PHYSICAL REVIEW VOLUME 101, NUMBER 3 FEBRUARY 1, 1956

Nucleon Anomalous Moments via Pion-Pion Attraction*

W. G. HOLLADAY†
Department of Physics, University of Wisconsin, Madison, Wisconsin
(Received August 5, 1955)

The importance of a pion-pion attraction for calculating nucleon anomalous moments is emphasized. It is found that reasonably strong pion-pion attraction is capable of giving large enough moments, even if the mirror condition of Sachs is imposed. It is pointed out that many effects, such as core recoil, interaction currents, and excited cores offer no relief in calculating large enough moments.

I. INTRODUCTION

THE general failure of the meson theories to predict the quantitative facts regarding the properties of nucleons has led Sachs[1] to approach the problem of understanding these properties from another point of view. Rather than make assumptions about the specific form of the interaction which gives rise to the pion field presumably surrounding the core of the nucleon, Sachs' efforts are directed towards correlating the data on the nucleon with the pionic structure of the nucleon. An important aspect of this approach is the hypothesis that the probability of the existence of the various numbers of pions in the cloud is finite; i.e., that the nucleon wave function is normalizable.

With this approach the nucleon magnetic moments, the neutron-electron interaction, the pion-nucleon scattering data,[2] and the neutron-proton mass difference[3] have been analyzed. The present paper is concerned with additional considerations on the magnetic moments.

Since the assumptions and conclusions of Sachs regarding the nucelon magnetic moments are basic to the work reported here, a brief discussion of them is in order. On the basis of a no-recoil, spin $\frac{1}{2}$, isotopic spin $\frac{1}{2}$ core and the assumptions of charge symmetry, Sachs showed that the sum of the moments is

$$\mathfrak{M}_n+\mathfrak{M}_p=\mathfrak{M}(c)[1-(4/3)P_1], \tag{1}$$

where $\mathfrak{M}(c)$ is the core magnetic moment and P_1 is the total probability that pions occur in the field with total angular momentum $L=1$. Equation (1) is referred to as the mirror theorem. For $\mathfrak{M}(c)=1$ nuclear magneton, $P_1=9\%$. The smallness of this number is a direct reflection of the fact that the proton and neutron *anomalous* moments are so close together in absolute value.

Then, to calculate the neutron magnetic moment Sachs argued that interaction currents are negligible and that the pions surrounding the core are in p-states.[4] For simplicity, he neglected altogether the kinetic energy of the pions, i.e., set $\langle 1/\omega\rangle=1/\mu$, where the bracket indicates the average over pion momenta, ω is the total pion energy and μ is the pion rest mass. It was then found [see Eq. (S-28)] that the only way a large enough moment could be calculated was to make the two-pion state much more probable than the one-pion state.[5] Even with this unusual requirement it is very difficult to fit the moments when pion kinetic energy is taken into account. From the cursory knowledge of the spatial extent of the pion cloud as provided by the range of nuclear forces, it would seem that $\langle 1/\omega\rangle$ could not be significantly greater than $1/2\mu$, in which case the neutron moment calculated from Eqs. (S-28) and (S-29) would be too small by a factor of 2. This result necessitates that the assumptions made by Sachs be reexamined for the purpose of finding a mechanism that will provide large enough moments when reasonable values of the pion kinetic energy are considered.

The essential dilemma to be overcome is that the nearness in absolute value of the anomalies constrains P_1 (via the sum of the moments) to be a rather small number, whereas the anomalies themselves are large, which is expected to require rather large pion probabilities. Now from Eq. (1), P_1 can be made larger than 9% if the core moment $\mathfrak{M}(c)$ is greater than 1 nuclear magneton. In an effort to increase $\mathfrak{M}(c)$ and P_1 we are led to examine the effects of core recoil, excited cores[6] (i.e., cores with spin and/or isotopic spin of $\frac{3}{2}$), and interaction currents. It is found that these effects increase the dilemma before us. Core recoil will be discussed in the next section and interaction currents have already been treated.[7] With regard to excited cores we merely mention that the contributions from them are quite small, largely because the orthogonality of the excited core states to the ordinary core state exterminates the no-pion, two-pion cross term[8] which for the spin $\frac{1}{2}$, isotopic spin $\frac{1}{2}$ core gives the essential contribution. It

* Supported in part by the U. S. Atomic Energy Commission and in part by the Wisconsin Alumni Research Foundation.

