided to measure resonance parameters with greater accuracy, and in particular to concentrate on the heavy elements.

II. ANALYSIS OF RESONANCES

The experimental determinations of resonance parameters were made with the Brookhaven fast chopper. This instrument consists of a high-speed rotor capable of producing neutron bursts of approximately 1 μsec duration, together with a neutron detector at the end of a 20-meter flight path and electronics for determining the time of flight of the detected neutrons. The best instrumental resolution obtainable during the course of these measurements was 0.18 μsec/m. The design and construction features of the fast chopper are discussed in detail by Seidl[12]; the instrumentation, control, operation, and method of taking data are described by Seidl *et al.*[13]

The equipment was used to determine sample transmission in good geometry, which gives the total cross section modified by the effects of instrumental resolution and Doppler broadening. On the assumption that the cross section can be described by a sum of single-level Breit-Wigner formulas[14] with interference between resonance scattering and potential scattering only, one can obtain the resonance parameters E_0, Γ, and $g\Gamma_n$, where E_0 is the neutron energy at exact resonance, Γ is the total width of the level, and $g\Gamma_n$ is the product of the statistical weight factor and the neutron width of the level. In order to determine Γ_n and g separately additional information is needed, for example the ratio of scattering to total cross section. For most of the levels studied, however, a knowledge of g is not important since Γ_n is much smaller than Γ and the determination of Γ_γ ($\Gamma_\gamma=\Gamma-\Gamma_n$) is therefore not appreciably affected by the uncertainty in Γ_n.

A. Shape Method

Three methods of analysis were employed to obtain resonance parameters from the total cross section data. The first of these, the "shape" method of analysis, makes use of the detailed shape of the observed transmission curve as a function of neutron time of flight. In this analysis, the shape of the observed transmission dip is first corrected for the effects of instrumental resolution. The maximum cross section σ_Δ and full width at half-maximum Γ_Δ of the resulting Doppler-broadened curve are then used to obtain the Breit-Wigner resonance parameters Γ and σ_0 (the cross section at exact resonance) through the use of prepared curves that relate these two sets of quantities. This method of analysis was useful for isolated levels in the energy region below about 10 ev, in those cases where the instrumental resolution and Doppler width Δ were smaller than the width of the resonance, Γ. The 5.2-ev level in Ag^{109}, 2.4-ev level in Hf^{177}, and 4.9-ev level in Au^{197} were analyzed by the shape method. No attempt was made to use this method of analysis on the 6.7-ev level in U^{238} because for this level $\Delta\approx 2\Gamma$ and therefore $\sigma_\Delta/\sigma_0\approx\frac{1}{3}$, which is a large correction.

B. Area Method

The second method of analysis, which was employed in most of those cases which were not well suited for shape analysis, consists of measurement of the area above transmission dips for samples of different thicknesses. These areas, between the observed transmission curves and the transmission due to potential scattering, are independent of the instrumental resolution, and are measurements of $\sigma_0\Gamma^p$, where the exponent p lies between 1 for a very thin sample ($n\sigma_\Delta<1$ where n is the number of atoms per cm^2 and σ_Δ the peak height of the Doppler-distorted resonance) and 2 for a very thick sample ($n\sigma_\Delta>10$). If the thicknesses of the samples used for two area determinations are sufficiently different so that the powers of Γ differ by about 1, then reasonably accurate values of σ_0 and Γ are obtained. If more than two sample thicknesses are used, the data can be treated as an over-determined system of equations and a least-squares solution obtained for the best values of the parameters. This "area" method of analysis is most useful in the energy region above 10 ev, where instrumental resolution and Doppler broadening distort the shape of a resonance so badly as to make shape analysis insensitive. It does, however, require sufficiently good resolution so that statistically significant measurements can be made of thin sample areas. Area analysis was used to determine the parameters of all the resonances measured except the 23-ev levels in Th^{232}, the 61-ev level in Au^{197}, and those levels mentioned previously in the discussion of shape analysis.

C. Interference Method

The third method of analysis, which was used to determine the parameters of the levels in Th and Au mentioned above, will be discussed in some detail for it has not been described previously. This analysis makes use of the interference between resonance and potential scattering to determine $g\Gamma_n$. In the off-resonance region, where the cross section is changing slowly and therefore is not affected by instrumental resolution and Doppler broadening, the observed cross section is given by

$$\sigma_T=\sigma_p+\sum_i\left[\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{\sigma_0\Gamma^2}{4(E-E_0)^2}+\frac{2\lambda g\Gamma_n(\pi\sigma_p)^{\frac{1}{2}}}{(E-E_0)}\right]_i. \quad (1)$$

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[12] F. G. P. Seidl, Atomic Energy Commission Report BNL-278, 1954 (unpublished).

[13] Seidl, Hughes, Palevsky, Levin, Kato, and Sjöstrand, Phys. Rev. 95, 476 (1954).

[14] J. M. Blatt and V. F. Weisskopf, reference 3, pp. 391–394, and 426.

The summation is taken over all nearby resonances. The first expression in the summation represents the resonance contribution of a level, the second term is the resonance-potential scattering interference of that level, and σ_p is the potential scattering. Interference between resonances has been neglected in Eq. (1); in the resonance term it has been assumed that $\Gamma_n \ll \Gamma$.

Near a reasonably isolated resonance only those terms involving the level itself are important, and the contributions of the other levels can be considered to be corrections to the cross section. In addition, the term that falls off as $(E-E_0)^{-2}$ will be small compared to the interference term, which decreases only as $(E-E_0)^{-1}$. One can therefore define the corrected cross section in the vicinity of the jth level as

$$\sigma_j=\sigma_T-\left[\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{\sigma_0\Gamma^2}{4(E-E_0)^2}\right]_j -\sum_{i\neq j}\left[\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{\sigma_0\Gamma^2}{4(E-E_0)^2}+\frac{2\lambda g\Gamma_n(\pi\sigma_p)^{\frac{1}{2}}}{E-E_0}\right]_i. \quad (2)$$

From Eq. (1) it follows that

$$\sigma_j=\sigma_p+\left(\frac{2\lambda g\Gamma_n(\pi\sigma_p)^{\frac{1}{2}}}{E-E_0}\right)_j. \quad (3)$$

If σ_j is plotted as a function of $(E-E_0)_j^{-1}$, the curve should thus be a straight line with slope $2\lambda(\pi\sigma_p)^{\frac{1}{2}}(g\Gamma_n)_j$ and intercept σ_p, from which $g\Gamma_n$ for the resonance can be determined.

Negative energy levels (bound levels) cannot be accounted for in calculating σ_j since their parameters are not known. However, for the cases of interest here the contribution of the resonance terms of negative levels is negligible, while the interference terms, although perhaps not negligible, change only slightly over the energy region considered. This neglect of negative energy levels will therefore not affect the slope of the line but will tend to alter the intercept and give too large a value for σ_p. Difficulties in normalization between open and sample runs, which tend to introduce additive errors in cross section which are not energy-dependent, also alter only σ_p. In order to calculate $g\Gamma_n$ from the slope it is therefore probably better to use σ_p as given by the optical model of the nucleus.[15] Fortunately σ_p enters only as the square root in the determination of $g\Gamma_n$.

The pair of resonances at about 23 ev in Th^{232} will serve as an illustration of this method of analysis. Transmission curves for samples of Th can be found in reference 13. The resonance-potential scattering interference is very prominent in Th because this element is monoisotopic and has zero spin ($g=1$); therefore the resonance scattering interferes with the entire potential scattering. At the time this measurement was made the resolution of the fast chopper was not sufficient to make good thin sample area measurements for these resonances; however, thick sample measurements gave reasonably good values of $\sigma_0\Gamma^2$ for these and other levels in Th up to 140 ev.[13]

From a preliminary analysis it was known that $\Gamma_n \ll \Gamma$ for the 22.1-ev and 23.8-ev levels in Th. On the assumption that $\Gamma=\Gamma_\gamma$ was the same for these two levels, one could write

$$(\lambda\Gamma_n)_1/(\lambda\Gamma_n)_2=\frac{(\sigma_0\Gamma^2)_1}{(\sigma_0\Gamma^2)_2}\frac{\lambda_2}{\lambda_1}, \quad (4)$$

where the subscripts 1 and 2 refer to the 22.1-ev and 23.8-ev levels, respectively. The numerical value of this ratio for these two resonances is 0.5. The total cross section in the energy ranges 14.6 to 19.2 ev and 27 to 46 ev was then corrected for the resonance contributions of the 22.1-, 23.8-, 60.5-, and 70.8-ev levels and the interference contribution of the 60.5-, 70.8-, 117-, 127-, and 133-ev levels, so that

$$\sigma_j=\sigma_p+2(\pi\sigma_p)^{\frac{1}{2}}(\lambda\Gamma_n)_2\left[\frac{0.5}{E-22.1}+\frac{1}{E-23.8}\right]. \quad (5)$$

A plot of σ_j as a function of $0.5(E-22.1)^{-1}+(E-23.8)^{-1}$ is shown in Fig. 1. The correction for the interference terms of the other levels varied from 1.2 b at 46 ev to 0.5 b at 14.6 ev; the resonance terms contributed 0.3 to 0.4 b to the two or three nearest points on each side of resonance and less elsewhere. (Points nearer resonance were not used because the resonance term was large and the effects of instrumental resolution became important.) The errors indicated on the points are standard deviations and include uncertainties in the corrections.

