Small-Order Effects in Beta Spectra*

A. V. POHM,† R. C. WADDELL, AND E. N. JENSEN
Institute for Atomic Research and Department of Physics, Iowa State College, Ames, Iowa

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An intermediate-image spectrometer and a thin-lens spectrometer were used to examine carefully the beta spectra of P^{32}, K^{42}, and As^{76}. The P^{32} beta spectrum was examined for a Fierz-type deviation, and within the experimental error none was found. The value of the parameter r, describing the magnitude of the Fierz deviation as defined by Davidson and Peaslee, was found to be 0.00 ± 0.03. The first forbidden transitions with $\Delta I=0$, "yes" of K^{42} and As^{76} were examined for deviations from a linear Kurie plot. Within experimental error no deviations were found. From the shape factors calculated by Mahmoud and Konopinski on the basis of an STP combination, one would expect deviations for these first-forbidden transitions if the best previous estimate of certain parameters are correct. A deviation of the K^{42} Kurie plot below 500 kev was attributed to an additional beta group.

I. INTRODUCTION

As the result of better experimental techniques and higher specific activity sources, it has become feasible to examine the Kurie plots of certain beta spectra for possible small-order deviations. With this in mind, the authors examined the activities of P^{32}, K^{42}, and As^{76}.

Mahmoud and Konopinski[1] have proposed an STP combination of interactions for the law of beta decay. Important arguments leading to this choice are based on the reported "allowed" shapes of first-forbidden beta transitions with spin changes of zero or one unit and on the apparent absence of Fierz interference effects in allowed transitions.

For allowed transitions the shape factor C_0 (which multiplies the statistical factor to give a linear Kurie plot) would contain Fierz interference terms if a TA (or SV) mixture were present. The apparent absence, within experimental error, of the Fierz effect has led to the tentative conclusion that the TA (or SV) terms are not present. The 1.7-Mev allowed transition of P^{32}, because of its low Z and high energy, would maximize Fierz effects if they were present and consequently is well suited for studying to set limits on possible TA (or SV) admixture. The shape factor C_1, for first forbidden transitions, resulting from the STP combination, gives linear Kurie plots for spin changes of zero and one unit if the Coulomb energy at the nuclear radius predominates over the electron-neutrino kinetic energies; i.e., $\alpha Z/2R \gg W_0-1$, where α is the fine structure constant, Z the atomic number, R the nuclear radius in $\hbar/mc$ units, and W_0 the maximum total energy of the beta spectrum in mc^2 units.[1] For this condition, the theoretical shape factor C_1 is essentially a constant. However, for low-Z and high-energy beta spectra, where W_0-1 approaches the value of $\alpha Z/2R$, the shape factor C_1 is not constant, but may be quite dependent on the beta energy. It thus becomes important to establish experimentally whether first-forbidden high-energy beta spectra from low-Z nuclei exhibit a positive deviation from a linear Kurie plot.

For the 2.0-Mev beta group of K^{42} and the 2.4-Mev beta group of As^{76}, which are presumably transitions with a spin change of zero and a parity change,[2-6] $\alpha Z/2R$ and W_0-1 are 7.3 mc^2 and 3.9 mc^2, respectively, for K^{42} and 10.6 mc^2 and 4.7 mc^2, respectively, for As^{76}. These two beta spectra are shown by the solid lines in the partial decay schemes shown in Figs. 5 and 7. The two beta spectra were examined carefully for the purpose of ascertaining whether or not the shape factors C_1 resulting from the STP combination, as given by Mahmoud and Konopinski,[1] are in agreement with the shape factors obtained experimentally.

The subtraction method was used to obtain the 2.0-Mev beta group of K^{42} because it gave better statistics than could be obtained by coincidence measurements with our present arrangement. The 2.4-Mev beta group of As^{76} was obtained by coincidence measurements and also by the substraction method.

In this investigation, both a thin-lens spectrometer[7,8] and an intermediate-image spectrometer,[9,10] with resolutions of about four percent, were employed. The coincidence measurements were performed with the intermediate-image spectrometer. Two different instruments were used to help detect possible instrumental errors. Also, as a check on the instruments the unique first-forbidden spectrum of Y^{90} was carefully examined.[11-17]

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† Now at Engineering Research Associates, St. Paul, Minnesota.

[1] H. M. Mahmoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952).

[2] Hamilton, Lemonick, and Pipkin, Phys. Rev. **92**, 1191 (1953).

[3] F. B. Shull and E. Feenberg, Phys. Rev. **75**, 1768 (1949).

[4] J. R. Beyster and M. L. Wiedenbeck, Phys. Rev. **79**, 728 (1950).

[5] D. T. Stevenson and M. Deutsch, Phys. Rev. **84**, 1071 (1951).

[6] E. P. Tomlinson and S. L. Ridgway, Phys. Rev. **88**, 170 (1952).

