for the heavy elements. It seems worth noting that if, on the assumption that the distribution of matter in the nucleus is the same as the distribution of charge, we calculate $(A / Z) \rho(r)$, as in Fig. 14(b), the central value of this "nucleon density" remains roughly constant from element to element. In the last column of Table III we give the electrostatic Coulomb energy of the nuclear charge distributions ( $E_{c}=\frac{1}{2} \int \rho(r) V(r) d^{3} r$ ). This turns out to be approximately the same as the Coulomb energy of a uniformly charged sphere of radius $R$.
These results may be summarized as follows: for seven elements between calcium-40 and bismuth-209 the nuclear charge distribution is found to have a radius $c$ (to the midpoint of the surface) of $(1.07 \pm .02) A^{\frac{1}{3}}$ $\times 10^{-13} \mathrm{~cm}$, and a surface thickness $t$ ( 0.9 to 0.1 distance) of $(2.4 \pm 0.3) \times 10^{-13} \mathrm{~cm}$.

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# Effects of a Ring Current on Cosmic Radiation 

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#### Abstract

A theoretical analysis of the effect of an equatorial ring current on the latitude variation of the primary cosmic radiation has been carried out. It has been found that a ring current of the size suggested by Schmidt should lead to observable effects on the latitude variation. In particular, if a ring current of radius equal to 7.5 earth radii and current strength sufficient to produce a field of $100 \gamma$ at the earth's equator exists, then the knee in the latitude variation is a feature of the rigidity cut-off curve rather than of the primary spectrum. The primary spectrum obtained with the use of geomagnetic theory which includes a ring current is satisfactorily fitted with a function of the form $J=0.29 E^{-0.9}\left(\mathrm{~cm}^{2} \mathrm{sec} \text { sterad }\right)^{-1}$, where $E$ is the total energy of a primary particle. Certain features of time variations of the cosmic-ray intensity apparently disagree with the theory.


## INTRODUCTION AND SUMMARY

IN their theory of magnetic storms and auroras, Chapman and Ferraro ${ }^{1}$ speculated that a westwardflowing ring current encircling the earth at the geomagnetic equator with a radius between five and ten earth radii might explain the decrease of the earth's field observed during the main phase of a geomagnetic storm, and perhaps as well serve as a reservoir of particles to produce the auroras during times of no magnetic storm. The present work is an attempt to submit this hypothesis to experiment by exploring the effect of such a ring current on the latitude variation

[^0]of the primary cosmic radiation. This investigation has been carried out in the Störmer approximation, ${ }^{2}$ and hence can be expected to be valid only in directions near the vertical and in a range of latitudes north of about $45^{\circ}$ geomagnetic. This range of validity is quite convenient, since it will turn out that the ring current has its greatest effect in those latitudes, and is of comparatively little importance nearer the equator.

The main results found are the following: If the primary spectrum is of the form $J=J_{0} E^{-\gamma}$, where $E$ is the total energy of the primary particle, ${ }^{3}$ then with $J_{0}=0.29$ and $\gamma=0.9$, a ring current with suitably chosen parameters will produce the observed latitude dependence of the vertical intensity (see Fig. 7). The

[^1]parameters cannot be chosen with great accuracy, but the radius must be about 7.5 earth radii, a value very appropriate for producing the observed location of the auroral zone, and the current strength must be such as to produce a magnetic field at the geomagnetic equator of about $100 \gamma$, a value sufficiently small to be consistent with Vestine's analysis of the earth's field. ${ }^{4}$ This ring current is, however, stronger than that felt to be necessary by Chapman and Ferraro. According to Chapman, on those occasions when a fall of $100 \gamma$ is observed near the equator, it is thought that the ring radius is greatly reduced, rather than that the current strength is increased. On the other hand, Schmidt has speculated that a ring of radius a few earth radii and current strength sufficient to reduce the equatorial field by as much as $250 \gamma$ might always be present, a speculation with which Chapman does not take issue. ${ }^{5}$
If a ring current does in fact exist, and if the latitude "knee" (which does not represent a sharp cutoff in the spectrum on this picture) is produced by this mechanism, then one would expect the knee to change position in correlation with the sunspot cycle. This feature has tentative experimental confirmation, but certain apparently related variations (changes in intensity below the knee and near the geomagnetic pole) are not explained by the theory.
Experiments have not so far produced data capable of deciding the existence of the ring current on these grounds, but some experimental tests of this theory are proposed in the present paper.