Least-squares fits to the data above the resonances (positive values of the abscissa), the data below the resonances, and all the data taken together gave the slopes and intercepts listed below.

	Slope (ev-b)	Intercept (b)	$(\Gamma_n)_{22.1}$ (mv)[a]	$(\Gamma_n)_{23.8}$ (mv)	Γ_γ (mv)
Above resonance	4.0±0.7	12.2±0.4	1.9±0.3	3.8±0.7	30±10
Below resonance	5.1±0.8	12.9±0.5	2.4±0.4	4.9±0.8	20±6
All data	2.9±0.2	12.4±0.1	1.4±0.1	2.8±0.3	40±10

[a] 1 mv $=10^{-3}$ ev.

The errors indicated were calculated from the errors in the individual points, not from the deviations of the points from the straight lines. The slopes obtained from the data above resonance and below resonance agree satisfactorily, as do the intercepts; however, because of the difference in the two intercepts the slope obtained using all the data does not agree favorably with the other slopes. The neutron widths and radiation widths listed above were based on $\sigma_p=10\pm1$ b and $\sigma_0\Gamma^2=7\pm2$ and 13 ± 4 ev^2 b for the 22.1-ev and 23.8-ev levels, respectively. The value for σ_p was computed from the equation $\sigma_p=(0.99)4\pi R^2$, where $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm. The factor 0.99 for atomic weight 232 is obtained from reference 15. Although the agreement of the values of

[15] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

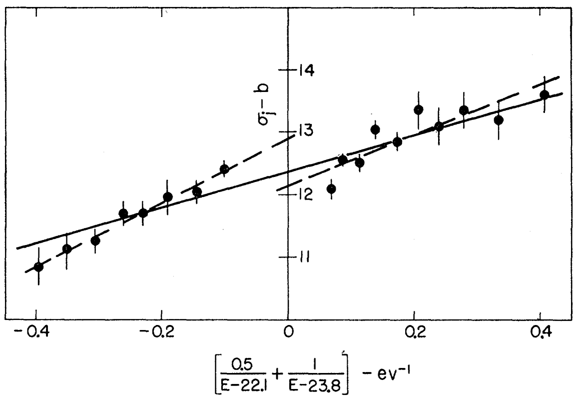

FIG. 1. Analysis of the pair of resonances near 23 ev in Th^{232} by the interference method. The slope is a measure of Γ_n and the intercept gives the potential scattering σ_p as explained in the text.

Γ_γ obtained by using different parts of the data leaves something to be desired, it seems safe to conclude that $\Gamma_\gamma=30\pm10$ mv.

III. RESULTS

With the exception of silver, the elements investigated in this study of radiation widths lie in the region of atomic weight $A>170$. Measurements were made on these heavy elements in order to extend the knowledge of Γ_γ over as large a range of atomic weights as possible. In addition, measurements were made on Hf^{177} to supplement the information available in that region of atomic weight.

A. Silver

The analysis of the 5.2-ev level in Ag^{109} is described in detail in reference 13. The value of 156 ± 8 mv obtained for the radiation width is in good agreement with the work of other experimenters.[16,17]

B. Hafnium

The 2.38-ev level in Hf^{177} was analyzed by the shape method. A transmission curve taken with a 0.12-g/cm^2 sample of zirconium containing 1.9 ± 0.4 weight percent of hafnium as an impurity is shown in Fig. 2. A sample of this form was the most convenient way to obtain a sufficiently thin sample of hafnium, and although the amount of hafnium present is not very well known, this information is not required to obtain a measurement of the total width Γ. The instrumental resolution used for this measurement is also shown in Fig. 2. Correction for this instrumental resolution (about 7 percent) and for Doppler broadening gives $\Gamma=64\pm10$ mv and $n\sigma_0=0.36\pm0.03$. A check on the value of Γ can be obtained by measuring the area above the transmission curve to obtain $n\sigma_0\Gamma^p$ and then dividing by the value obtained for $n\sigma_0$ to get Γ^p. This area of 0.039 ± 0.002 ev, which contains a 13 percent correction for the area in the wings of the resonance,[13] implies $n\sigma_0\Gamma^{1.016}=0.026\pm0.002$, therefore $\Gamma^{1.016}=0.072\pm0.007$ and $\Gamma=74\pm7$ mv.

The two values of Γ obtained above are not independent measurements, however they do serve as a check on the self-consistency of the data and the methods of analysis. It can be concluded that 70 ± 7 mv is a good estimate of the total width of the level. An area measurement of a thicker sample of Hf^{177} for which $n=(8.2\pm0.4)\times10^{19}$ atoms/cm^2 gave an area of 0.257 ± 0.005 ev, from which it follows that $\sigma_0\Gamma^{1.51}=960\pm60$. For the above value of Γ, one can calculate $g\Gamma_n=3.4\pm0.3$ mv. Since the spin of Hf^{177} is $I\leq\frac{3}{2}$,[18] the best estimate of the statistical weight factor is $g=\frac{1}{2}$, and thus $\Gamma_n=7$ mv. The best estimate of the radiation width of this level is then $\Gamma_\gamma=63\pm8$ mv. The results of this measurement are in satisfactory agreement with the work of Bollinger *et al.*[19] who concluded that $\Gamma<0.1$ ev. They do not agree well with the measurements of Egelstaff and Taylor,[20] who quoted a value of $\Gamma=0.17\pm0.05$ ev, which is even larger than the observed width of the uncorrected transmission curve as measured with the Brookhaven fast chopper. This discrepancy is probably due to an incorrect estimate of the resolution of the crystal spectrometer used in the work of Egelstaff and Taylor.

The parameters of the 6.5-ev level in Hf^{177} were determined by area analysis of transmission curves obtained from measurements with normal samples of

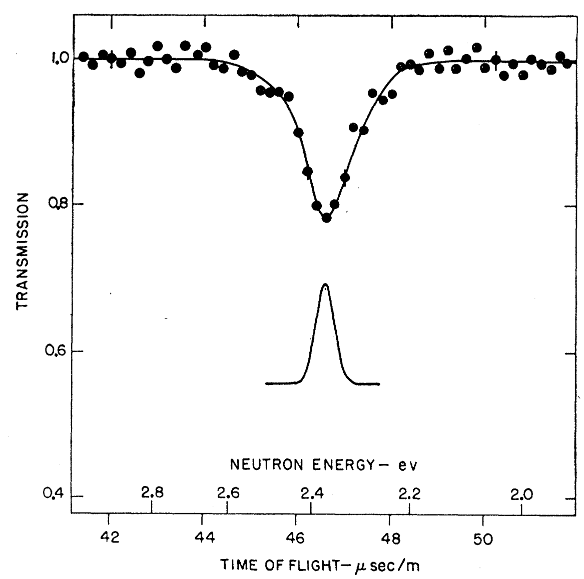

FIG. 2. The thin sample transmission curve used for analysis of the 2.38-ev level in Hf^{177} by the shape method. The sample used is a 0.12-g/cm^2 sample of zirconium containing a 1.9 percent hafnium impurity. The resolution function is shown in the lower curve.

[16] W. Selove, Phys. Rev. **84**, 869 (1951).
[17] R. E. Wood, Phys. Rev. **95**, 644 (1954).
[18] E. Rasmussen, Naturwiss. **23**, 69 (1935).
[19] Bollinger, Harris, Hibdon, and Muehlhause, Phys. Rev. **92**, 1527 (1953).
[20] P. A. Egelstaff and B. T. Taylor, Nature **167**, 896 (1951).

TABLE I. The area analysis of the 6.5-ev resonance in Hf^{177}.

Sample	$(1/n)_{177}\times10^{24}$ (cm²/atom)	Area[a] (ev)	Wing correction (% of total area)	p	$\sigma_0\Gamma^p$	Parameters	
$Hf^{180}O_2$[b]	27 000±5000	0.092±0.010	11	1.08	1500±400	$E_0=6.5\pm0.1$ ev	$\sigma_0\Gamma=2200\pm500$ ev b
$Hf^{179}O_2$[c]	26 000±5000	0.14 ±0.02	8	1.13	2100±500	$\sigma_0=40\,000\pm20\,000$ b	$\sigma_0\Gamma^2=120\pm30$ ev² b
Hf metal	3990± 40	0.36 ±0.04	11	1.66	360± 80	$\Gamma=55\pm20$ mv	$g\Gamma_n=5.5\pm1.1$ mv
$Hf^{177}O_2$[d]	1080± 50	0.60 ±0.06	20	1.80	200± 40	$\Gamma_\gamma=44\pm20$ mv	

[a] This area includes the wing correction.
[b] Sample enriched in Hf^{180}, containing 0.7 percent Hf^{177}.
[c] Sample enriched in Hf^{179}, containing 2 percent Hf^{177}.
[d] Sample containing 62 percent Hf^{177}.

hafnium metal and isotopically enriched samples of HfO_2. A typical transmission curve is shown in Fig. 3. The 6.5-ev level is not completely separated from the 5.7-ev level in Hf^{179}, the 5.9-ev level in Hf^{177} and the 7.8-ev level in Hf^{178}. The measured areas require considerable correction to remove the effects of these other levels, and consequently the errors are rather large. In Table I is listed the pertinent information obtained from measurements of four different samples, together with the resonance parameters calculated from these data. The lack of agreement of areas measured in the two thin sample runs is probably due to the difficulty in determining the small amount of Hf^{177} present in the enriched samples of Hf^{179} and Hf^{180}. The radiation width obtained from the data is 44±20 mv.