† National Science Foundation Predoctoral Fellow 1952–1954, now at Vanderbilt University, Nashville, Tennessee.

[1] R. G. Sachs, Phys. Rev. **87**, 1100 (1952). Equations referred to in this paper are designated by an S.

[2] R. G. Sachs, Phys. Rev. **95**, 1065 (1954).

[3] W. G. Holladay and R. G. Sachs, Phys. Rev. **96**, 810 (1954).

[4] The consideration of only p-state pions involves the assumption that angular correlation between pions is negligible. For simplicity, radial correlations were also omitted.

[5] The greater effectiveness of the two-pion state to produce a magnetic moment resides in the fact that its contribution depends on the *amplitude* not the probability of the state.

[6] M. Sugawara, Progr. Theoret. Phys. (Japan) **8**, 549 (1952). F. J. Belinfante, Phys. Rev. **92**, 994 (1953).

[7] G. F. Chew (private communication) and R. H. Capps and W. G. Holladay (Phys. Rev. **99**, 931 (1955), have shown that interaction currents do not affect P_1 and give either no anomalous moment or small anomalous moments of the wrong sign.

[8] This cross term will be referred to as the annihilation term. It involves the annihilation of a $\pi^+-\pi^-$ pair.

should be pointed out that Miyazawa,[9] who considered the effect of nucelon-antinucleon pairs on the effective mass of the nucleon core and hence on $\mathfrak{M}(c)$, has shown how P_1 may be increased to 15–20%. This is a worthwhile increase, but seems insufficient to resolve the difficulty, for from (S-28) with $\langle 1/\omega\rangle = 1/2\mu$ rather than $1/\mu$, the probabilities that yield the correct moments need to be so large (a typical set being[10] $P_1(1)=0.40$, $P_1(2)=0.15$, $P_0(2)=0.15$, $P(0)=0.30$) that the strong-coupling result very nearly holds, namely, that the nucleon *moments* have about the same magnitude, in contrast to the known fact that the anomalies have about the same magnitude. If we take $\langle 1/\omega\rangle = 1/3\mu$, then no values of the probabilities can be found on the basis of Eq. (S-28) that yield a large enough neutron moment.

Since the considerations above have proved inadequate to resolve the difficulties associated with the magnetic moment problem, other possibilities must be investigated. One such possibility involves the assumption of an attraction between pions.[11] To see that such an interaction can increase the moment, recall that the main contribution to the moment comes from the annihilation current. The probability for annihilation of two pions is greater if the pions tend to be near one another. A positive correlation[12] between two pions, which would result from a pion-pion attraction, would therefore enhance the annihilation process, and would thus increase the moments.

An investigation with an explicit form of the two-pion function including a pion-pion correlation is carried out in Sec. III. The result shows that the moments can be increased sufficiently to compensate for the reduction of the moments due to pion kinetic energy. The introduction of correlation between pions does not change the mirror property, and thus does not alter the value of P_1.

For arbitrarily large correlation, the correct moments can be obtained with an arbitrarily small two-pion probability, so that the one-pion state can have a larger probability than the two-pion state. For this to happen, however, the correlation has to be so strong that the two pions very nearly form a single particle. More reasonable values of the correlation parameter are obtained if the two-pion state appears with a probability comparable to or greater than the one-pion probability. It is interesting to note that a strong pion-pion attraction might lead to just this result, i.e., the interaction would tend to stabilize the two-pion state as compared to the one-pion state, where the attraction, of course, is not operative.

II. EFFECT OF CORE RECOIL

Since core effects do not satisfy the mirror property, i.e., do not cancel in the sum of the nucleon moments, it is of special interest to see whether the recoil of the core can significantly change the condition on P_1. To estimate the recoil effect, we consider first just the one-pion state. The orbital magnetic moment operator of the system we take to be

$$\mathfrak{M}_z{}^o = \frac{1}{2c}\left[e_\pi(\mathbf{r}_\pi\times\mathbf{v}_\pi)_z + e_c(\mathbf{r}_c\times\mathbf{v}_c)_z\right],$$

where the subscripts π and c refer to the pion and to the core. The reference frame is chosen so that the total momentum of the pion and the core is zero, i.e.,