[7] Jensen, Laslett, and Pratt, Phys. Rev. **75**, 458 (1949).

[8] Pratt, Boley, and Nichols, Rev. Sci. Instr. **22**, 92 (1951).

[9] H. Slätis and K. Siegbahn, Arkiv Fysik **1**, 339 (1949).

[10] Nichols, Pohm, Talboy, and Jensen, Rev. Sci. Instr. **26**, 580 (1955).

[11] W. E. Meyerhof, Phys. Rev. **74**, 621 (1948).

[12] J. Moreau and J. Perzy Jorba, Compt. rend. **235**, 38 (1952).

[13] Braden, Slack, and Shull, Phys. Rev. **75**, 1964 (1949).

[14] L. M. Langer and H. C. Price, Jr., Phys. Rev. **76**, 641 (1949).

[15] E. N. Jensen and L. J. Laslett, Phys. Rev. **75**, 1949 (1949).

[16] Laslett, Jensen, and Paskin, Phys. Rev. **79**, 412 (1950).

[17] L. E. Glendenin and C. D. Coryell, *Radiochemical Studies:*

The careful examination of this spectrum also served to indicate the validity with which this type of spectrum could be subtracted from a total beta spectrum.

The single 1.7-Mev allowed beta group of P^{32} has been examined by many investigators[18–29]; however, it is only recently that a careful study of its spectral shape has been made to detect possible Fierz-type effects. Davidson and Peaslee[30] analyzed published P^{32} data of Langer and Price[14] and concluded that in the expression for a Kurie plot with a Fierz-type deviation,

$$(N/IF)^{\frac{1}{2}}=A(W_0-W)[1+(r/W)]^{\frac{1}{2}}, \qquad (1)$$

the parameter r, expressing the magnitude of the Fierz-type deviation, had a value of 0.00 ± 0.07. The quantity N represents the number of counts per minute, I the current, which is proportional to the momentum, W_0 the end-point energy, W the total energy of the beta particle, F the Fermi function, and A a constant of proportionality.

The beta spectra of K^{42} [2–5,31–34] and As^{76} [6,35–43] have been investigated by a number of groups, and it is found that both have a complex beta spectrum. Both presumably contain about a 2-Mev beta transition from a 2− to a 2+ state.

The 2.0-Mev transition of K^{42} has been examined by Koerts *et al.*[33] using coincidence measurements. They found it to have a linear Kurie plot to 0.5 Mev, which then deviates upward.

The transitions from the 2− to 2+ states were of particular interest, for any deviation from a linear Kurie plot which might be found would aid in selecting the form of the interactions in beta decay.

II. APPARATUS AND PROCEDURE

Most sources used in the intermediate-image spectrometer were prepared by vacuum evaporation onto thin aluminized collodion films. The formula of Hamilton and Gross[44] was used in estimating when source effects would become appreciable and no data were used in subsequent calculations below the energy value so obtained. Sources for the thin-lens spectrometer were either prepared as described above or were made by evaporating off the water from a drop of solution placed on a thin aluminized collodion backing.

In processing the data a number of corrections were generally made. These were for (1) counter dead time, which was of the order of 10 μsec, (2) background, (3) decay, and (4) the finite resolution of the spectrometer. Where necessary the electron screening effect was compensated for by corrections obtained by interpolating in the table of Reitz.[45] The corrections necessary to the simple distribution function of Fermi were obtained from the values tabulated by Rose, Perry, and Dismuke.[46] These values were in turn corrected for the new value of the nuclear radius r_0 ($r_0=1.15\times10^{-13}A^{\frac{1}{3}}$ cm) obtained by Pidd.[47] The effect of the cutoff of the Coulomb potential at the nuclear radius is negligible for values of $Z\lesssim60$; hence, this correction was neglected.

The beta spectrum of Y^{90}, examined as an instrument check, consists of a single beta group with an energy of 2.27 Mev and has the unique shape of a first-forbidden transition with a spin change of two units.[11–17] As the shape factor for this transition is uniquely known, any deviations of the observed spectrum from that predicted by theory should result from instrumental errors.

To facilitate the detection of small deviations from a linear Kurie plot, a shape-factor plot was made. By manipulating the usual equation of the Kurie plot, one

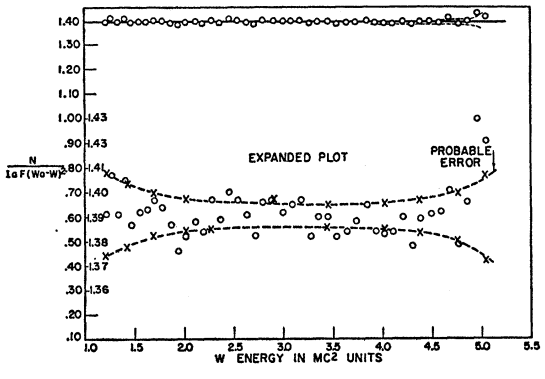

FIG. 1. Shape-factor plot of Y^{90} data obtained with the intermediate-image spectrometer.