C. Gold

The shape analysis of the 4.9-ev level in Au^{197} is discussed in reference 13. This analysis yielded a value of Γ_γ of 144±15 mv, which is in good agreement with the work of other experimenters.[17,21] A value of $g\Gamma_n$ for the 61-ev level in Au^{197} was obtained in the present work by an analysis of the interference between resonance and potential scattering, as described in Sec. II.C. The cross section between 20 ev and 50 ev, as measured with a 9.8-g/cm² sample of metallic gold, was corrected for the resonance contributions of the 80-, 61-, and 4.9-ev levels and the interference effects of the 80- and 4.9-ev levels by using the resonance parameters given in references 13 and 21. The general features of the gold cross section in the 20- to 50-ev energy region are shown in reference 13.

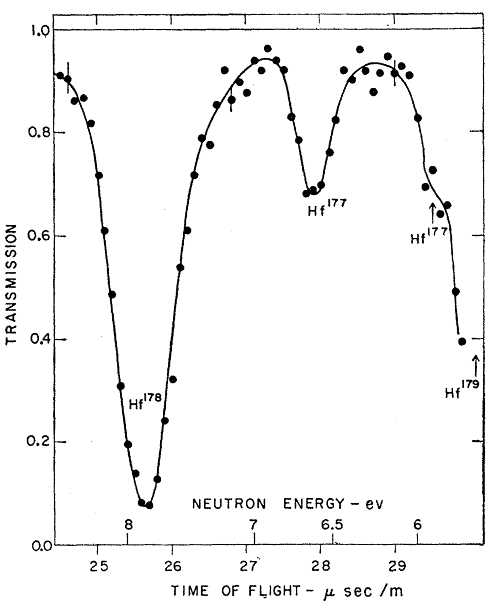

FIG. 3. A transmission curve of Hf metal used for analysis of the 6.5-ev level in Hf^{177}. Similar measurements with samples of varying isotopic constitution were used to estimate the effect of nearby levels on the 6.5-ev level.

In this energy region the contribution of the interference term for the 61-ev level varies from $\frac{1}{3}$ as large as, to 2 times larger than, the corrections made for other levels. Only the low-energy side of the resonance was analyzed since the presence of the 80-ev level necessitates excessively large corrections in the region above the resonance. The parameters obtained from this analysis are $2\lambda g\Gamma_n(\pi\sigma_p)^{\frac{1}{2}}=29\pm8$ ev-b and $\sigma_p=10.8\pm0.4$ b. This value of potential scattering is in good agreement with those obtained by Wood[17] and Landon and Sailor,[21] who used a similar analysis in the region of the 4.9-ev level; it is also in agreement with the theory of Feshbach *et al.*[15] which predicts a value about 10 percent larger than the 9.0 b that would be calculated by using $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm. The value of $g\Gamma_n$ for this level is therefore 43±12 mv. A measurement with a 1.21 g/cm² sample of gold gave an area of 2.40±0.10 ev for the 61-ev level, from which it follows that $\sigma_0\Gamma^{1.97}=510\pm40$. The total width of the level is therefore 260±80 mv. Since $I=\frac{3}{2}$ for gold, $g=\frac{1}{2}$ is a good approximation, and $\Gamma_\gamma=170\pm80$ mv.

D. Mercury

Parameters were obtained for the 23.3-ev level of Hg^{198} and the 34.0-ev level of Hg^{199}.[22] A transmission curve for a 0.90-g/cm² sample of normal mercury in the form of HgO is shown in Fig. 4. The areas measured in this and other runs, and the parameters thereby deduced are given in Tables II and III. The radiation width of the level in Hg^{198} is 145±20 mv. Since the spin of Hg^{199} is $\frac{1}{2}$ and $g\Gamma_n/\Gamma=0.11$ for the 34.0-ev level,

[21] H. H. Landon and V. L. Sailor, Phys. Rev. 93, 1030 (1954).
[22] L. M. Bollinger and R. R. Palmer, Atomic Energy Commission Report ANL-5031, 1953 (unpublished).

TABLE II. The area analysis of the 23.3-ev resonance in Hg^{198}.

$(1/n)_{198}\times10^{24}$ (cm²/atom)	Area (ev)	Wing correction (% of total area)	p	$\sigma_0\Gamma^p$	Parameters
3720±100	0.225±0.013	6	1.14	500±40	$E_0=23.3\pm0.2$ ev; $\sigma_0\Gamma=650\pm60$ ev b
386±4	0.91 ±0.04	7	1.85	130±10	$\sigma_0=4300\pm1000$ b; $\sigma_0\Gamma^2=98\pm10$ ev² b
					$\Gamma=150\pm20$ mv; $\Gamma_n=5.8\pm0.5$ mv
					$\Gamma_\gamma=145\pm20$ mv

TABLE III. The area analysis of the 34.0-ev resonance in Hg^{199}.

$(1/n)_{199}\times10^{24}$ (cm²/atom)	Area (ev)	Wing correction (% of total area)	p	$\sigma_0\Gamma^p$	Parameters
27 600±600	0.18±0.02	8	1.07	3100±400	$E_0=34.0\pm0.3$ ev; $\sigma_0\Gamma=3000\pm200$ ev b
8700±300	0.44±0.02	7	1.18	2450±160	$\sigma_0=8400\pm1400$ b; $\sigma_0\Gamma^2=1090\pm60$ ev² b
2200±70	1.19±0.05	7	1.62	1640±120	$\Gamma=360\pm40$ mv; $g\Gamma_n=39\pm3$ mv
761±7	2.10±0.06	8	1.93	1170±60	J — Γ_n — Γ_γ
					0 — 160±10 mv — 200±40 mv
					1 — 50±4 mv — 310±40 mv

the value obtained for Γ_γ is very much dependent on whether $J=0$ or 1 for this level. The radiation width of this level is therefore 200±40 mv if $J=0$ or 310±40 mv if $J=1$.

E. Thorium

The analysis of the pair of resonances at 23 ev in thorium is discussed in Sec. II. The radiation width deduced is 30±10 mv.

F. Uranium

Parameters of the 6.7-, 21.1-, and 37.1-ev levels in U^{238} were determined by area analysis. Areas measured for various sample thicknesses and the parameters deduced are given in Table IV. Figure 5, which is a plot of σ_0 as a function of Γ for the 6.7-ev level as determined with four different sample thicknesses, shows the internal consistency typical of these measurements. The radiation widths obtained for these three levels are 24±2, 30±6, and 40±17 mv, in order of increasing E_0. The value obtained for σ_0 of the 6.7-ev level is considerably larger than that quoted by von Dardel and Persson.[23]

IV. SUMMARY OF VALUES OF Γ_γ

A. Directly Measured Values

In the past two years instruments with improved resolution have greatly increased the information available on neutron resonance parameters, including radiation widths. In the energy region below about 5 ev the resolution of these instruments is generally sufficiently good to permit analysis of the shape of the observed resonances; however, at higher energies, area methods of analysis are used almost universally to obtain resonance parameters. Table V is a summary of those radiation widths that have been measured with an accuracy of about 20 percent or better, including those of the present work. Where several experimenters have obtained values for the same level, the number quoted is a weighted average. In most cases the errors stated are intended to be standard deviations; however, some authors do not make clear the meaning of their error estimate. Many of the values of Γ_γ listed in this table have been taken from the compilation by Hughes and Harvey.[24]

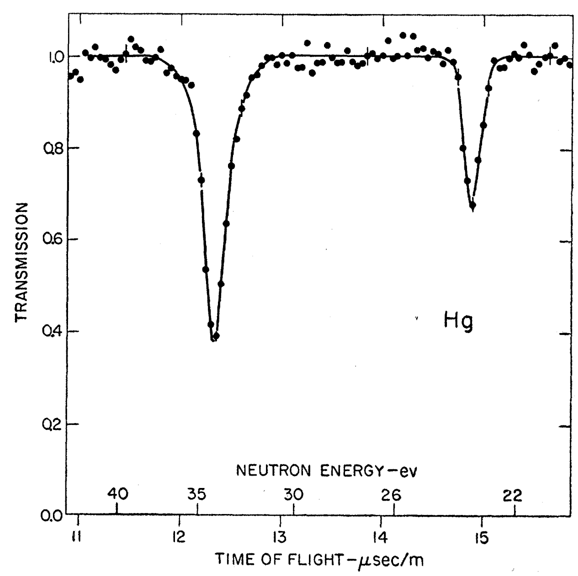

FIG. 4. A transmission curve for a 0.90-g/cm² sample of normal mercury used for the analysis of the 23.3-ev level of Hg^{198} and the 34.0-ev level of Hg^{199}.

[23] B. von Dardel and R. Persson, Nature **170**, 1117 (1952).

[24] D. J. Hughes and J. A. Harvey, Nature **173**, 942 (1954).

TABLE IV. The area analysis of the 6.7-, 21.1-, and 37.1-ev resonance in U^{238}.