$$\mathbf{p}_\pi = -\mathbf{p}_c = \mathbf{p},$$

or

$$E_\pi\mathbf{v}_\pi = -E_c\mathbf{v}_c.$$

The origin of the coordinate system is taken at the center of inertia, i.e.,

$$E_\pi\mathbf{r}_\pi = -E_c\mathbf{r}_c.$$

We then have from the above equations

$$\mathfrak{M}_z{}^o = \frac{c}{2}(\mathbf{r}\times\mathbf{p})_z\frac{E_\pi E_c}{E_\pi+E_c}\left(\frac{e_\pi}{E_\pi{}^2}+\frac{e_c}{E_c{}^2}\right),$$

where $\mathbf{r}$ is the radius vector between the two particles. If we add to this the spin core moment, the sum of the moments in nuclear magnetons is found by taking the expectation value of

$$\sigma_z + L_z\frac{M_nE_\pi}{E_c(E_\pi+E_c)},$$

since the pion terms cancel. Here M_n is the nucleon rest energy. Hence

$$\mathfrak{M}_n+\mathfrak{M}_p = 1-4/3P_1+2/3P_1\frac{M_nE_\pi}{E_c(E_\pi+E_c)}. \qquad (2)$$

To make the effect of recoil and the value of P_1 as large as possible we suppose that $E_\pi = E_c = M_n$, from which the probability P_1 is found to be 12%.

With recoil the neutron moment in the one-pion state is given by [an extension of Eq. (S-24)]

$$\mathfrak{M}_n(1) = -\frac{2}{9}\left[1+2\left(\frac{M_n}{E_\pi}-\frac{M_n}{E_c}\right)\right]P_1(1).$$

Since the orbital moment from the positive core is opposite to that produced by the negative pion, there is a considerable reduction in the calculated moment from the one-pion state when $E_\pi \approx E_c$.

[9] H. Miyazawa, Phys. Rev. **97**, 1413 (1955). Because of lower threshold energy, it might be expected that hyperons and heavy mesons would be more likely to affect $\mathfrak{M}(c)$ than pairs.

[10] The subscript refers to the total angular momentum of the pion field and the other number to the number of pions present. Notice that this set has the one-pion probability larger than the two-pion probability.

[11] W. G. Holladay and R. G. Sachs, Phys. Rev. **98**, 1155 (1955).

[12] Such a correlation would introduce angular momentum states higher than p-states into the two-pion wave function. This is another way to see that the moments can be increased by pion-pion attraction.

For states with higher numbers of pions we can still use Eq. (2) by supposing that all the pions act as a unit with total energy E_π against which the core recoils. With this procedure recoil effects are maximized and Eq. (2) gives an upper limit to P_1, which we have seen is 12%. Suppose that all this 12% is put into the two-pion state. The largest contribution to the moment comes from the annihilation term in which P_1 appears to the $\frac{1}{2}$ power, so that the core recoil provides through the small increase of P_1 an even smaller increase in the moment.

At the same time core recoil leads to effects that reduce the moment derived from the cross term. In addition to the reduction of the annihilation term by the high pion kinetic energies which large core recoil implies, a further decrease will occur because the overlap integral in the space variables of the core will be less than one, the value assumed in the no-recoil approximation. We therefore conclude that the consideration of core recoil does not alleviate the dilemma associated with nucleon magnetic moments, and it is neglected in the remainder of the paper.

III. EFFECT OF CORRELATION

In this section we examine the results of the postulate of strong pion correlations, which makes the annihilation contribution to the moments larger by stimulating the annihilation process. To consider this effect in detail, it is convenient to work with an explicit form of the two-pion wave function, the relevant part of which has quantum numbers $L=1$, $T=1$, $T_3=0$, with a π^+, π^- pair present. The functional for this state can be written

$$|N=2, L=1, T=1, (+,-)\rangle = \sum_{\mathbf{k},\mathbf{k}'} \frac{f(\mathbf{k},\mathbf{k}')}{\sqrt{2}} \left[\frac{\Phi(\mathbf{k}+,\mathbf{k}'-)-\Phi(\mathbf{k}-,\mathbf{k}'+)}{\sqrt{2}}\right], \quad (3)$$

where $\Phi(\mathbf{k}+,\mathbf{k}'-)$ refers to a positive pion with momentum $\hbar\mathbf{k}$ and a negative pion with momentum $\hbar\mathbf{k}'$. The function $f(\mathbf{k},\mathbf{k}')$ has the general form

$$f(\mathbf{k},\mathbf{k}')=\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{k}')F(k^2,k'^2,|\mathbf{k}+\mathbf{k}'|^2), \quad (4)$$

which insures that the pions have total angular momentum $L=1$, coupled to the core spin to give a total angular momentum $\frac{1}{2}$. Since the isotopic spin function for this state is antisymmetric, $f(\mathbf{k},\mathbf{k}')$ itself must be antisymmetric, which requires that F be symmetric.