The Fission Products (McGraw-Hill Book Company, Inc., New York, 1951), Paper No. 78, National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Div. IV.

[18] M. Duquesne, Compt. rend. **236**, 598 (1953).
[19] H. M. Agnew, Phys. Rev. **77**, 655 (1950).
[20] Jensen, Nichols, Clement, and Pohm, Phys. Rev. **85**, 112 (1952).
[21] Warshaw, Chen and Appleton, Phys. Rev. **80**, 288 (1950).
[22] K. Siegbahn, Phys. Rev. **70**, 127 (1946).
[23] E. M. Lyman, Phys. Rev. **51**, 1 (1937).
[24] R. S. Caswell, Phys. Rev. **86**, 82 (1952).
[25] Kurie, Richardson, and Paxton, Phys. Rev. **49**, 368 (1936).
[26] J. L. Lawson, Phys. Rev. **56**, 131 (1939).
[27] H. C. Paxton, Phys. Rev. **51**, 170 (1937).
[28] C. W. Sherwin, Phys. Rev. **82**, 52 (1951).
[29] Sheline, Holtzman, and Fan, Phys. Rev. **83**, 919 (1951).
[30] J. P. Davidson and D. C. Peaslee, Phys. Rev. **91**, 1232 (1953).
[31] B. Kahn and W. S. Lyon, Phys. Rev. **91**, 1212 (1953).
[32] K. Siegbahn, Arkiv. Mat. Astron. Fysik **34B**, No. 4 (1948).
[33] Koerts, Schwarzschild, Gold, and Wu, Phys. Rev. **95**, 612 (1954).
[34] N. H. Lazar and P. R. Bell, Phys. Rev. **95**, 612 (1954).
[35] K. Siegbahn, Arkiv. Mat. Astron. Fysik **34A**, No. 7 (1947).
[36] N. Marty-Wollman, Ann. phys. **6**, 662 (1951).
[37] K. Philipp and F. Rehbein, Z. Physik **124**, 225 (1948).
[38] Wu, Havens, and Rainwater, Phys. Rev. **74**, 1248 (1948).
[39] Marty, Labeyrigue, and Langevin, Compt. rend. **228**, 1722 (1949).
[40] Walter, Huber, and Zünti, Helv. Phys. Acta **23**, 697 (1950).
[41] P. Hubert, J. phys. radium **12**, 823 (1951); Ann. phys. **8**, 662 (1953).
[42] J. K. Bair and F. Maienschein, Phys. Rev. **81**, 463 (1951).
[43] Sakai, Murray, and Kurbatov, Phys. Rev. **96**, 862 (1954).
[44] D. R. Hamilton and L. Gross, Rev. Sci. Instr. **21**, 912 (1950).
[45] J. R. Reitz, Phys. Rev. **77**, 10 (1950).
[46] Rose, Perry, and Dismuke, Oak Ridge National Laboratory Unclassified Report ORNL-1459, 1953 (unpublished).
[47] R. W. Pidd (private communication); Pidd, Hammer, and Raha, Phys. Rev. **92**, 436 (1953).

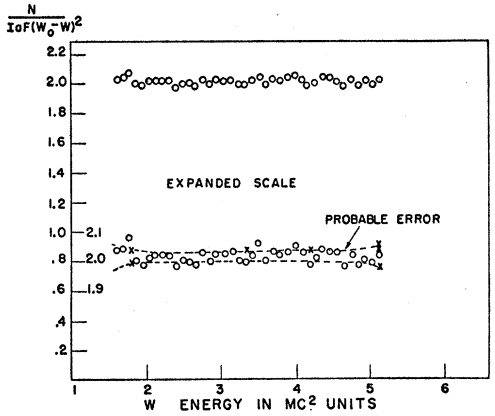

FIG. 2. Shape-factor plot of Y^{90} data obtained with the thin-lens spectrometer.

obtains

$$\frac{N_i}{I_iF_i(W_0-W_i)^2}=B^2, \tag{2}$$

where W_i is the energy in mc^2 units at which the ith measurement is performed and B is a quantity describing the slope of the Kurie line and containing any correction factors if present. As B^2 is determined from experimental points, it will have roughly a mean value of the various $N_i/I_iF_i(W_0-W_i)^2$ quantities and will be constant within experimental error for a linear Kurie plot.

A shape factor plot of the Y^{90} spectrum obtained with the intermediate-image spectrometer is given in Fig. 1, and that obtained with the thin-lens instrument in Fig. 2. The probable error of the various points was assigned by consideration of the measured number of counts and the uncertainty of the background. It is seen that neither instrument introduces deviations beyond statistical error. Figure 3 is a Kurie plot of Y^{90} obtained with the intermediate-image spectrometer. The source used in the intermediate-image spectrometer was prepared by vacuum evaporation of yttrium chloride and had a surface density of 20 μg/cm^2 on an aluminized collodion film of 30 μg/cm^2. The source used in the thin-lens spectrometer was formed by placing a drop of the material on the film and evaporating off the water. Its average surface density was less than 1 mg/cm^2. The end-point energy of the Y^{90} beta spectrum was found to be 2.275$\pm$0.010 Mev.

