# Isotopic Spin of Antiparticles*

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The eigenvalues of $\tau_3$, the isotopic spin operator describing protons and neutrons, are usually taken as $+1$, $-1$ respectively. At first sight there appears to be some arbitrariness in the corresponding assignments for antiprotons and antineutrons, since either $+1$, $-1$ or $-1$, $+1$ seem to be logical possibilities; however, it is shown that in order to construct a theory involving nucleons, mesons, and the electromagnetic field which is invariant under electric charge conjugation only the first possibility is allowed.

## I. NUCLEONS IN AN ELECTROMAGNETIC FIELD

THE wave equation describing a nucleon in an electromagnetic field is[1]

$$[\gamma_\mu(\partial_\mu+ieQA_\mu)+M]\psi=0. \tag{1}$$

The charge operator $Q$ is given by

$$Q=\tfrac{1}{2}(1+\tau_3), \tag{2}$$

provided that $e$ is the protonic charge and one adopts the convention

$$\begin{aligned} p&\rightarrow(\tau_3=+1),\\ n&\rightarrow(\tau_3=-1). \end{aligned} \tag{3}$$

From the wave function of a particle of charge $e$, one can construct the wave function of its charge conjugate. With $\bar{\psi}=\psi^+\gamma_4$, one has

$$\psi'=C\bar{\psi}. \tag{4}$$

The equation for $\psi'$ is

$$[\gamma_\mu(\partial_\mu-ieQ'A_\mu)+M]\psi'=0. \tag{5}$$

The matrix $C$ satisfies

$$C^{-1}\gamma_\mu C=-\tilde{\gamma}_\mu. \tag{6}$$

In electron-positron theory, $C$ depends only on the Dirac matrices; in a theory with isotopic spin, however, it may also be taken to depend on the isotopic spin matrices, since this would in no way violate Eq. (6). Suppose it is taken as

$$C=C_0\tau_1, \tag{7}$$

where $\tau_1$ is the 1-component of the isotopic spin. One then finds

$$Q'=\tfrac{1}{2}\tau_1(1+\tau_3)\tau_1=\tfrac{1}{2}(1-\tau_3). \tag{8}$$

The wave function $\psi'$ thus describe particles with charges $-e$ or 0 according to the scheme

$$\begin{aligned} p'&\rightarrow(\tau_3=-1),\\ n'&\rightarrow(\tau_3=+1), \end{aligned} \tag{9}$$

whereas in the usual theory ($C$ independent of isotopic spin)

$$Q'=Q, \tag{10}$$

so that

$$\begin{aligned} p'&\rightarrow(\tau_3=+1),\\ n'&\rightarrow(\tau_3=-1). \end{aligned} \tag{11}$$

Thus, so far as only nucleon-electromagnetic couplings are concerned, the operation of charge conjugation applied to the electromagnetic field,

$$C^{-1}A_\mu C=-A_\mu, \tag{12}$$

shows that the theory is invariant under charge conjugation regardless of which alternative is used.

To sum up the results so far we may note the following table. Here $q$ is the value of the charge on the particle in question.

*Alternative A.*—

$q=eQ=\tfrac{1}{2}e(1+\tau_3)$, holds for particles,
$q=-eQ=-\tfrac{1}{2}e(1+\tau_3)$, holds for antiparticles,
$p\rightarrow(\tau_3=+1)$, $p'\rightarrow(\tau_3=+1)$,
$n\rightarrow(\tau_3=-1)$, $n'\rightarrow(\tau_3=-1)$.

*Alternative B.*—

$q=eQ=\tfrac{1}{2}e(1+\tau_3)$, holds for particles,
$q=-eQ'=-\tfrac{1}{2}e(1-\tau_3)$, holds for antiparticles
$p\rightarrow(\tau_3=+1)$, $p'\rightarrow(\tau_3=-1)$,
$n\rightarrow(\tau_3=-1)$, $n'\rightarrow(\tau_3=+1)$.

It is important to notice that in a process such as production of a pair by a photon there is no question of the violation of isotopic-spin conservation, since in a theory of type $A$ the rule for composition of isotopic spins is

$$T_3=T_3(\text{particles})-T_3(\text{antiparticles}), \tag{13}$$

while for the case $B$ one has

$$T_3=T_3(\text{particles})+T_3(\text{antiparticles}). \tag{14}$$

To see that these rules hold one needs only to observe that under a rotation through angle $\epsilon$ about the 3 axis in isotopic spin space $\psi$ transforms as

$$\psi\rightarrow e^{\frac{1}{2}i\tau_3\epsilon}\psi \tag{15}$$

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[1] The $\tau$ matrices are the usual Pauli matrices for half-integral isotopic spin. The notation for the Dirac matrices is that of J. Schwinger, Phys. Rev. 74, 1439 (1949).