For F we choose the form

$$F(k^2,k'^2,|\mathbf{k}+\mathbf{k}'|^2)=\frac{\pm iC(K,\gamma)}{(k_0k_0')^{3/2}} \times\left(\frac{K^2}{K^2+k^2}\right)^{5/4}\left(\frac{K^2}{K^2+k'^2}\right)^{5/4}\frac{\gamma}{\gamma+(\mathbf{k}+\mathbf{k}')^2/kk'}, \quad (5)$$

where C is the positive normalization constant, depending on the parameters K and γ, and $\pm i$ is a phase factor whose occurrence may be established by time reversal arguments. The quantity $k_0=(\mu^2+k^2)^{\frac{1}{2}}$. We are led to this form of F by the following considerations: The creation operators which produce the two-pion state contain $(k_0k_0')^{-\frac{1}{2}}$, the energy denominators of perturbation theory give rise to $(k_0k_0')^{-1}$, the quantities containing K^2 are inserted as cut-off functions for the pion momenta, the exponent 5/4 causing F to die off rapidly enough that the calculation in the next paper on the $n-p$ mass difference converges, and the term in γ contains the correlation between the two-pions, a small γ indicating a large correlation. The correlation disappears as $\gamma\rightarrow\infty$.

To calculate the magnetic moment of the nucleon in this state, it is convenient to expand the pion field into plane waves. Then, the pion part of the magnetic moment can be written

$$\begin{aligned}\mathfrak{M}_z(\pi)&=\frac{icM}{\hbar}\int[\psi(\mathbf{r}\times\nabla)_z\psi^*-\psi^*(\mathbf{r}\times\nabla)_z\psi]d^3r\\&=iM\sum_{\mathbf{k},\mathbf{k}'}\frac{1}{(k_0k_0')^{\frac{1}{2}}}(a_{-\mathbf{k}'}{}^*+b_{\mathbf{k}'})(a_{\mathbf{k}}+b_{-\mathbf{k}}{}^*)\mathbf{i}_z\\&\qquad\cdot(\mathbf{k}\times\nabla_k)\delta(\mathbf{k}+\mathbf{k}')\end{aligned}$$

in units of the nuclear magneton, with M the reciprocal of the nucleon Compton wavelength.

For the no-pion, two-pion matrix element of $\mathfrak{M}_z(\pi)$ (annihilation term), we have

$$\begin{aligned}(0|\mathfrak{M}_z(\pi)|2)&=2(N=0,\chi^{\frac{1}{2}}|\mathfrak{M}_z(\pi)|L=1,T=1,(+,-)\chi^{\frac{1}{2}})\\&=iM\left(\frac{P_1(2,1)P(0)}{3}\right)^{\frac{1}{2}}\left\{\chi^{\frac{1}{2}},\int\int\frac{d^3kd^3k'}{(k_0k_0')^{\frac{1}{2}}}f(\mathbf{k},\mathbf{k}')\mathbf{i}_z\right.\\&\qquad\left.\cdot[\mathbf{k}\times\nabla-\mathbf{k}'\times\nabla']\delta(\mathbf{k}+\mathbf{k}')\chi^{\frac{1}{2}}\right\},\end{aligned}$$

where $P(0)$ is the no-pion probability, $P_1(2,1)/3$ is the probability in the two-pion state of the π^+, π^- pair with $L=1$, $T=1$. The curly bracket in the latter expression refers to the sum over nucleon core spin variables. Integration by parts and the fact that f is antisymmetric lead to

$$(0|\mathfrak{M}_z(\pi)|2)=-i2M\left(\frac{P_1(2,1)P(0)}{3}\right)^{\frac{1}{2}}\times\left\{\chi^{\frac{1}{2}},\int\int d^3kd^3k'\mathbf{i}_z\cdot\left[\mathbf{k}\times\nabla\frac{f(\mathbf{k},\mathbf{k}')}{(k_0k_0')^{\frac{1}{2}}}\right]\delta(\mathbf{k}+\mathbf{k}')\chi^{\frac{1}{2}}\right\}.$$