E_0 (ev)	$(1/n)_{238}\times10^{24}$ (cm²/atom)	Area (ev)	Wing correction (% of total area)	p	$\sigma_0\Gamma^p$	Parameters	
6.70±0.06	22 500±300	0.039±0.003	2	1.03	550±50	$\sigma_0=23\,000\pm3000$ b	$\Gamma=26\pm2$ mv
	7460±80	0.093±0.005	3	1.10	410±30	$\sigma_0\Gamma=600\pm40$ ev b	$\Gamma_n=1.54\pm0.10$ mv
	284±3	0.44 ±0.01	5	1.96	17.9±0.8	$\sigma_0\Gamma^2=15.3\pm0.5$ ev² b	$\Gamma_\gamma=24\pm2$ mv
	141±2	0.61 ±0.01	9	1.99	15.3±0.9		
21.1±0.2	7460±80	0.164±0.010	2	1.12	690±50	$\sigma_0=27\,000\pm6000$ b	$\Gamma=38\pm6$ mv
	7190±80	0.165±0.012	2			$\sigma_0\Gamma=1020\pm80$ ev b	$\Gamma_n=8.3\pm0.7$ mv
	284±3	0.72 ±0.04	5	1.95	46±5	$\sigma_0\Gamma^2=39\pm5$ ev² b	$\Gamma_\gamma=30\pm6$ mv
37.1±0.4	7191±80	0.32 ±0.03	2	1.15	1500±200	$\sigma_0=30\,000\pm10\,000$ b	$\Gamma=70\pm20$ mv
	3230±30	0.46 ±0.02	2	1.40	680±70	$\sigma_0\Gamma=2100\pm300$ ev b	$\Gamma_n=30\pm4$ mv
	284±3	1.34 ±0.07	5	1.96	170±20	$\sigma_0\Gamma^2=150\pm20$ ev² b	$\Gamma_\gamma=40\pm20$ mv

B. Values Inferred from Thermal Cross Sections

In a few special cases where the energy interval between zero neutron energy and the first resonance excited by $l=0$ neutrons is much less than the average level spacing, it may be assumed that the thermal capture cross section is determined primarily by the first level. In these instances a knowledge of the thermal capture cross section and $\sigma_0\Gamma$ (or $\sigma_0\Gamma^2$ if the level is not predominantly capture) enables one to obtain a reasonable estimate of the radiation width by use of the relation

$$\Gamma_\gamma=\sigma_{th}(E_{th})^{\frac{1}{2}}\frac{4E_0^{\frac{3}{2}}}{\sigma_0\Gamma}, \qquad (6)$$

where σ_{th} is the capture cross section at thermal energy, E_{th}. In the sections which follow those cases which readily lend themselves to this treatment are discussed. Wherever specific references are not quoted for the information used, the data can be found in the compilation[25] of neutron cross sections.

1. Sodium, Silicon, and Sulfur

The first level in Na^{23} is at 2.9 kev, whereas the average spacing of all levels excited by $l=0$ neutrons is about 200 kev. This resonance, which is almost entirely scattering, is known to have $\sigma_0\Gamma=12\times10^4$ ev b [26] and

TABLE V. A summary of the radiation widths of slow neutron resonances. Also tabulated are $\bar{\Gamma}_\gamma$, the average value of the radiation width for each isotope; D, the average spacing for each spin state of levels excited by $l=0$ neutrons; and E_B, the neutron binding energy for the compound nucleus.

Target isotope	Target spin (I)	Resonance energy (ev)	Γ_γ (mv)	Ref.	$\bar{\Gamma}_\gamma$ (mv)	D (ev)	Ref.	E_B (Mev)
$_{11}Na^{23}$	3/2	3000	400		400	~400 000	aa, bb	6.96[rr]
$_{14}Si^{28}$	0	200 000	<9000		<9000	~500 000	cc, dd	8.47[rr]
$_{16}S^{32}$	0	110 000	<25 000		<25 000	~200 000	ee, ff	8.65[rr]
$_{25}Mn^{55}$	5/2	346	500		500	4000	gg	7.2[rr]
$_{27}Co^{59}$	7/2	134	500		500	10 000	aa	7.7[rr]
$_{30}Zn^{68}$	0	530	170		170	~3000	hh	6.6
$_{42}Mo^{95}$	5/2	45	210±60		210±60	~200	ii	9.2
$_{45}Rh^{103}$	1/2	1.26	155±5	a	155±5	100	jj	6.8[rr]
$_{47}Ag^{109}$	1/2	5.20	141±5	b, c, d	141±5	33		6.8
$_{48}Cd^{113}$	1/2	0.18	112±5	e, f	112±5	~150	kk	9.0[rr]
$_{49}In^{113}$	9/2	14.7	60±20	g	70±20	15	g, ll	7.3
		25.2	110±40	g				
$_{49}In^{115}$	9/2	1.46	72±2	h, i, j	77±5	16	g, ll	6.6[rr]
		3.85	81±4	g, j				
		9.1	80±40	g				
		12.1	140±60	g				
		40	140±50	g				
$_{52}Te^{123}$	1/2	2.33	104±9	k	104±9	~100	mm	9.0
$_{55}Cs^{133}$	7/2	5.90	115±20	l	115±17	50	g	6.7[rr]
		22.6	110±40	g				
		48	120±60	g				
$_{62}Sm^{149}$	7/2	0.096	65±2	m, n, o, p	65±2	4	o	⩾7.9[rr]
$_{63}Eu^{151}$	5/2	0.33	70±10	j, q	85±8	2.5	q	5.8
		0.46	93±3	j, q				
		1.06	94±3	j, q				
$_{64}Gd^{157}$	7/2	0.03	100±30	m, n, r	100±30	20	kk	7.0
$_{64}Gd$		2.58	70±10	s				

[25] D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report 325 (Office of Technical Services, Department of Commerce, Washington, D. C., 1955).
[26] V. E. Pilcher (private communication).

TABLE V.—*Continued.*

Target isotope	Target spin (I)	Resonance energy (ev)	Γ_γ (mv)	Ref.	$\bar{\Gamma}_\gamma$ (mv)	D (ev)	Ref.	$\bar{E}_B$ (Mev)
$_{67}Ho^{165}$	7/2	18.2	170±80	t	87±12	12	t	5.7
		36	60±20	t				
		40	60±30	t				
		49	80±20	t				
		73	130±50	t				
$_{69}Tm^{169}$	1/2	14.4	90±30	t	90±15	15	t	5.9
		17.6	60±20	t				
		35	120±40	t				
$_{71}Lu^{175}$	7/2	11.3	40±20	t	85±15	8	t, u	5.9
		20.7	160±50	t				
		23.7	70±20	t				
		37	90±30	t				
		42	80±30	t				
$_{71}Lu^{176}$	⩾7	0.14	63±5	u	63±5	3	t, u	7.1
$_{72}Hf^{177}$*	⩽3/2	1.08	43±10	v	56±6	9	v	7.4
		2.38	63±8					
		6.5	44±20					
$_{73}Ta^{181}$	7/2	4.28	49±5	w, x	49±4	12	x	6.1[rr]
		10.4	50±8					
$_{74}W^{182}$	0	4.15	70±20	b	70±20	~20	b	6.4
$_{75}Re^{185}$	5/2	2.18	90±20	y	90±20	10	y, nn	6.4
$_{77}Ir^{191}$	1/2	0.65	80±5	z	80±5	~10	e, z	6.4
$_{77}Ir^{193}$	3/2	1.31	94±5	z	94±5	~10	e, z	6.2
$_{79}Au^{197}$	3/2	4.93	125±3	d, l	125±3	60		6.5
		61	170±80					
$_{80}Hg^{198}$	0	23.3	145±20		145±20	~100	kk	6.6
$_{80}Hg^{199}$	1/2	34	$J=0$ 200±40		250±50	~100	kk	7.8
			$J=1$ 310±40					
$_{81}Tl^{203}$	1/2	240	800		800	15 000	oo, pp	6.5[rr]
$_{83}Bi^{209}$	9/2	810	44		44	10 000	gg, qq	4.6[rr]
$_{90}Th^{232}$	0	22.1, 23.8	30±10		30±10	22		5.0[rr]
$_{92}U^{238}$	0	6.70	24±2		25±2	20		4.7[rr]
		21.1	30±6					
		37.1	40±20					