in either theory. The behavior of $\psi'$ is, however, different; in fact, one finds

*Alternative A.*—

$$\psi' \rightarrow e^{-\frac{1}{2}i\tau_3\epsilon}\psi', \tag{16}$$

*Alternative B.*—

$$\psi' \rightarrow e^{\frac{1}{2}i\tau_3\epsilon}\psi'.$$

Thus, one sees that if a state involving particles and antiparticles is formed by operating on the vacuum state $\Psi_0$ with appropriate creation operators, such a state $\Psi$ will transform, under a rotation about the 3-axis in isotopic spin space, in a way determined by factors of $e^{-\frac{1}{2}i\tau_3\epsilon}$, which appear for each particle, and factors of $e^{\pm\frac{1}{2}i\tau_3\epsilon}$, which appear for each antiparticle, the plus sign to be taken for Alternative $A$ and the minus for Alternative $B$.

## II. SYSTEM OF NUCLEONS, MESONS, AND ELECTROMAGNETIC FIELD

Since the operation of conjugation according to Alternative $B$ involves an exchange of isotopic spin states, one may suspect that the introduction of a meson field will rule out one of the two alternatives for nucleon charge conjugation. This is in fact the case. Suppose for example we deal with pseudoscalar mesons. This corresponds to introducing a term if $\gamma_5\tau_\alpha\phi_\alpha\psi$ in the nucleon equation:

$$[\gamma_\mu(\partial_\mu+ieQA_\mu)+if\gamma_5\tau_\alpha\phi_\alpha+M]\psi=0. \tag{1$'$}$$

The corresponding charge conjugate wave function $\psi'$ satisfies, for Alternative $A$,

$$[\gamma_\mu(\partial_\mu-ieQA_\mu)+if\gamma_5\tau_\alpha{}^*\phi_\alpha+M]\psi'=0, \tag{5$'$}$$

while for Alternative $B$ one finds

$$[\gamma_\mu(\partial_\mu-ieQ'A_\mu)+if\gamma_5\tau_1\tau_\alpha{}^*\tau_1\phi_\alpha+M]\psi'=0. \tag{5$''$}$$

If one now demands that the theory be invariant under charge conjugation, the substitutions $A\rightarrow -A$ and $\psi'\rightarrow\psi$ leave the nucleon field equation unchanged provided that

$$\begin{aligned} &\phi_1\rightarrow\phi_1, \quad \phi_2\rightarrow-\phi_2, \quad \phi_3\rightarrow\phi_3 \quad \text{(Alternative } A\text{)},\\ &\phi_1\rightarrow\phi_1, \quad \phi_2\rightarrow\phi_2, \quad \phi_3\rightarrow-\phi_3 \quad \text{(Alternative } B\text{)}. \end{aligned} \tag{17}$$

On the other hand, the electromagnetic interaction of mesons depends on

$$A_\mu j_\mu = ie(\phi_1\partial_\mu\phi_2-\phi_2\partial_\mu\phi_1)A_\mu. \tag{18}$$

Thus, if $A\rightarrow -A$, then $\phi_2\rightarrow-\phi_2$ is the only acceptable alternative. This conclusion, of course, does not depend on the special form of coupling chosen.

## III. CONCLUSION

In a theory involving nucleons, mesons, and electromagnetic field, the charge operator that describes nucleons is

$$q=\pm\tfrac{1}{2}e(1+\tau_3). \tag{19}$$

The upper sign is appropriate to particles, the lower to antiparticles. Thus the eigenvalue of the isotopic spin operator $\tau_3$ associated with protons or antiprotons is $+1$, while that associated with either neutral particle is $-1$. On the other hand, because of the rule for compounding isotopic spins of a system of particles and antiparticles, one sees that the antiproton carries an "effective" value of isotopic spin $-1$, and the antineutron an effective value of $+1$.

It is of some interest to compare Eq. (19) with the formula for the charge of a nucleon proposed by Gell-Mann[2]:

$$Q=\tfrac{1}{2}e(I_3+n). \tag{20}$$

In this formula, $n=1$ for nucleons and $n=-1$ for antinucleons and therefore corresponds to the following assignment:

$$\begin{aligned} p&\rightarrow(I_3=+1), \quad p'\rightarrow(I_3=-1),\\ n&\rightarrow(I_3=-1), \quad n'\rightarrow(I_3=+1). \end{aligned}$$

If this is compared with the foregoing results one sees that $I_3$ is the "effective" value of the isotopic spin variable $\tau_3$.

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[2] M. Gell-Mann and A. Pais, Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics (Pergamon Press, London, 1955).