Insertion of Eq. (4) into this expression yields

$$(0|\mathfrak{M}_z(\pi)|2)=i2M\left(\frac{P_1(2,1)P(0)}{3}\right)^{\frac{1}{2}}\times\left\{\chi^{\frac{1}{2}},\int d^3k\mathbf{i}_z\cdot\frac{\mathbf{k}\times(\mathbf{k}\times\boldsymbol{\sigma})}{k_0}F(k^2,k^2,0)\chi^{\frac{1}{2}}\right\},$$

TABLE I. Values of the pion momentum cutoff, the correlation parameter and the corresponding two-pion function normalization constant that yield the correct neutron moment.

	K/μ	γ	C'
$P_1(2,1)=9\%$	3	0.80	4.90
	5	0.60	5.30
	7	0.50	5.52
$P_1(2,1)=20\%$	3	1.23	4.10
	5	1.04	3.80
	7	0.88	3.80

where only the derivative of $[\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{k}')]$ remains after the integration over the δ function. Note, in addition that the correlation drops out in this integral and affects the moments only through the normalization of the pion function. When the angular integral and spin sum are performed and Eq. (5) for F inserted, it is found that

$$(0|\mathfrak{M}_z(\pi)|2)=\pm C'(\gamma,K)\frac{16}{3^{\frac{3}{2}}}\frac{M}{K}[P_1(2,1)P(0)]^{\frac{1}{2}}I, \quad (6)$$

where

$$I=\int_0^\infty \frac{dkk^4}{Kk_0^4}\left(\frac{K^2}{K^2+k^2}\right)^{5/2}$$

$$=\frac{K^4\alpha^2}{\mu^4}\left\{\frac{3}{2}-\frac{3}{2}\alpha^{\frac{1}{2}}\tan^{-1}\alpha^{-\frac{1}{2}}+\frac{5}{2}\alpha-\frac{5}{2}\alpha^{\frac{3}{2}}\tan^{-1}\alpha^{-\frac{1}{2}}\right\}$$

with $\alpha\equiv\mu^2/(K^2-\mu^2)$; $C'(\gamma,K)=K^2\pi C$ is dimensionless. In Eq. (6) the negative sign should be chosen for the neutron and the positive for the proton.

Some values of the parameters and the normalization constant which yield -1.91 for the neutron moment are given in Table I. Since the effect of heavy mesons or considerations such as those of Miyazawa might double the value of P_1, values of these parameters for both $P_1=9\%$ and 20% are included for completeness.

The evaluation of the normalization integral to obtain C' has not been exact, but the approximation's used are believed to be accurate to better than 10%.

Of course, other pionic states contribute to the moment but to a much smaller extent than the states considered here, so that their effect on the above parameters will be minor.

Note that the correlation parameter has values $\gamma\sim 1$. A correlation parameter of this magnitude implies that the amplitude of the two-pion function when the two pions are coincident is about 5 times the amplitude of the function when they are on opposite sides of the core. Therefore, the pion-pion attraction is not weak.

The effect of the correlation can be seen from the fact that a neutron moment of -0.42 is calculated for $P_1=9\%$, $K/\mu=7$, if correlation is omitted ($\gamma\rightarrow\infty$).

IV. CONCLUSION

Although the mirror theorem places rather stringent limitations on the probability P_1 with which pions exist in the cloud surrounding the nucleon, it is still possible to calculate correct nucleon moments by assuming a pion-pion correlation such as might result from a strong pion-pion attraction. The positive correlation enhances the annihilation of a π^+, π^- pair, and therefore increases the contribution from the annihilation term (no-pion, two-pion cross term). The correlation which is required in order that large enough moments be calculated seems more reasonable if the two-pion state enters with a probability comparable to that of the one-pion state. This does not necessarily imply strong coupling, but could result from the pion-pion attraction itself.

The existence of a strong pion-pion attraction should manifest itself in other physical processes, and may account for some of the discrepancies that now exist between theory and experiments on pion-nucleon phenomena. Further evidence for the effect is presented in the following paper.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the counsel of Professor R. G. Sachs, with whom many discussions were held.