## $R_{c}$ AS A FUNCTION OF LATITUDE

The exact nature of our problem is the following: given the magnetic field in the vicinity of the earth as being due to a dipole situated at the earth's center ${ }^{6}$ of moment $M=8.1 \times 10^{25}$ gauss- $\mathrm{cm}^{3}$ and a ring current flowing in the geomagnetic equatorial plane with radius $a$ and equivalent dipole moment $M_{r}=\pi a^{2} I / c$ (where $I$ is the current in statamperes), what rigidity ( $R=p c / Z e$, $p$ the particle momentum, $Z e$ the charge, $c$ the velocity of light) must a charged particle have in order to arrive at a given latitude from the vertical direction? The solution of this problem is very difficult, requiring machine calculations. However, it is quite easy to find a necessary (but not necessarily sufficient) condition for arrival (the Störmer cone) and in practice this condition differs little from sufficiency above some latitude, about $45^{\circ}$ geomagnetic in directions near the vertical. Since the latitude knee is farther north than this, we shall use the Störmer cone in this paper.

The motion of a charged particle in a magnetic field

[^2]

Fig. 1. Coordinate System. N and S denote the north- and south-seeking poles of the earth's dipole.
is specified by the Lorentz force equation

$$
\begin{equation*}
m d^{2} \mathbf{x} / d t^{2}=(Z e / c)(d \mathbf{x} / d t) \times \mathbf{H} \tag{1}
\end{equation*}
$$

where $m=m_{0}\left[1-(v / c)^{2}\right]^{-\frac{1}{2}}, m_{0}$ is the rest mass of the particle, $\mathbf{x}$ the position vector, and $\mathbf{H}$ the magnetic field. As is well known, $v$ and $p$ are constants of the motion if $\mathbf{H}$ does not depend explicitly on time, and therefore $m$ is also a constant. Putting $d s=v d t$, and $p=m v$, (1) becomes

$$
\begin{equation*}
\mathbf{x}^{\prime \prime}=R^{-1} \mathbf{x}^{\prime} \times \mathbf{H}, \quad R=p c / Z e, \tag{2}
\end{equation*}
$$

where the prime denotes differentiation with respect to path length, $s$. Note that the physical properties of the particle are all contained in $R$, the so-called "rigidity." Let us introduce the spherical coordinates $r, \lambda, \varphi$ (see Fig. 1). It is well known ${ }^{7}$ that the $\varphi$ component of Eq. (2) has a first integral which is

$$
\begin{equation*}
r^{2} \varphi^{\prime} \cos ^{2} \lambda+A R^{-1} r \cos \lambda=2 \gamma \tag{3}
\end{equation*}
$$

$A$ is the $\varphi$ component of the usual vector potential (the only component which exists in our case) and $2 \gamma$ is an integration constant having the dimension of length. Physically it is proportional to the impact parameter. If we assume, as is customary, that the cosmic radiation is isotropic and homogeneous at infinity, then $\gamma$ may vary throughout the range $-\infty \leqslant \gamma \leqslant+\infty$ without varying the intensity of particles being considered.
Now imagine a right circular cone with vertex at the observer and axis along the east-west direction. Denote the angle between the eastward direction and an element of the cone by $\omega$. Then it is easy to see that for a particle arriving at the observer with its trajectory tangent to the surface of the cone,

$$
\begin{equation*}
\cos \omega=r \varphi^{\prime} \cos \lambda \tag{4}
\end{equation*}
$$

Then Eq. (3) becomes

$$
\begin{equation*}
r \cos \lambda \cos \omega+A R^{-1} r \cos \lambda=2 \gamma \tag{5}
\end{equation*}
$$

[^3]

Fig. 2. Störmer plots for a dipole field.
(This equation holds for positive particles. For negative particles, the sign of $\cos \omega$ should be reversed.) The vector potential of a dipole is

$$
A_{1}=M \cos \lambda / r^{2}
$$

while that of a ring current is

$$
A_{2}=\left(2 M_{r} / \pi a\right)(a r \cos \lambda)^{-\frac{1}{2}} F(k),
$$

where

$$
\begin{aligned}
F(k) & =(2 / k)(K-E)-k K \\
k^{2} & =\frac{4 a r \cos \lambda}{r^{2}+2 a r \cos \lambda+a^{2}},
\end{aligned}
$$

and $K$ and $E$ are the complete elliptic integrals

$$
\begin{aligned}
& K=\int_{0}^{\pi / 2} d \theta\left(1-k^{2} \sin ^{2} \theta\right)^{-\frac{1}{2}} \\
& E=\int_{0}^{\pi / 2} d \theta\left(1-k^{2} \sin ^{2} \theta\right)^{\frac{1}{2}}
\end{aligned}
$$