[a] V. L. Sailor, Phys. Rev. **91**, 53 (1953).
[b] W. Selove, Phys. Rev. **84**, 869 (1951).
[c] C. Sheer and J. Moore, Phys. Rev. **98**, 565 (1955).
[d] R. E. Wood, Phys. Rev. **95**, 644 (1954).
[e] L. J. Rainwater *et al.*, Phys. Rev. **71**, 65 (1947).
[f] B. N. Brockhouse, Can. J. Phys. **31**, 432 (1953).
[g] R. S. Carter and J. A. Harvey, Phys. Rev. **95**, 645 (1954).
[h] B. D. McDaniel, Phys. Rev. **70**, 832 (1946).
[i] L. B. Borst, Phys. Rev. **90**, 859 (1953).
[j] H. H. Landon and V. L. Sailor, Phys. Rev. **98**, 1267 (1955).
[k] H. L. Foote, Jr., Phys. Rev. **94**, 790 (1954).
[l] H. H. Landon and V. L. Sailor, Phys. Rev. **93**, 1030 (1954).
[m] L. B. Borst *et al.*, Phys. Rev. **70**, 557 (1946).
[n] W. J. Sturm, Phys. Rev. **71**, 757 (1947).
[o] V. L. Sailor (private communication).
[p] A. W. McReynolds and E. Anderson, Phys. Rev. **93**, 195 (1954).
[q] Sailor, Landon, and Foote, Phys. Rev. **93**, 1292 (1954).
[r] T. Brill and H. V. Lichtenberger, Phys. Rev. **72**, 585 (1947).
[s] L. M. Bollinger and R. R. Palmer, Atomic Energy Commission Report ANL-5080, 1953 (unpublished).
[t] Pilcher, Carter, and Stolovy, Phys. Rev. **95**, 645 (1954).
[u] Foote, Landon, and Sailor, Phys. Rev. **92**, 656 (1953).
[v] Bollinger, Harris, Hibdon, and Muehlhause, Phys. Rev. **92**, 1527 (1953).
[w] R. L. Christensen, Phys. Rev. **92**, 1509 (1953).
[x] H. L. Foote and R. E. Wood (private communication).
[y] Melkonian, Havens, and Rainwater, Phys. Rev. **92**, 702 (1953).
[z] H. H. Landon, Phys. Rev. **100**, 1414 (1955).
[aa] Hibdon, Langsdorf, and Holland, Phys. Rev. **85**, 595 (1952).
[bb] P. H. Stelson and W. M. Preston, Phys. Rev. **88**, 1354 (1952).
[cc] R. E. Fields and M. Walt, Phys. Rev. **83**, 479 (1951).
[dd] G. Freier *et al.*, Phys. Rev. **78**, 508 (1950).
[ee] Adair, Bockelman, and Peterson, Phys. Rev. **76**, 308 (1949).
[ff] Peterson, Barschall, and Bockelman, Phys. Rev. **79**, 593 (1950).
[gg] Bollinger, Palmer, and Dahlberg, Phys. Rev. **95**, 645 (1954).
[hh] D. A. Dahlberg and L. M. Bollinger, Phys. Rev. **95**, 645 (1954).
[ii] S. P. Harris, Atomic Energy Commission Report ANL-5031, 1953 (unpublished).
[jj] W. W. Havens, Jr. *et al.* (unpublished).
[kk] L. M. Bollinger and R. R. Palmer, U. S. Atomic Energy Commission Report ANL-5031, 1953 (unpublished).
[ll] V. L. Sailor and L. B. Borst, Phys. Rev. **87**, 161 (1952).
[mm] C. Heindl and I. W. Ruderman, Atomic Energy Commission Report CU-117, 1953 (unpublished).
[nn] S. P. Harris and L. M. Bollinger, Atomic Energy Commission Report ANL-4659, 1951 (unpublished).
[oo] Wu, Rainwater, and Havens, Phys. Rev. **71**, 174 (1947).
[pp] J. H. Gibbons and H. W. Newson, Phys. Rev. **95**, 644 (1954).
[qq] J. H. Bibbons and H. W. Newson, Phys. Rev. **91**, 209 (1953).
[rr] Experimentally determined values. Binding energies of other isotopes are calculated from the empirical mass formula.

$g=\frac{5}{8}$.[27] The thermal cross section is 0.50±0.01 b, from which it follows that $\Gamma=220$ ev and $\Gamma_\gamma=0.4$ ev.

The first level in Si^{28} is at 200 kev, the average spacing of $l=0$ levels is at least 500 kev, and $\sigma_{th}=0.08 \pm 0.03$ b. From the shape of the resonance, which has been measured with good resolution, the total width is observed to be about 40 kev. In addition it is known that the statistical weight is unity, and $\Gamma_n/\Gamma\approx 1$. From this information one can compute that $\Gamma_\gamma\approx 9$ ev. Since in this case other levels probably make appreciable contributions to σ_{th}, a safer statement would be $\Gamma_\gamma<9$ ev.

The first $l=0$ level in S^{32} is at 100 kev and the average spacing of these levels is about 200 kev; $\sigma_{th}<0.5\times 10^{-3}$ b. The observed width of the level is 18 kev, $g=1$, and $\Gamma_n/\Gamma\approx 1$. From Eq. (6) it follows that $\Gamma_\gamma<25$ ev.

2. *Manganese, Cobalt, and Zinc*

The analysis of the 346-ev level in Mn^{55} is described in reference 13. The average spacing of levels in this energy region is about 2 kev, and the parameters of the 1080-ev and 2360-ev levels[28] indicate that their contribution to the 13.3±0.3 b thermal cross section is

[27] Hibdon, Muehlhause, Selove, and Woolf, Phys. Rev. **77**, 730 (1950).

[28] Bollinger, Palmer, and Dahlberg, Phys. Rev. **95**, 645 (1954).

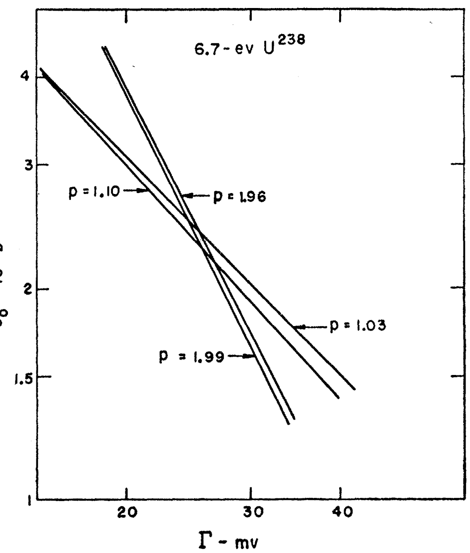

FIG. 5. A plot of σ_0 as a function of Γ for four (two thin and two thick) samples for the 6.7-ev level in U^{238} to obtain the peak cross section σ_0 and the half-width Γ. The internal consistency of the measurements is obvious from the figure.

small compared to that of the 346-ev level. The value[13] of $\sigma_0\Gamma^2$ of $(3.0\pm0.6)\times10^6$ ev^2 b for this resonance and the fact that it is predominantly scattering therefore indicate that $\Gamma_\gamma=0.5$ ev.

The first level in Co^{59} is at 134 ev, and with an average level spacing of about 5 kev the contribution of this level predominates in the thermal capture cross section. By using the parameters of this resonance as given in reference 13, one can calculate its radiation width in two ways. One means of calculation is to use the values $\Gamma=5\pm1$ ev and $\Gamma_n/\Gamma=0.94$ [29] to obtain $\Gamma_\gamma=0.3$ ev. The other is to make use of the quantities $\sigma_0\Gamma=(51\pm3)\times10^3$ ev b and $\sigma_{th}=37\pm2$ b to obtain $\Gamma_\gamma=0.7$ ev. A good estimate of the radiation width is then $\Gamma_\gamma=0.5$ ev.

Zn^{68} has its first level at 530 ev and an average level spacing of about 3 kev. The total width of this resonance is 10 ev[30] and is almost entirely neutron width. The thermal cross section of 1.1 ± 0.2 b thus implies a radiation width of 0.17 ev.

3. *Thallium and Bismuth*

The 240-ev level in Tl has been identified as belonging to Tl^{203}, and has a $\sigma_0\Gamma^2$ of 1.5×10^5 ev^2 b.[31] The spacing of all levels excited by $l=0$ neutrons in this isotope is about 8 kev. The thermal capture cross section of 11 ± 1 b implies the following sets of parameters for this level:

J	σ_0	Γ	Γ_γ
0	2300 b	8 ev	1.4 ev
1	6700 b	5 ev	0.8 ev

[29] Harris, Muehlhause, and Thomas, Phys. Rev. **79**, 11 (1950).
[30] D. A. Dahlberg and L. M. Bollinger, Phys. Rev. **95**, 645 (1954).
[31] A. Stolovy and J. A. Harvey (to be published).

A peak cross section of 4100 b has been observed in a thin-sample measurement with no correction made for instrumental resolution.[31] Since $\frac{1}{4}(4\pi\lambda\!\!\!^{-2})=2700$ b at 240 ev is the maximum cross section possible with $l=0$ neutrons for $I=\frac{1}{2}$ and $J=0$, it seems certain that $J=1$ is the proper assignment, and $\Gamma_\gamma=0.8$ ev.

The first two levels in Bi^{209} have resonance energies of 810 ev and 2370 ev and total widths of 5.3 and 19 ev, respectively,[28] whereas the average spacing of levels is about 5 kev. The small thermal capture cross section of $(32\pm3)\times10^{-3}$ b implies a radiation width of only 44 mv if the contributions of these two levels are assumed to be additive.

V. DISCUSSION

A. Variation of Γ_γ in a Single Nuclide

The radiation width of a highly excited level of a compound nucleus is the sum of the partial widths for transitions to various lower levels. For most nuclides it is assumed that the number of these lower levels to which transitions of various multipole orders are possible is very large. Therefore, small differences in excitation energy and spin of neighboring resonances in a given nuclide should not cause much variation in the observed radiation widths of those resonances. It is possible to investigate the validity of this argument by using the data in Table V for nuclides in which Γ_γ has been measured for more than one level.