The P^{32} source material was carrier-free phosphorus obtained from the Oak Ridge Isotope Division and was made by a neutron-proton reaction on a sulfur sample. The sources used in both spectrometers were prepared by vacuum evaporation and had a surface density of less than 30 μg/cm^2 on a 30 μg/cm^2 collodion backing. As a small amount of P^{33} activity was present, with an end-point energy of 249 kev, only points taken at energies above 300 kev were used.

To statistically analyze the data, a least-squares development was used. If Fierz interference effects are present, the quantity $(N/IF)^{\frac{1}{2}}$ is described by Eq. (1), and the Kurie plot is slightly curved. This three-parameter problem in A, W_0, and r was reduced to a one-parameter problem in r to a good approximation.

Consider an actual Kurie plot in which a least-squares line is determined from the points on the high-energy end of the distribution between W_1 and W_2, where W_1 is some arbitrary point and W_2 is the highest-energy point that is used. Approximately one-third of the spectrum is included between points W_1 and W_2. A value of S, the slope of the least-squares line, and W_0 are determined from this line. One can show that

$$A\cong S/(1+\gamma r-\beta r^2),$$

where

$$\gamma=W_0/2W_1W_2$$

and

$$\beta=(W_0W_2+W_0W_1-W_1W_2)/8W_1^2W_2^2.$$

The error in W_0, obtained from this least-squares line, is almost negligible even if some Fierz-type deviation is present. If a value of r outside of statistical error were found, this value so obtained could be used to recompute W_0. It can be shown that the error in W_0 is less than one part per thousand for an assumed value of $r=0.1$, in a typical case.

Upon minimizing the square of the deviation of the experimental points from the Fierz line through the points having energies less than W_1 and solving for r, one obtains to a good approximation

$$r=\sum_i\left(\frac{Y_{ie}}{S^2}-1\right)\left(\frac{1}{W_i}-2\gamma\right)\Big/\sum_i\left(\frac{1}{W_i}-2\gamma\right)^2, \tag{3}$$

where

$$Y_{ie}=N_i/[I_iF_i(W_0-W_i)^2].$$

To determine the probable error in r, one notes that in Eq. (3) only the term $[(Y_{ie}/S^2)-1]$ is subject to large error. The other terms are in error at most by two percent, whereas this term may have errors of the order of one hundred percent. A consideration of this term

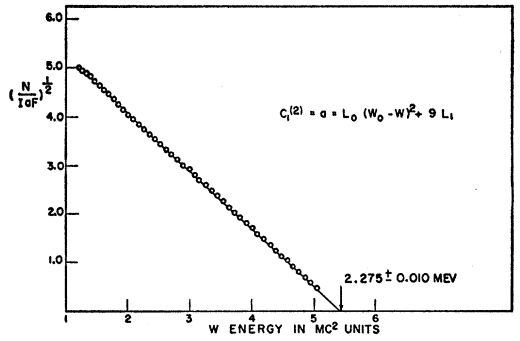

FIG. 3. Kurie plot of Y^{90} beta spectrum obtained with the intermediate-image spectrometer. The values of L_0 and L_1 have been tablulated by Rose, Perry, and Dismuke.[46]

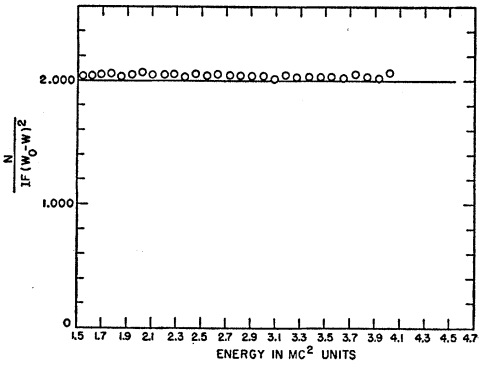

FIG. 4. Shape-factor plot of P^{32} data obtained with the intermediate-image spectrometer.

results in the computational formula for the probable error in r.

The 12.4-hour activity of K^{42} was examined with both instruments. The radioactive K^{42} was obtained by irradiating in the CP-5 pile at the Argonne National Laboratory a sample of KCl in which the potassium was enriched to 99.21 percent in K^{41}. The KCl sample was obtained from the Isotope Research and Production Division of the Oak Ridge National Laboratory. The source for the intermediate-image spectrometer was prepared by vacuum evaporation and had a surface density of 20 μg/cm^2 on a backing of 30 μg/cm^2 of collodion while that used in the thin-lens spectrometer, formed by evaporating off the water from a drop of the liquid placed on a 30-μg/cm^2 collodion film, had an average surface density of 0.5 mg/cm^2.