Also,

$$
M_{r}=\pi a^{2} I / c
$$

This expression for the vector potential corresponds to a filamentary ring current. Giving the ring a finite cross section would complicate the problem excessively without much altering the results obtained. Putting these expressions into (5), we obtain, after a few
algebraic transformations,

$$
\cos \omega=\frac{2 \gamma}{r \cos \lambda}-\frac{M}{R}\left[\frac{\cos \lambda}{r^{2}}+\frac{M_{r}}{M} \frac{2 \gamma}{\pi a} \frac{F(k)}{(a r \cos \lambda)^{\frac{1}{2}}}\right]
$$

$(M / R)^{+\frac{1}{2}}$ has the dimensions of a length. If we use it as the unit of length, we have

$$
\cos \omega=\frac{2 \gamma}{r \cos \lambda}-\frac{\cos \lambda}{r^{2}}-\frac{2 M_{r}}{\pi a} \frac{F(k)}{(a r \cos \lambda)^{\frac{1}{2}}},
$$

where we have also used the earth's dipole moment as unit dipole moment. If we now put $\rho=r / a, \gamma^{\prime}=\gamma / a$,

$$
\begin{equation*}
\cos \omega=\frac{2 \gamma^{\prime}}{\rho \cos \lambda}-\frac{1}{a^{2}}\left[\frac{\cos \lambda}{\rho^{2}}+\frac{2 M_{r}}{\pi} \frac{F(k)}{(\rho \cos \lambda)^{\frac{1}{2}}}\right] . \tag{6}
\end{equation*}
$$

This is the most convenient form of the equation for computational work. In the discussion to follow, lengths will be in units of $(M / R)^{\frac{1}{2}}$ rather than units of $a$. $\left[(M / R)^{\frac{1}{2}}\right.$ is called the Störmer unit.]
Let us first consider briefly the result first given by Störmer, the case when $M_{r}=0$. (For a fuller discussion see reference 2 and references therein.) In Fig. 2 in the shaded regions of the various figures, Eq. (6) yields a value of $\cos \omega$ such that $|\cos \omega|>1$. In the unshaded regions, $|\cos \omega|<1$. Thus no particle can travel in the shaded regions. The surface of the earth, if plotted on these diagrams, is a circle with center at the origin and radius which depends on the rigidity of the particle being considered (since the size of the Störmer unit does). For $R=59.2 \mathrm{Bv}$, the earth's radius is unity. From a study of diagrams like those in Fig. 2, one can see that for $R>59.2 \mathrm{Bv}, \gamma$ may be chosen so that the particle can reach any point on the earth.

Now, confining our attention to rigidities less than 59.2 Bv (hence earth radii less than unity), we see from Fig. 2 that if $\gamma \geqslant 1$ no particles can reach the earth anywhere. Then let us choose $\gamma=1$ as the critical condition for preventing particles from reaching the earth. Putting $\gamma=1$ and $M_{r}=0$ in Eq. (6), as well as $\cos \omega=0$, and any convenient number for $\rho$, one obtains $a$ as a function of $\lambda . \rho$ is now interpreted as the ratio of the earth's radius to the ring radius. Assigning a numerical value to $\rho$ amounts to choosing the ring radius. Since in this case no current flows in the ring, it does not matter where one chooses it to be. Using the value chosen for $\rho$ and the cgs values of the earth's dipole moment and radius, one can then interpret $a$ as rigidity. That is, $a(\lambda)$ specifies the rigidity which a particle must exceed to reach the earth at the latitude $\lambda$ from the vertical direction. Actually, at latitudes below about $45^{\circ}$, there are appreciable forbidden regions in the region allowed by this discussion. However, we confine ourselves to latitudes where this approximation is valid.

