Elastic Scattering of Protons from Li^7†

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Differential cross sections for the elastic scattering of protons from Li^7 were measured over the energy range 1350 kev–3000 kev at the center-of-mass angles 70°, 90°, 110°, 130°, 150°, and 167°. The various proton groups scattered from different elements in the lithium target and target backing were separated by a magnetic analyzer. The 167° data agreed well with the earlier work at 166.3° by Bashkin and Richards. At all six angles the curves showed a maximum at about 2000 kev, a dip at 2230 kev, and an anomaly near the 1882 kev $Li^7(p,n)Be^7$ threshold. Exclusive of the anomaly, the energy dependence of the data suggests the presence of two interfering levels in Be^8.

I. INTRODUCTION

MEASUREMENTS of the differential cross sections for the elastic scattering of particles by nuclei as a function of energy and angle have proven very fruitful for the determination of the properties of the energy levels of the compound nuclei involved. The present experiment concerns the elastic scattering of protons from Li^7. In this case the compound nucleus, Be^8, is formed in highly excited states. Warters, Fowler, and Lauritsen[1] have measured this elastic scattering cross section at seven angles in the bombarding energy range between 360 kev and 1400 kev. A yield curve for one angle at higher energies has been obtained by Bashkin and Richards.[2] This curve exhibited interesting variations near the threshold and the first resonance for the $Li^7(p,n)Be^7$ reaction. The following experiment was undertaken to obtain data above 1400 kev at several angles in order to assist in determining the structure of the Be^8 nucleus.

II. EXPERIMENTAL PROCEDURE

The proton beam was supplied by the State University of Iowa statitron. A magnet which deflected the beam by 25° before it entered the target chamber prescribed the proton energy. A second magnet was used to separate the protons elastically scattered by the lithium from the protons scattered by other elements in the target and target backing.

The target chamber and magnetic analyzer are shown schematically in Fig. 1. The deflected proton beam enters from the left and then passes through the slits S_1 and S_2, the tube E, the $\frac{1}{8}$-in. diameter aperture A_1, and the target T, being finally caught in a Faraday cage F. The protons emerging from the target passing downward into the tube to the right are subjected to analysis by the magnet M; the selected group passes on down into the thin end-window proportional counter C_1. Possible particle paths are determined by the settings of the slits S_4, S_7, and S_8. The other slits (S_3, S_5, and S_6) and the apertures (A_2, A_3, A_4, A_5, A_6, and A_7) do not interfere with these paths, but minimize spurious counts due to protons scattering off the walls and surfaces of this analyzer section. As shown in the detail, there is a sliding joint, sealed by the "O" ring O, between the upper and lower halves of the cylindrical target chamber. This allows rotation of the lower half of the target chamber together with the magnetic analyzer about a vertical axis through the target center while the upper part remains fixed. In this manner, the angle of detection with respect to the beam direction may be varied between 15° and 165°.

Not shown on the drawing are two proportional counters in the upper half of the target chamber facing the target which were used for detecting the alpha particles from the $Li^7(p,\alpha)\alpha$ reaction. This reaction was used for comparing different targets, for measuring variations of targets with time, and for monitoring target thicknesses during their preparation.

The target was a thin lithium layer evaporated from a furnace onto the backing while it is in place in the target chamber. The target backing was a thin Zapon film upon which a layer of aluminum had been evaporated to prolong its life under bombardment. The total target, including backing, was only a few kilovolts thick to the proton beam. The post on which the target holder was mounted was rotated to minimize the total path length in the target and backing for the scattered protons.

The 85° angle between the entrance and exit edges of the magnet causes a focusing action in the direction of bending. Because of this focusing, a particle group passing through slit S_4 will be smaller in width than the opening of slit S_7 in front of the counter. Thus there is a small flat portion on the top of the resolution curve in which the counting rate is independent of the exact magnet current. With the particular settings of the slits used, the acceptance solid angle of the magnet was 10^{-4} steradian and the resolution in momentum was 2.6%.

A spectrum of the protons from a natural lithium target may be seen in Fig. 2. A clear separation of the

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* Work performed in part while an Atomic Energy Commission Predoctoral Fellow. Present address: Naval Research Laboratory, Washington 25, D. C.
[1] Warters, Fowler, and Lauritsen, Phys. Rev. **91**, 917 (1953).
[2] S. Bashkin and H. T. Richards, Phys. Rev. **84**, 1124 (1951).