To observe the photoelectric spectrum of K^{42} in the intermediate-image spectrometer, a 0.063-inch copper cap was placed over the source material and a 17.2-mg/cm^2 thorium radiator was placed over it. The data of Davisson and Evans[48] were used to calculate the attenuation of the gamma rays in passing through the copper cap and to determine the photoelectric cross section for the gamma rays.

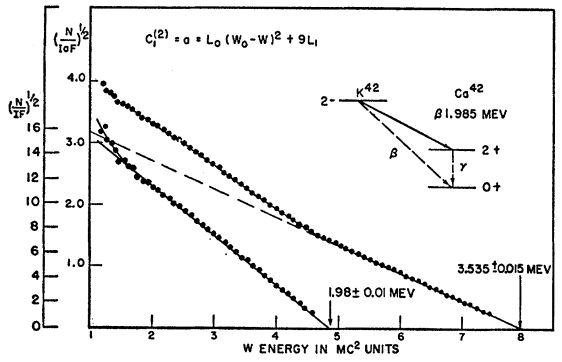

FIG. 5. Kurie plots and maximum beta energies of the total beta spectrum of K^{42} obtained with the intermediate-image spectrometer. The values of L_0 and L_1 have been tabulated by Rose, Perry, and Dismuke.[46]

[48] C. M. Davisson and R. D. Evans, Revs. Modern Phys. 24, 79 (1952).

The upper end of the total beta spectrum of As^{76} was examined with the intermediate-image spectrometer, the lower energy end being obscured by the superposition of the various beta groups. The source was prepared by evaporating the water from a drop of solution, placed on a 30-μg/cm^2 aluminized collodion film. Its average surface density was about 0.5 mg/cm^2. Coincidence data were also taken.

III. RESULTS

The Kurie plot of the P^{32} beta spectrum obtain by using the intermediate-image spectrometer gave an end-point energy of 1.712±0.005 Mev, and within experimental error was found to be straight. Figure 4 is the shape-factor plot obtained with this instrument. The intermediate-image spectrometer data gave a value for r of 0.030±0.040.

The data from the thin-lens spectrometer gave an end-point energy of 1.712±0.006 Mev and within experimental error the Kurie plot was linear. The shape-factor plot was also straight within the probable error. The value of r obtained was −0.032±0.045.

Averaging the values of the end-point energies gives 1.712±0.004 Mev. Averaging the two values of r gives $r=0.00\pm0.03$. This average value of r should be reliable as precautions were taken to have uniform thin sources and backings and two separate instruments were used to obtain consistent values of r.

These data indicate that for a TA mixture either $G_T^2/G_A^2 \lesssim 2\times10^{-4}$ or $G_A^2/G_T^2 \lesssim 2\times10^{-4}$, where G_T and G_A are Fermi coupling constants giving respectively the magnitudes of the T and A interactions in the correct beta decay law.

K^{42} and As^{76}

As indicated in the Kurie plot of Fig. 5, the decay of K^{42} involves two and perhaps three beta groups. The average values for the maximum energies of the two higher energy beta groups, as obtained with both spectrometers, were found to be 3.545±0.010 Mev and 1.985±0.010 Mev. The possible weak third beta group has a maximum energy of about 0.5 Mev. The data shown in Fig. 5 were obtained with the inter-

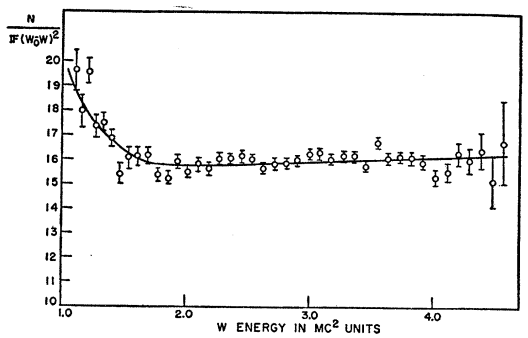

FIG. 6. Shape-factor plot of the 1.985-Mev beta group of K^{42} obtained with the intermediate-image spectrometer.

mediate-image spectrometer. Within statistical error identical results were obtained with the thin-lens spectrometer.