For the ith resonance in a compound nucleus, the values of Γ_γ are random samples taken from populations with a mean $\Gamma_{\gamma i}$ and standard deviations given by the estimates of error. Although these errors are generally quoted as plus or minus given quantities, for radiation widths measured by the area and interference methods it is more nearly true that the measured values are apt to be too large or too small by the same *factors;* for example, measured values twice as large as the true value or half as large as the true value have almost equal probabilities. It is therefore a better approximation to assume that the measured values are samples taken from populations in which the *logarithms* of Γ_γ are normally distributed, rather than to assume normal distributions for the Γ_γ's themselves. If the fractional errors in the Γ_γ's are small, as is generally true of measurements by the shape method, this distinction is of no importance; however, for large fractional errors the difference is significant. For those cases in which the radiation width of a level has been measured by several experimenters, the values given in the fourth column of Table V are calculated by averaging the logarithms of the values of Γ_γ, each being weighted inversely as the square of the standard deviation of the logarithm.

In order to test the hypothesis that $\Gamma_{\gamma i}$, the best estimate of the radiation width of the ith level, has the same value $\bar{\Gamma}_\gamma$ for each resonance in a given nuclide, one calculates the statistical quantity $\chi^2=\sum_i (\ln\Gamma_{\gamma i}-\ln\bar{\Gamma}_\gamma)^2/\sigma_i^2$, where σ_i is the standard deviation of $\ln\Gamma_{\gamma i}$, and $\ln\bar{\Gamma}_\gamma$ is the weighted average of the $\ln\Gamma_{\gamma i}$'s. If $\ln\Gamma_{\gamma i}$ is normally distributed for each i, the quantity χ^2 has a chi-square distribution with $n-1$ degrees of freedom, where n is the number of levels considered. Those nuclides for which the radiation widths have been measured for more than one level are listed in Table VI. The third column of this table gives P, the probability of observing a larger value of χ^2, that is, the probability of observing a larger scatter in the $\Gamma_{\gamma i}$ if the measurements were repeated with the same accuracy. If P is not too small, then it is reasonable to conclude that the radiation widths are the same from level to level and the observed spread in value is due just to experimental errors.

Of the 11 target isotopes in which two or more radiation widths have been measured two, In^{115} and Lu^{175}, have values of P less than 0.05. The data for these two nuclides, therefore, show large scatter relative to the assigned errors and do not seem to agree with the assumption that radiation widths are the same from level to level. In Lu (97.4 percent Lu^{175}, 2.6 percent Lu^{176}) the isotopic identification of the levels is based on the relative sizes of the resonances as observed in the normal element,[26,32] not on measurements with separated isotopes. Since variations in size ($\sigma_0\Gamma/E_0^{\frac{1}{2}}$ or $\Gamma_n/E_0^{\frac{1}{2}}$) as large as a factor of 200 have been observed for levels in a single nuclide,[33] it is possible that some of the resonances assigned to Lu^{175} are really large resonances in Lu^{176}. As the radiation width of the 0.14-ev level in Lu^{176} seems to be somewhat smaller than the widths observed for the levels assigned to Lu^{175}, such an error in isotopic assignment could account for the large scatter in the Lu^{175} radiation widths.

The small value of P for In^{115} is primarily due to the large difference in the well-measured values of Γ_γ for the 1.46-ev and 3.85-ev levels. It is known that the ratio of the population of the isomeric state of In^{116} to that of the ground state of In^{116} is also different for these two levels.[34] With this additional information it is reasonable to conclude that the difference in radiation widths of these levels is real. This difference is probably attributable to a difference in the spins of the two levels, however, the spins of these resonances have not been measured. A similar situation exists in Eu^{151}, where the radiation width of the 0.33-ev level appears to be significantly different from that of the 0.46-ev and 1.06-ev levels. Here it is also known that the first level populates the isomeric state of Eu^{152} differently than do the other two levels.[35] Again, the spins of these resonances have not been measured.

[32] Foote, Landon, and Sailor, Phys. Rev. **92**, 656 (1953).
[33] Harvey, Hughes, Carter, and Pilcher, Phys. Rev. **99**, 10 (1955).
[34] V. L. Sailor (private communication).
[35] R. E. Wood, Phys. Rev. **95**, 453 (1954).

TABLE VI. The probability P of observing greater deviations in Γ_γ, calculated on the assumption that Γ_γ is constant from level to level in a given isotope.

Target isotope	Number of levels	Probability P of observing greater deviations in Γ_γ
$_{49}In^{113}$	2	0.15
$_{73}Ta^{181}$	2	0.92
$_{79}Au^{197}$	2	0.42
$_{55}Cs^{133}$	3	0.98
$_{63}Eu^{151}$	3	0.12
$_{69}Tm^{169}$	3	0.43
$_{72}Hf^{177}$	3	0.21
$_{92}U^{238}$	3	0.36
$_{49}In^{115}$	5	0.03
$_{67}Ho^{165}$	5	0.14
$_{71}Lu^{175}$	5	0.02

The other nine target isotopes listed in Table VI have values of P greater than 0.1. Taken separately, the value of P for each of these nuclides does not disagree with the assumption that the radiation widths are the same from level to level. However, only two cases have values of P greater than 0.5, whereas one would expect values greater than 0.5 to occur in about half of the cases. It seems probable, therefore, that the radiation widths of neighboring levels in a given nuclide are not *exactly* the same. The fact that P is greater than 0.1 for nine of the eleven target isotopes investigated shows, however, that the variation of Γ_γ in any one isotope is of the same order as the experimental errors, that is, about 20 percent. The cases of In^{115} and Eu^{151} indicate that this variation in radiation width may be attributable to differences in the spin of different levels. Since, of the 11 target isotopes listed in Table VI, only the levels of U^{238} have just one spin state, one cannot reject the hypothesis that for levels of a given spin Γ_γ is exactly the same.

B. Dependence of $\bar{\Gamma}_\gamma$ on Spin

As discussed in Sec. V.A, there is some experimental evidence that Γ_γ is slightly different for levels of different spin in the same nuclide. It is also of interest to see whether there is any correlation between the magnitude of $\bar{\Gamma}_\gamma$ and that of J for different nuclides. It is not easy to investigate this since very few level spins have been measured; however, since $J=I\pm\frac{1}{2}$ for levels excited by $l=0$ neutrons, it is almost as informative to look for correlations between $\bar{\Gamma}_\gamma$ and the spin of the target isotope, I. This has been done by Hughes and Harvey[24] for about 20 of the isotopes listed in Table V. The additional values listed in this table do not alter their conclusion—there appears to be no dependence of $\bar{\Gamma}_\gamma$ on J. This indicates that the $(2J+1)$ factor introduced by some authors[4,36,37] when comparing measured radiation widths with theoretical estimates is not correct, a point that has been discussed in greater detail by Hughes.[38]

[36] M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951).
[37] D. H. Wilkinson, Phil. Mag. **44**, 450 (1953).
[38] D. J. Hughes, Phys. Rev. **94**, 740 (1954).

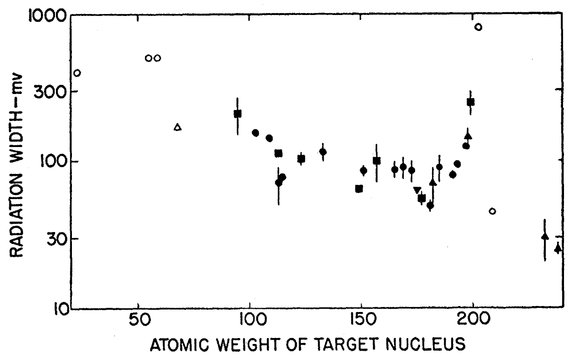

FIG. 6. The average width for various nuclides as a function of atomic weight of the target nucleus. The open points are values computed from thermal cross sections while the others are measurements of actual resonances. The symbols refer to the type of target nucleus: ▲△=even Z-even N; ▼=odd Z-odd N; ●○=odd Z-even N; ■=even Z-odd N.

C. Variation of $\bar{\Gamma}_\gamma$ with Atomic Weight

In Fig. 6 the average radiation width $\bar{\Gamma}_\gamma$ listed in Table V is plotted as a function of the atomic weight A of the target nucleus. The over-all trend indicates a slow decrease of $\bar{\Gamma}_\gamma$ with increasing A, as was found by Heidman and Bethe[7] and Hughes and Harvey.[24] In addition there is an increase in $\bar{\Gamma}_\gamma$ by a factor of 10 just before $A=200$, followed by a decrease of a factor of 20 between $A=203$ and $A=209$. This behavior seems to be related to the closed neutron and proton shells at $A=208$. A similar but much less pronounced behavior has also been observed near the closed neutron shell at $A\approx 140$ by Stolovy and Harvey[31] with the Brookhaven fast chopper.