Now let us see how these results are modified by the addition of a ring current. A ring current will appear on these diagrams as a point on the line $z=0$, whose
distance from the origin depends on the ring radius in centimeters and the rigidity of the particle for which the diagram is drawn. Let us discuss the case when the ring has a radius of 7.5 earth radii and $M_{r}=1$. Then for a rigidity $R=59.2 \mathrm{Bv}$, we obtain Fig. 3 (a). Note that the presence of the ring has caused the "pass point" at the equator to open slightly. In order to just close it again, the value of $\gamma$ must be increased. As the rigidity decreases, if one keeps $\gamma$ adjusted so that the pass point is always just closed, one obtains successively Figs. 3(b) and 3(c). In Fig. 3 (c), there are two pass points. The inner one is the one which has been there all along. The outer one is the result of the gradual distortion of the shape of the forbidden region by the increasing value of $\gamma$. If one were to continue keeping the inner pass point just closed, the outer one would become a solid block much like the equatorial block in Fig. 2(d). Therefore when the condition shown in Fig. 3(c) is reached, as the rigidity is further decreased it is the outer pass point which is kept just closed, as shown in Fig. 3(d).

Now it is desirable to have a systematic method of calculating the critical value of $\gamma$. Consider Fig. 4, drawn for $\lambda=0$. Since the ring radius is a fixed number of centimeters, this is a plot of a quantity proportional to $R^{\frac{1}{2}}$ against the radial coordinate in Störmer units.


Fig. 3. Störmer plots for a combined dipole and ring current field. Figures (a) and (c) were calculated. Figures (b) and (d) were sketched using Fig. (c) and Fig. 2 as well as figures like Fig. 4 as guides. The scale on Figs. (b) and (d) therefore indicates the general magnitudes of the quantities involved but has no precision.


Fig. 4. Forbidden regions in the equatorial plane as a function of ring radius in Störmer units. Combined dipole and ring current fields.

The shaded regions are the forbidden regions. A comparison of this figure with the sequence of diagrams in Fig. 3 will immediately show the relation between the two kinds of plot. From Fig. 4 it is apparent that the value of $\gamma$ for which the pass point just closes may be obtained by putting

$$
\begin{equation*}
(d a / d r)_{\lambda=0}=0 \tag{7}
\end{equation*}
$$

putting in this equation the desired value of $a$ (which serves as rigidity), and solving for $\gamma$. The result of this procedure may be denoted $\gamma_{c}(a)$. (Note that one does not obtain $\gamma_{c}(a)$ by putting $\cos \omega=1$ in Eq. (6) and finding the value of $\gamma$ for which the two roots intersect and become complex, since these points are those denoted by $B$ in Fig. 4.) After carrying out these operations, one obtains

$$
\begin{align*}
2 \gamma_{c}= \pm \rho_{c}+ & \frac{1}{a_{c}{ }^{2} \rho_{c}} \\
& \times\left[3-\frac{2 M_{r}}{\pi} \rho_{c}\left(1-\rho_{c}\right)\left(\frac{1+\rho_{c}{ }^{2}}{\left(1-\rho_{c}\right)^{2}} E_{c}-K_{c}\right)\right] \\
a_{c}^{2}=\mp & \frac{1}{\rho_{c}^{2}}\left[1-\frac{2 M_{r}}{\pi} \rho_{c}^{3}\left(\frac{E_{c}}{1-\rho_{c}}+\frac{K_{c}}{1+\rho_{c}}\right)\right] \tag{8}
\end{align*}
$$

where all lengths except $a_{c}$ are in units of the ring radius and $M_{r}$ is in units of the earth's dipole moment, just as as in Eq. (6). The subscript $c$ means that the quantity so labelled is appropriate to just closing the pass point. $\rho_{c}$ is the ratio of the radial coordinate of the pass point to the ring radius. $a_{c}$ is in Störmer units. By calculating both $a_{c}$ and $\gamma_{c}$ for a range of $\rho_{c}$, one obtains $\gamma_{c}\left(a_{c}\right)$, the quantity of interest.


Fig. 5. Critical values of $\gamma$ as a function of ring radius in Störmer units. Combined dipole and ring current fields.