Because it was necessary to apply the a correction factor (this shape factor is designated as $C_1^{(2)}$ by Mahmoud and Konopinski[1]) to the 3.545-Mev beta group to obtain a straight Kurie plot and because of its $\log(W_0^2-1)ft$ value of about 9.7, this group was identified as a first-forbidden transition with a spin change of 2 units. The 1.985-Mev beta group, which was obtained by performing a subtraction, has approximately an allowed shape. The excellent Kurie plot obtained for Y^{90}, Fig. 3, indicates that the high-energy beta group of K^{42}, which has the same shape factor as Y^{90}, can be subtracted from the total spectrum without introducing additional errors except those due the statistics. A plot of the shape factor for the 1.985-Mev beta group of K^{42} is indicated in Fig. 6. The deviation at the low-energy end is attributed to the presence of a weak third beta group. Assuming that it represents a weak beta group, it is found that the intensity is approximately one percent of the 1.985-Mev beta group. The shape factor might possibly have a slight positive slope above 0.5 Mev. The log ft value of 7.5 for this beta transition agrees well with the average value for transitions from 2− to 2+ states.[49] Coincidence counts were detected between the 1.985-Mev beta group and the 1.53-Mev gamma ray; however, the coincidence counting rate was insufficient to give an accurate spectrum shape.

Using a 17.2-mg/cm^2 thorium radiator, the authors found two gamma rays with energies of 1.53±0.01 Mev and 320±5 Mev. The 320-kev gamma ray has an intensity of $0.8^{+0.8}_{-0.4}$ percent of that of the 1.53-Mev gamma ray.

A Kurie plot of the higher energy end of the total beta spectrum of As^{76} is shown in Fig. 7. The a ($C_1^{(2)}$) correction factor was applied to the points of the highest energy beta group in order to obtain a straight Kurie plot. This is in agreement with the assignment of a first-forbidden transition with a spin change of two units. This beta group has a maximum energy of 2.97±0.01 Mev. The two other groups shown, of energies 2.43±0.02 and 1.79±0.05 Mev, were obtained by the subtraction method. Figure 8 displays the Kurie plots obtained by coincidence measurements with gamma rays having energies above 0.5 Mev. Maximum beta energies were found to be 1.76±0.02 and 2.41±0.015 Mev. The average maximum energies of these beta-ray groups, 1.77±0.02 and 2.42±0.01 Mev, are in agreement with those cited by Hubert.[41]

Shape-factor plots of the 2.42-Mev beta group of As^{76} are shown in Fig. 9 by the open and solid circles for the coincidence and subtraction spectra respectively. The theoretical correction factors $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$ are also shown by the broken lines as computed with the best previous estimates of certain parameters. The dashed lines indicate the shape factors the experimental procedure would give if these shape factors were present. The experimental shape factors shown in Fig. 9 indicate that over the range examined, which was limited by the presence of other beta groups, this beta group has an

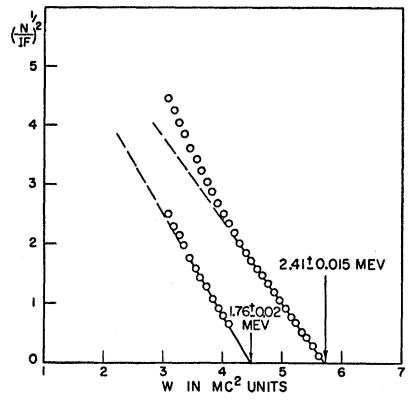

FIG. 8. Kurie plots and maximum beta energies of the coincidence spectrum of As^{76} obtained with the intermediate-image spectrometer.

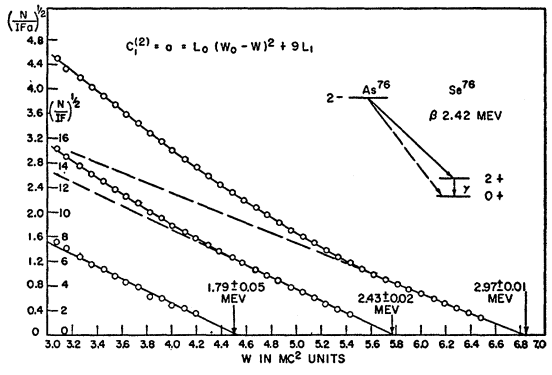

FIG. 7. Kurie plots and maximum beta energies of the high-energy portion of the total beta spectrum of As^{76} as obtained with the intermediate-image spectrometer.

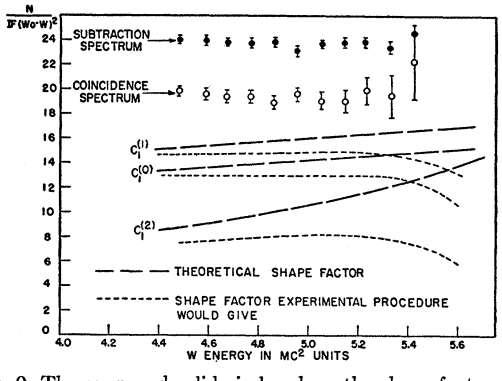

FIG. 9. The open and solid circles show the shape-factor plots obtained experimentally and the broken lines show the theoretical shape-factor plots of $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$ for the 2.43-Mev beta spectrum of As^{76}. The dashed lines indicate the modifications of the theoretical shape factors expected from the experimental procedure and analysis of the data.

[49] R. W. King and D. C. Peaslee, Phys. Rev. 94, 1284 (1954).