It is not at all surprising that $\bar{\Gamma}_\gamma$ shows effects that are related to nuclear shell structure. Similar effects are also present in neutron binding energies[39] and nuclear level densities.[5] Hughes *et al.*,[5] who used the radiation widths given by Heidman and Bethe[7] to interpret neutron capture cross sections measured at an effective energy of 1 Mev, concluded that level densities remained constant up to a closed neutron shell, dropped *abruptly* at the shell, and then gradually returned to their normal value for nuclides with about 10 neutrons outside the closed shell. Reinterpreting this capture cross section data in the light of the dependence of $\bar{\Gamma}_\gamma$ on A shown in Fig. 6, one finds that the level density shows a gradual decrease before the closed shell as well as a gradual increase after the shell; this is in agreement with the findings of Newson and Rohrer.[40]

D. Dependence of $\bar{\Gamma}_\gamma$ on Excitation Energy and Level Spacing

By using a modified independent-particle model, Blatt and Weisskopf[3] have estimated the partial radiation width for transitions of order l between a highly excited state a and a lower state b. They obtained

$$\Gamma_{El}(a,b)\approx\frac{18(l+1)(2l+1)}{l(l+3)^2(1\cdot3\cdots2l+1)^2}\frac{e^2}{\hbar c}\left(\frac{R}{\hbar c}\right)^{2l}\times\frac{D(E_a)}{D_0}E^{2l+1},\quad(7)$$

and

$$\Gamma_{Ml}(a,b)\approx10(\hbar/McR)^2\Gamma_{El}(a,b),\quad(8)$$

where E is the energy of the emitted gamma-ray, $D(E_a)$ is the spacing of levels of the same spin and parity as a near E_a, D_0 is the spacing of low-lying *single-particle* levels of the same spin and parity that can combine with the ground state by transitions of the type considered, R is the nuclear radius, and M is the mass of a nucleon. The total radiation width for transitions of order l of state a is given by $\Gamma\equiv\sum_b\Gamma(a,b)$. Since for the resonances to be considered the number of states b to which transitions are possible is large, the summation can be replaced by an integral, giving

$$\Gamma_{El}\approx\frac{18(l+1)(2l+1)}{l(l+3)^2(1\cdot3\cdots2l+1)^2}\frac{e^2}{\hbar c}\left(\frac{R}{\hbar c}\right)^2\frac{D(E_a)}{D_0}\times\int_0^{E_a}\frac{E^{2l+1}}{D_l(E_a-E)}dE,\quad(9)$$

where $D_l(E_a-E)$ is the spacing of levels near $E_a-E=E_b$ that can combine with level a by emission of radiation of the type considered. The ratio of Γ_{El} to Γ_{Ml} is again given by Eq. (8), which predicts $\Gamma_{Ml}/\Gamma_{El}\approx10^{-2}$ for $A\approx160$.

Admittedly the estimate of radiation width given by Eqs. (7), (8), and (9) is very rough; in particular, the statistical factors are not the correct ones. However, these equations do provide a basis for comparing radiation widths measured in nuclides having very different level spacings and excitation energies. The level spacings $D(E_a)$ needed for such a comparison are listed in the seventh column of Table V; the references from which this information was obtained are listed in the eighth column. (The data from almost all these references can be found in reference 25.) In some cases only one or two levels have been observed and consequently the estimated level spacings are very approximate; however, it will be seen that the variation of Γ_γ with changing $D(E_a)$ is very slow, therefore such rough estimates are adequate. For all but the lightest target nuclei in Table V, the excitation energies E_a are the same as the neutron binding energies E_B that are listed in the last column of this table. Binding energies not determined experimentally were calculated by means of the empirical mass formula[41] with corrections for shell

[39] J. A. Harvey, Phys. Rev. 81, 353 (1951).
[40] H. W. Newson and R. H. Rohrer, Phys. Rev. 94, 654 (1954).
[41] N. Metropolis and G. Reitwiesner, Atomic Energy Commission Report NP-1980, 1950 (unpublished).

effects taken from the summaries of Harvey[39] and Wall.[42]

In order to proceed with a comparison of radiation widths of nuclei having different excitation energies and level spacings, it is necessary to assume a form for $D_l(E)$, the spacing of final levels for the first gamma-ray transition. Very little is known, either experimentally or theoretically, about the behavior of this level spacing as a function of energy. The statistical model of the excited nucleus predicts a nuclear level density (the reciprocal of the level spacing) of the form

$$\rho(E)=c\exp[(aE)^{\frac{1}{2}}], \tag{10}$$

where $\rho(E)$ is the density of levels of *all* spins and parities, and a and c are parameters which vary with atomic weight and must be estimated from the scant experimental data available. Since better information is not available, this functional form will be assumed. Because the model does not predict the spacing of levels of a single spin and parity, it will also be assumed that for the spins of interest ($J\lesssim 8$), and over the energy range of interest (0 to 8 Mev), the spacing is the same for levels of every spin and either parity. Under the above assumptions Eq. (9) reduces to

$$\Gamma_{El}\approx\frac{18(l+1)(2l+1)}{l(l+3)^2(1\cdot 3\cdots 2l+1)^2}\frac{e^2}{(\hbar c)^{2l+1}}\times\frac{R^{2l}}{D_0}E_B{}^{2(l+1)}Gf_l(cD(E_B)), \tag{11}$$

where

$$f_l(cD(E_B))=\frac{2cD(E_B)}{[\ln cD(E_B)]^{4(l+1)}}\int_0^{-\ln cD(E_B)}\times\{[\ln cD(E_B)]^2-u^2\}^{2l+1}ue^u du. \tag{12}$$

The factor G is the lesser of the quantities $(2J+1)$ and $(2l+1)$. It arises from the fact that $D_l(E)$ is the spacing of all levels that can combine with the initial state by radiation of order l, whereas the parameter c will be taken to refer to levels of a single spin and parity. (For example, a level of spin $\frac{1}{2}$ can have $E1$ transitions only to states of spin $\frac{1}{2}$ or $\frac{3}{2}$ and therefore has $G=2$, whereas a level of spin $\geqslant 1$ has $G=3$ for this type transition.) It should be noted that the parameter a of Eq. (10) does not appear in Eqs. (11) and (12) because, for a given c, a is selected to give the experimentally observed level spacing at excitation energy E_B.

If it is assumed that all the radiation widths listed in Table V represent transitions of the same multipole order and parity change, then, under the assumptions that lead to Eqs. (11) and (12), the experimentally observed $\bar{\Gamma}_\gamma$'s are functions of R, E_B, G, and $cD(E_B)$. In particular, for nuclides with the same $cD(E_B)$, the quantity $\bar{\Gamma}_\gamma/GR^2$ for electric multipole radiation or $\bar{\Gamma}_\gamma/GR^{2(l-1)}$ for magnetic multipole radiation should be

[42] N. S. Wall, Phys. Rev. **96**, 664 (1954).

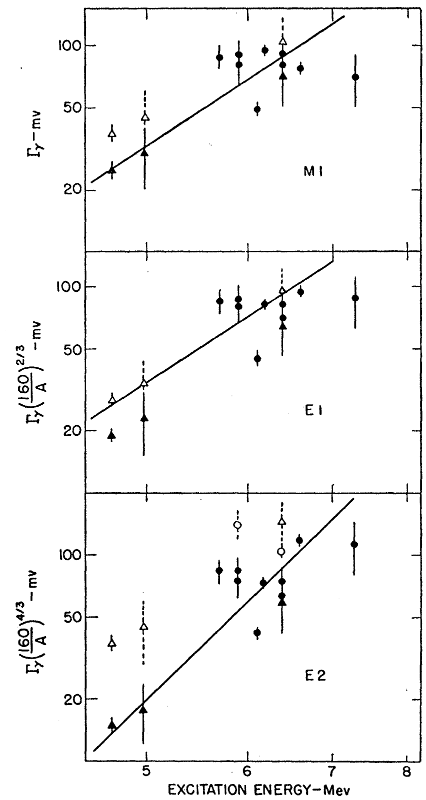

FIG. 7. Radiation widths plotted against excitation energy of compound nucleus. The slopes of the lines are those expected for $M1$, $E1$, and $E2$ transitions. The open points represent values of Γ_γ that have been normalized to $G=2l+1$ as explained in the text. The symbols have the same meaning as in Fig. 6.

proportional to $E_B{}^{2(l+1)}$. An investigation of the dependence of this quantity on E_B should yield information on the type of transition involved. Fortunately $f_l(cD(E_B))$ is a slowly varying function of its argument; over the range of $cD(E_B)$ to be considered f_l for $l=1$ or 2 changes only by a factor of 2 or 3 for a factor of 10 change in $cD(E_B)$. Therefore for this investigation nuclides with values of $cD(E_B)$ lying with a range of a factor of 3 or 4 of each other can be considered together without introducing fluctuations larger than the experimental errors in their $\bar{\Gamma}_\gamma$'s.

In order to select those target nuclei having approximately the same value of $cD(E_B)$ it is necessary to know the dependence of the parameter c on atomic weight. Two different forms of variation will be considered: c independent of atomic weight and c of the form obtained by Blatt and Weisskopf[43] from experimental data. Both of these are very crude. For a c independent of atomic weight, the 14 target isotopes having $D(E_B)$ between 8 ev and 22 ev can be considered as a group. For the c varying with atomic weight these 14 nuclides form a group in which $cD(E_B)$ falls within a range of a factor of 4. In Fig. 7 the quantities $\bar{\Gamma}_\gamma$, $\bar{\Gamma}_\gamma(160/A)^{2/3}$, and $\bar{\Gamma}_\gamma(160/A)^{4/3}$, which are related to $M1$, $E1$, and $E2$ radiation, respectively, are plotted logarithmically as functions of E_B for 12 of these 14 target isotopes. The two cases of even-even

[43] J. M. Blatt and V. F. Weisskopf, reference 3, pp. 371–374.

compound nuclei are omitted since it will be seen later that nuclides of this type have a behavior different from that of other nuclides. The open points in Fig. 7 occur in those cases where the factor G is expected to be less than $(2l+1)$. These points represent the same target nuclei as the solid points below them; they differ in that for the open points the $\bar{\Gamma}_\gamma$'s have been normalized to $G=2l+1$. The solid lines drawn in Fig. 7 indicate the expected values of the slopes, 4 for $M1$ and $E1$ radiation and 6 for $E2$ radiation. Unfortunately any fit to the data must give considerable weight to the two points with the lowest excitation energies, U^{238} and Th^{232}. Since these two target nuclei are the heaviest for which $\bar{\Gamma}_\gamma$'s are available, the positions of these points relative to the rest of the data are very much changed by the different dependences on nuclear radius. Figure 7 does demonstrate that for the assumptions made the experimental data show energy dependences which do not differ from those predicted for $E1$ transitions by more than ± 1 in the exponent of E_B.