The result of this procedure is shown in Fig. 5. for two values of $M_{r}$. The dotted portions of the top curve show the result of keeping the "wrong" pass point just closed. It should be noted that the "kinks" in these curves are not due to the filamentary approximation to the ring current. Even if a ring of finite cross section were used, one would still shift from one pass point to the other as a criterion. It may be that current distributions can be constructed in such a way that this shift in criteria may take place while keeping the first derivative of these curves continuous, but there is no reason why this break is unrealistic. In particular, it is not due to the singularity in the vector potential.
Now, just as before, we put this result back in Eq. (6), at the same time putting $\cos \omega=0$ in order to get the vertical cutoff. The value of $\rho$ to be used is the ratio of the earth's radius to the ring radius; hence this is the number specifying the ring radius, and is otherwise constant throughout the problem. The other parameter needed in order to specify the ring completely is the value of $M_{r}$, the equivalent dipole moment of the ring in units of the earth's dipole moment. For convenience in visualizing the magnitude of these parameters, Table I gives the value of the field measured at the earth's surface at the equator produced by a ring having the listed properties. $1 \gamma=10^{-5}$ gauss. The total field at the equator, due almost entirely to the dipole, is about $30000 \gamma$.
$500 \gamma$ is, from data on the earth's field, probably excluded as being too large. As we shall see from a study of the rigidity cutoff, this set of ring parameters is excluded by cosmic ray data as well. The other sets of

Table I. Magnetic field at equator due to ring current.

| $M_{r}^{1 / \rho_{e}}$ | 5 | 7.5 |
| :---: | :---: | :---: |
| 1 | $247 \gamma$ | $72 \gamma$ |
| 2 | $494 \gamma$ | $144 \gamma$ |

values seem consistent with knowledge of the earth's field. ${ }^{4}$
The resulting vertical rigidity cutoffs are shown in Fig. 6. Note the "break" in each curve with a nonzero value of $M_{r}$. This is a direct result of the break occurring in the curves shown in Fig. 5.

## APPLICATION TO EXPERIMENTAL RESULTS

Figure 7 shows the significance of these breaks. In this figure are plotted some of the available data concerning vertical intensities obtained in latitude surveys. ${ }^{8}$ The rigidity plotted along the horizontal axis is derived from geomagnetic theory. Assuming the earth's field to be that of a dipole alone, and taking account of the theory of the main cone of Lemaitre and Vallarta, ${ }^{2}$ one


Fig. 6. Critical rigidity for arrival in the vertical direction as a function of latitude, with and without ring current.
obtains the points on the solid curve. The latitude knee occurs at $R=1.5 \mathrm{Bv}$. The dotted curve shows the spectrum obtained for one set of the ring current parameters. This dotted curve is a plot of the function $J=0.29 E^{-0.9}$, where $E=\left(R^{2}+0.8668\right)^{\frac{1}{2}}$ is the total energy of a proton in Bev. If there is indeed a ring current, and if this mechanism is the dominant one in producing the observed latitude variation, then the latitude knee is produced by the "break" in the curves shown in Fig. 6. Hence within the framework of this theory we can study variations in the latitude knee by studying the behavior of the "kink" in the critical

[^4]rigidity curves of Fig. 6. Figure 8 shows the result of such a study. By comparing $8(\mathrm{a})$ with $8(\mathrm{~b})$, one sees that as the ring radius increases, or as the current strength decreases ( $M_{r}$ is proportional to the current strength) the kink moves to higher latitudes. Now it is of interest to try to correlate changes in the ring parameters with the sunspot cycle.

The two immediate consequences of changes in the ring (apart from consequences for cosmic radiation) are changes in the strength of the magnetic field measured at the earth's surface and changes in the position of the auroral zone (assuming the ring current to be the reservoir of auroral particles). Variations in the earth's field having a cycle of 11 years have been found at several stations. ${ }^{9}$ The analysis of the data is quite difficult, and the results not unique, as may be seen by studying the discussion of Vestine et al. The result given by Chapman and Bartels is not in phase with the sunspot cycle, a result not in accordance with the interpretation we will suggest below. The result favored by


Fig. 7. Primary cosmic ray rigidity spectrum as deduced from latitude surveys using geomagnetic theory with and without ring current.

Vestine, however, is moderately well in phase. One can perhaps conclude that these data are not inconsistent with the earth's field being lowered during sunspot maxima by an enhancement of an equatorial ring current (either an increase in the current flowing, or a decrease of the radius, or both) and being allowed to rise again during sunspot minima by a decay of the current.

It is commonly stated that the auroral zone drifts north as sunspot numbers decline. ${ }^{10}$ This result is also explainable qualitatively as being due to the enhancement of a ring current during times of solar activity, as discussed above.
Provided that this interpretation is correct, one then has a mechanism for partially explaining certain recent cosmic-ray results.