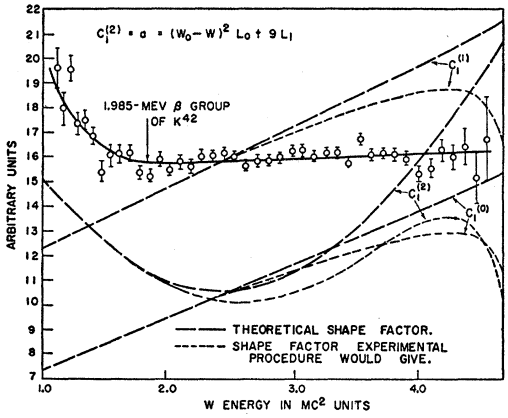

FIG. 10. The broken lines show the theoretical shape-factor plots of $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$ for the 1.985-Mev beta group of K^{42} and the dashed lines indicate the theoretical shape factors expected as a result of the experimental procedure and analysis of the data. The experimentally determined shape factor for the 1.985-Mev beta group of K^{42} is also shown for the purpose of comparison.

allowed shape. This beta group is presumably also a transition from a 2− state to a 2+ state.

IV. DISCUSSION

Mahmoud and Konopinski,[1] whose notation is used in what follows, derived an expression for the shape factors for first forbidden transitions in the approximation that $(\alpha Z)^2 \ll 1$.

They separated the first forbidden shape factor C_1 into three parts

$$C_1 = C_1^{(0)} + C_1^{(1)} + C_1^{(2)},$$

where $C_1^{(0)}$ consists of the only terms which contribute to $0 \rightarrow 0$ transitions ($C_1^{(0)}$ can also contribute to $\Delta I = 0$ transtions), $C_1^{(1)}$ consists of terms which contribute to $\Delta I = \pm 1$ transitions but not the $0 \rightarrow 0$ or $\Delta I = \pm 2$ transitions ($C_1^{(1)}$ can also contribute to $\Delta I = 0$ transitions), and $C_1^{(2)}$ consists of terms which contribute to $\Delta I = \pm 2$ transitions ($C_1^{(2)}$ can also contribute to $\Delta I = 0$, ± 1 transitions). The $C_1^{(2)}$ shape factor is the approximate parabolic shape factor which characterizes first-forbidden transitions with a spin change of two units with a parity change.

If one uses the shape factor for the STP combination, which present information seems to indicate is probably the correct one, and assumes that the real parameters x_1 and y_1 have a value of one and z_3 has a value of zero for the approximation that $G_x/G_x' \rightarrow 1$, one finds that noticeable deviations from a constant shape factor are expected for the 1.985-Mev and 2.42-Mev beta transitions of K^{42} and As^{76}, respectively. The real parameters x_1, y_1, and z_3 enter in the shape factor C_1 and are defined in reference 1. The values of the parameters x_1, y_1, and z_3 indicated above are the best estimates given by King and Peaslee[49] and Ahrens and Feenberg.[50]

For the 1.985-Mev beta group of K^{42} which presumably is a transition with $\Delta I = 0$ and a parity change, the shape factor C_1 consists of the sum of the three shape factors $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$. Shown in Fig. 10 for the 1.985-Mev beta group of K^{42} are the shape factors $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$, and how they would be modified by the experimental procedure used, together with the shape factor obtained experimentally. The experimentally determined shape factor for this transition is almost constant within experimental error above 0.5 Mev. If one assumes that the contribution from pure tensor terms ($C_1^{(2)}$) is negligible, the combination of $C_1^{(0)}$ and $C_1^{(1)}$ cannot give rise to a constant shape factor since both have nearly the same dependence on energy. It is clear that the shape factors $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$, as calculated with the assumed values of x_1, y_1, and z_3 given above, cannot account for the nearly constant shape factor found experimentally for the 1.985-Mev beta group of K^{42}. The shape factors $C_1^{(0)}$ and $C_1^{(1)}$ were calculated for a nuclear radius of $1.1 \times 10^{-13} A^{\frac{1}{3}}$ cm, where A is the mass number. Using the smallest reasonable value for the nuclear radius favors the constant Coulomb energy term $\alpha Z/2R$ compared with the energy-dependent parts of the shape factors $C_1^{(0)}$ and $C_1^{(1)}$.

It may be argued that there is no experimental evidence at present which demonstrates conclusively that the 1.53-Mev energy level of Ca^{42} is a 2+ state. However, it should be pointed out that if the 1.985-Mev beta group is a first-forbidden transition, which the logft value of 7.5 indicates, then the shape factors $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$ taken individually or in any combination (with the assumed values of x_1, y_1, and z_3) cannot account for the shape factor found experimentally.

Figure 9 also shows the shape-factor plots of $C_1^{(0)}$, $C_1^{(1)}$, and $C_1^{(2)}$ for the 2.42-Mev beta group of As^{76} and how they would be modified by the experimental procedure.