Having demonstrated roughly the expected energy variation one can remove the dependence on excitation energy from all of the $\bar{\Gamma}_\gamma$'s that have been measured in order to investigate the dependence on level spacing, $cD(E_B)$. Figure 8 shows the data treated in this manner for the assumption that the radiation is electric dipole and that c is independent of atomic weight. The solid points represent those radiation widths that have been measured directly; the open points are those inferred from thermal capture cross sections. For even-even *target* nuclei the $\bar{\Gamma}_\gamma$'s have been multiplied by $\frac{3}{2}$ to normalize them to $G=3$. The results for the even-even *compound* nuclei have been plotted separately since these nuclei seem to have values of the ordinate that average a factor of 2.5 smaller than the values that are found for other types of nuclei.

The lower curve in Fig. 8 represents a normalizing factor, K, times $f_l(cD(E_B))$, where f_l is calculated from Eq. (12) for $l=1$. The upper curve is this same f_l with K 2.5 times smaller. This normalizing factor and the value of the parameter c were not selected *a priori;* they were chosen to give a good fit of the theoretical

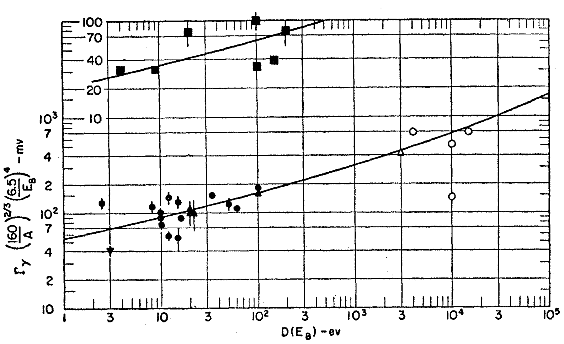

FIG. 8. Radiation widths adjusted for nuclear sizes and binding energies plotted against level spacing. The even-even *compound* nuclei are plotted separately from the others; the shape of the curves is obtained from Eq. (12) but the magnitude is arbitrary.

curve to the experimental data. Since $f_l(cD(E_B))$ is a slowly varying function of its argument and the experimental points occur in two widely separated clusters, this curve is essentially required only to pass through the centers of these two clusters of points. It then follows from the fact that the curve is so nearly linear that K and the parameter c are not each uniquely determined: c can be increased or decreased by about an order of magnitude and K then decreased or increased by about a factor of 2 without seriously worsening the fit. For the fit as shown in Fig. 8 the equation of $\bar{\Gamma}_\gamma$ is

$$\bar{\Gamma}_\gamma = KG\left(\frac{A}{160}\right)^{\frac{2}{3}}\left(\frac{E_B}{6.5}\right)^4 f_1(0.1D(E_B)), \qquad (14)$$

where $\bar{\Gamma}_\gamma$ is in ev, E_B and $D(E_B)$ are in Mev, and K is 10 ev if the compound nucleus is even-even, and 24 ev otherwise.

The removal of the effects of differences in nuclear radii, excitation energies, and level spacings from the radiation widths has greatly diminished the fluctuations observed in the experimental data. Whereas in Fig. 6 the experimental points spread over a range of a factor of 30, in Fig. 8 only one of the points deviates from the curves by more than a factor of 2, and the average deviation is only a factor of about 1.3. The one point which does not agree with the curve is that for Bi^{209}, and it was pointed out earlier that this radiation width could easily be in error by a large factor. In addition, the low values of $\bar{\Gamma}_\gamma$ for U^{238} and Th^{232} and the increase in $\bar{\Gamma}_\gamma$ near $A=200$ can now be understood in terms of the observed level spacings and excitation energies.

It should not be concluded, however, that this agreement between the experimental values and the theoretical prediction definitely proves that electric dipole radiation is the most probable mode of decay of these highly excited states, or that it exactly determines the dependence of level spacing on excitation energy and atomic weight. It should be noted that the radiation widths of the even-even compound nuclei can be brought into agreement with those observed for other types if one assumes that for the even-even nuclei the parameter c is about 30 times smaller than it is for the others. Since most of the other nuclei are odd-Z—odd-N in the compound state, this assumption is not unreasonable.

VI. CONCLUSIONS

Several general conclusions can be drawn from the behavior of radiation widths discussed in Sec. V. Perhaps the most important of these is that the variation of radiation widths of highly excited levels with excitation energy and level spacing seems to be in agreement with a statistical model of the compound nucleus. The fact that Γ_γ is approximately constant from level to level in a given nuclide indicates that the de-excitation proceeds by gamma-ray emission to *many* lower levels, with the choice for any given nucleus determined statistically. The observation that the variation of the average radiation width from nuclide to nuclide does not show a random dependence on excitation energy also indicates that the decay does not go just to a few favored levels, for in such a case there would be no reason to expect the energy of the first gamma-ray transition to show any marked correlation with the total excitation energy available. Evidence that in the same nuclide the radiation widths may differ slightly for levels of different spin does not contradict the assumption of statistical behavior. This may just indicate that the variation with excitation energy of the nuclear level spacing is different for levels of different spins.

It can also be concluded that the level density formula predicted by the statistical model of the nucleus [Eq. (10)] is a reasonable approximation. Weisskopf[44] has estimated the matrix elements for electric and magnetic multipole transitions for a nuclear model in which the radiation is caused by a transition of a single nucleon which moves independently within the nucleus. For highly excited levels in medium and heavy nuclides, Blatt and Weisskopf[3] have modified the single-particle formulas to take into account the complexity of the emitting state. Kinsey and Bartholomew[4] studied the partial widths for emission of radiation to the ground state from such levels and found that the energy dependence and nuclear size dependence predicted by the modified formulas agree with the experimental results, although the absolute magnitude of the theoretical widths is too large. The functional dependence of the single-particle formulas and their modification therefore seem to be well verified by the experimental data. The fact that the additional assumption of the variation of level density with energy, Eq. (10), permits a satisfactory fit to the dependence of Γ_γ on excitation energy, level spacing, and nuclear size, indicates that this level density formula cannot be grossly in error. Furthermore, Critchfield and Oleksa,[45] by assuming $E1$ transitions and using a level density formula obtained by exact counting of levels, as well as one derived from statistical theory, have had considerable success in calculating the energy and nuclear size dependence of neutron capture cross sections (which are proportional to $\bar{\Gamma}_\gamma/D$) for elements between Na and Cu and neutron energies between 0.1 and 0.8 Mev—an additional verification of the reasonableness of these formulas.

The information that has so far been obtained from the total radiation widths of highly excited levels in medium and heavy nuclides is not by itself sufficient to enable one to determine which mode of decay is most important in these transitions. However, when taken with other experimental data it seems almost certain that most of the radiation emitted by these levels is electric dipole. If Eq. (7) is used to calculate D_0, from the partial widths of those transitions observed by Kinsey and Bartholomew[4] that are known to be $E1$, one obtains values ranging from about 6 Mev to 600 Mev, with a mean value of about 150 Mev. If one now considers the total radiation widths of the present work, the values of D_0 obtained by fits of the theoretical curve [Eqs. (11) and (12)] to the experimental data are about 500, 2 or 0.2 Mev for the assumption of $E1$, $M1$, or $E2$ radiation, respectively. Within experimental accuracy, the value of D_0 of 500 Mev agrees with the 150-Mev value obtained from the partial widths of Kinsey and Bartholomew where the transitions are known to be electric dipole. On the other hand, the values of D_0 obtained from the total widths if the radiation is assumed to be $M1$ or $E2$ are very different. This fact is a strong indication that the assumption of $E1$ transitions for the total radiation widths is correct.

It seems rather surprising that the experimental value of D_0 is so large—a value of about 15 Mev would be calculated for a single particle in a nuclear potential well. The factor of 300 discrepancy between Blatt and Weisskopf's[3] calculations of $E1$ radiation widths and the experimentally observed values arose from their assumption of 0.5 Mev for D_0, rather than this larger value. The observed partial radiation widths of Kinsey and Bartholomew and total radiation widths of this paper are thus about an order of magnitude smaller than predicted by the modified single-particle formulas. Wilkinson[37] also found that Weisskopf's single-particle formula overestimates electric dipole radiation widths by about a factor of 10.

ACKNOWLEDGMENTS

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[44] V. F. Weisskopf, Phys. Rev. 83, 1073 (1951).
[45] C. L. Critchfield and S. Oleksa, Phys. Rev. 82, 243 (1951).