[^5]

Fig. 8. (a) Critical rigidity at the kink as a function of the ring radius in earth radii (with ring current). (b) The latitude of the kink as a function of the ring radius in earth radii (with ring current).

Both Neher ${ }^{11}$ and Meyer and Simpson ${ }^{12}$ report that the cosmic-ray latitude knee moved north during the period from 1948 to 1954 . As can be seen from Fig. 8 this is just the result the ring current would be expected to produce as solar activity decreased (assuming that a decrease in solar activity either allows the ring radius to increase or the ring current to decrease, or both, as discussed above), and the years 1948-1954 are indeed a period of declining sunspot activity. These data cannot, of course, be taken as confirmation of the existence of a ring current, since the effects may be produced by some entirely different mechanism. Nevertheless, they are consistent with such a hypothesis.

There exist other data which one might expect to be explained by the ring current which, at this level of analysis, at least, contradict it. The results obtained here, in agreement with those of Treiman, ${ }^{13}$ predict the cosmic-ray intensity at latitudes below the knee during the main phase of magnetic storms to be higher than normal. Actually, it is low. ${ }^{14}$ Also, the data of Neher ${ }^{11}$ and of Meyer and Simpson ${ }^{12}$ indicate that the intensity has increased at all latitudes even up to the geomagnetic pole during the period 1948-1954. Neither of these results can be explained by the theory developed in the present paper. Actually, the situation is more complicated than this. Both sets of data were taken with omnidirectional detectors, which means that the

[^6]

Fig. 9. Critical value of particle velocity ( $\beta=v / c$ ) with and without ring current.
behavior of the shadow cone at high latitude may be important. It would seem from qualitative considerations that the shadow cone should not be much affected by the ring current, but this is not certain. In addition, the results of Simpson are at a low altitude, and he believes that atmospheric absorption restricts his measurements to primaries sufficiently near the vertical that the shadow cone has no effect. In the absence of a significant shadow-cone variation, the results would suggest that either the ring current hypothesis is not correct, at least in its present form, or else some additional mechanism exists which increases the intensity at all latitudes. Vertical-intensity measurements near and above the knee as a function of time should help settle this question. (To be useful, they should be carried out at least within a gram or two of the top of the atmosphere.)

## PROPOSED EXPERIMENTS

The theory presented in this paper suggests the following experiments. In the first place, as shown in Fig. 6, the existence of a ring current will reduce the value of the rigidity cutoff at all latitudes but particularly in the vicinity of the latitude knee. Measurements in the vertical direction with, for instance, a Čerenkov counter, should in principle detect the differences necessary to produce the latitude knee, if this is the mechanism involved. If photographic emulsions can be flown to sufficiently high altitudes, measurements of multiple Coulomb scattering of heavy primaries
should also help decide whether particles with less than the Störmer cut-off rigidity reach the earth. The results of such measurements could help in the determination of the ring parameters if the effect were found. Figure 9 is a plot of the critical value of $\beta(=v / c)$ in the vertical direction as a function of latitude for protons and $\alpha$ particles assuming only a dipole field and then a ring current with representative values of $a$ and $M_{r}$. This figure shows the general magnitude of the effect for a Čerenkov detector.
Secondly, vertical measurements very near (within a gram of two) of the top of the atmosphere from below the knee to the geometric pole should settle the question concerning the change in absolute intensity as discussed near the end of the previous section. To give the best information on this point, the measurements should be repeated several times over a sunspot cycle with homogeneous, or at least carefully intercalibrated, techniques.
Finally, it is quite conceivable that a ring current of the sort discussed here can have large effects on the impact zones for charged particles leaving the vicinity of the sun and arriving at the earth. These zones have been studied in connection with the solar flare effect by Firor. ${ }^{15}$ We are now doing work on trajectories of such solar particles in order to assess the importance of this effect.

## CONCLUSION

If a ring current with the characteristics postulated by Schmidt exists, then it should lead to easily observable effects on the latitude variation of cosmic rays, producing a latitude knee even if there is no low rigidity cut off in the primary spectrum. If this effect is absent, then it must be concluded that the ring, if it exists, is much weaker than Schmidt's value. Some features of recently observed long period variations in the cosmic radiation are consistent with the Schmidt values. Experiments are proposed which should help settle the question more definitely.

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[^7]
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