If one examines the parameters x_1, y_1, and z_3 in the expressions for $C_1^{(0)}$ and $C_1^{(1)}$ for the STP combination (see reference 1), one finds that a constant shape factor can only be reasonably accounted for by assigning a large value to y_1 or z_3.

For the 1.985-Mev beta group of K^{42}, if one assumes that the major contribution to the transition comes from the $C_1^{(1)}$ term, then the parameter y_1 must have a value greater than 10 in the $(\alpha Z)^2 \ll 1$ approximation to account for the experimental results. On the other hand, if the major contribution comes from the $C_1^{(0)}$ term, then z_3 must have a value greater than 10 to account for the experimental results.

Over the energy range measured, the 2.42-Mev beta group of As^{76} also has an allowed shape. However, because it was possible to measure only part of this beta group by itself, it was not possible to get any accurate measurement of the shape factor.

King and Peaslee[49] have suggested that on the average

[50] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).

10 percent of the transitions from 2− to 2+ states should result from the tensor interaction which gives rise to the shape factor term $C_1^{(2)}$. Assuming that the deviation from a constant shape factor for the 1.985-Mev beta group in K^{42}, Fig. 10, is due only to the $C_1^{(2)}$ term, it is estimated that this shape-factor term cannot contribute more than two percent of the transitions.

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Perturbation Treatment of the Many-Body Problem

W. J. SWIATECKI
Institute for Mechanics and Mathematical Physics and The Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden
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In the conventional perturbation treatment of the many-body problem of interacting particles, the zero-order Hamiltonian corresponds to independent particles moving in a static over-all potential. A discussion of the effects of particle interactions or 'correlations' shows that if one starts instead with a Hamiltonian representing noninteracting particles in a *velocity-dependent* over-all potential, deeper for slow, and shallower for fast particles, then part of the correlation effect is included already in zero order. In addition, a velocity-dependent over-all potential may be called for by velocity dependent interparticle forces or by exchange forces. The degree of the improvement in the convergence of a perturbation expansion based on a Hamiltonian with a velocity-dependent over-all potential is discussed and illustrated by a simple example in which the velocity dependence of the potential gives rise to a reduced "effective mass" of the particles.

The many-body problem of a large, uniform system of interacting particles (e.g., the case of a heavy nucleus without surface effects) is formulated in detail in perturbation theory, starting with a velocity-dependent potential constant in space. The work involved in such a calculation turns out to be essentially the same as with a velocity-independent potential, the effect of the velocity dependence being to reduce each term in the perturbation expansion by a constant factor raised to a power equal to the number of "energy denominators" E_m-E_n in the term in question. A simple equation is deduced for the optimum degree of velocity dependence of the over-all potential which ensures the most rapid convergence.

1. INTRODUCTION

THERE are two principal difficulties in giving a quantitative description of the properties of nuclei: our limited knowledge concerning the fundamental interactions between nucleons, and the mathematical difficulties associated with the solution of a many-body problem. In recent years, some progress has been made in the nuclear many-body problem.[1] The present paper describes a method of dealing with the many-body problem which shows many similarities with those investigations and may turn out to be equivalent in many respects as regards the physical effects considered. The present method is, however, formulated according to conventional perturbation theory, which makes the interpretation of the results in terms of familiar concepts especially easy and makes clear the relation to earlier work along these lines.

The many-body problem of interacting particles has been treated with considerable success in the case of atomic electrons by a method in which the zero-order solution corresponds to particles moving in a common potential and the difference between the actual interaction energy and that part of it which is represented by the common potential is treated by perturbation theory. The same method has been applied, with much more limited success, to the case of nuclei.[2–4] The magnitude of the second-order term (representing correlation effects) in the perturbation expansion of the total energy indicates that, in the case of typical nuclear interactions, the convergence is rather slow. The expansion has never been carried beyond the second order.

The success of a perturbation treatment of any problem depends on how close the unperturbed system is to the exact solution, i.e., on the success of the initial choice of the zero-order Hamiltonian H_0 in the division of the exact Hamiltonian H into H_0 and the perturbation $W=H-H_0$. The method to be described in this article differs from the older perturbation expansions in the first place only in the choice of H_0. As will be shown, it is possible, by a proper choice of H_0, to include already in the zero-order solution part of the correlations normally appearing in higher orders. The result is that the second- and higher order terms in the new expansion are smaller and the convergence is improved.

[1] See, for example, the article by K. A. Brueckner and C. A. Levinson, Phys. Rev. **97**, 1344 (1955) and the series of articles by Brueckner, Watson, Levinson, Mahmoud, Eden, and Francis, referred to in the above article.

[2] H. Euler, Z. Physik **105**, 553 (1937).
[3] W. Heisenberg, Z. Physik **96**, 473 (1935).
[4] R. Huby, Proc. Phys. Soc. (London) **A62**, 62 (1949).