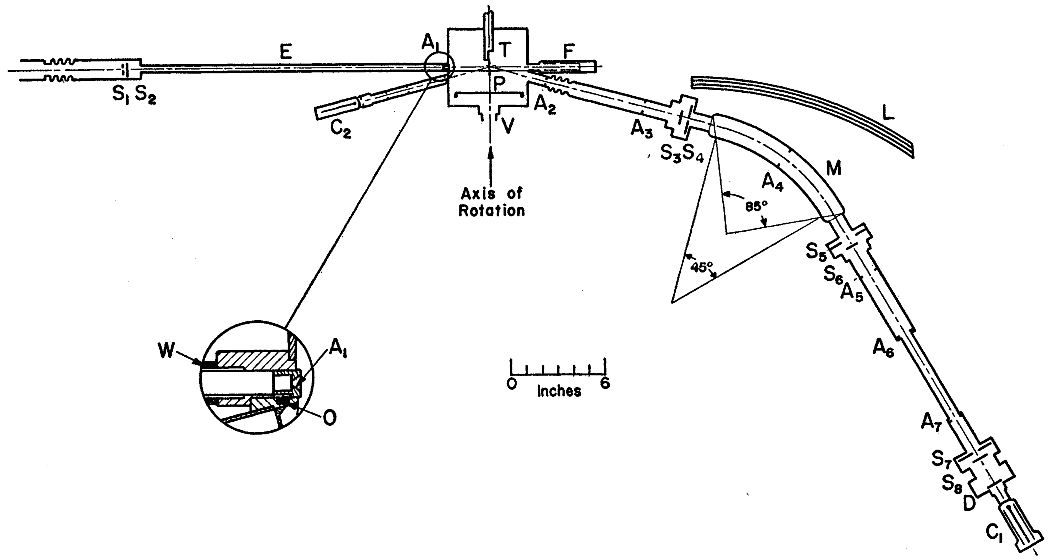

FIG. 1. Target chamber and analyzer path. Protons entering through tube E are scattered in the target T, analyzed by magnet M, and counted in the counter C_1. C_2 is a counter used for auxiliary measurements. The laminated shield L attached to the magnet and the high permeability tape wrap W minimize the influence of the fringing field of the magnet on the entering beam. D is a disk interposable in front of the counter for background measurements, P is a cooled plate used as a cold trap, O is an "O" ring, F is a Faraday cage, and V represents a vacuum lead to a diffusion pump. The A's and S's are various apertures and slits.

proton groups in this curve at the maximum backward angle available was obtained; at the more forward angles they moved closer together limiting the minimum angle for the present experiment to 70°. To minimize the Li^6 contribution at the forward angles, isotopically enriched Li^7 was used for some of the data.

In taking a datum point at a given angle and energy, the current in the analyzer magnet was adjusted to select the protons elastically scattered from the Li^7 and counts were then taken for a given amount of beam charge as measured by a current integrator connected to the Faraday cage. Points were taken at closely spaced proton energies for six different angles. A run at one angle took several days. A target usually lasted through such a run without breaking. The beam currents used were between 0.17 and 0.35 microampere.

The apparatus was checked for possible systematic errors by measuring the elastic scattering of protons from thin gold evaporated targets. This gave agreement within 2% of the expected dependence on energy and angle as predicted by the Rutherford law.

III. RESULTS

Since the thickness of the targets varied with time and between different targets, the data using the $Li^7(p,\alpha)\alpha$ counters were used to convert the $Li^7(p,p)Li^7$ data for all angles and energies to the same relative cross section scale. Absolute target thicknesses were not measured because the lithium content of the very thin targets used could not be determined with adequate precision. Therefore, the factor used in converting to an absolute cross section scale was obtained by a comparison with the data of Warters, Fowler, and Lauritsen[1] in the small region of overlap in energy.

Figure 3 shows the data for the six angles at which measurements were made. The maximum error in energy of a point is considered to be $\pm 1\%$. Several factors enter into errors in the values of the cross sections. Some of the counts recorded were from protons corresponding to tails of other groups in the spectrum curve, some due to particles from other reactions (e.g., $Li^6(p,\alpha)He^3$) which get through the analyzer at certain current settings, some from knock-on protons in the methane counter gas due to the neutron background in the room. Corrections for these background contri-

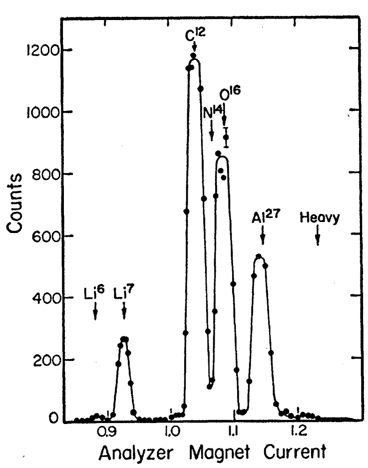

FIG. 2. Momentum spectrum curve for protons scattered from a lithium target. Proton energy was 1075 kev and angle of analyzer was 165°. Arrows indicate expected peak positions for the protons elastically scattered from the indicated isotope.

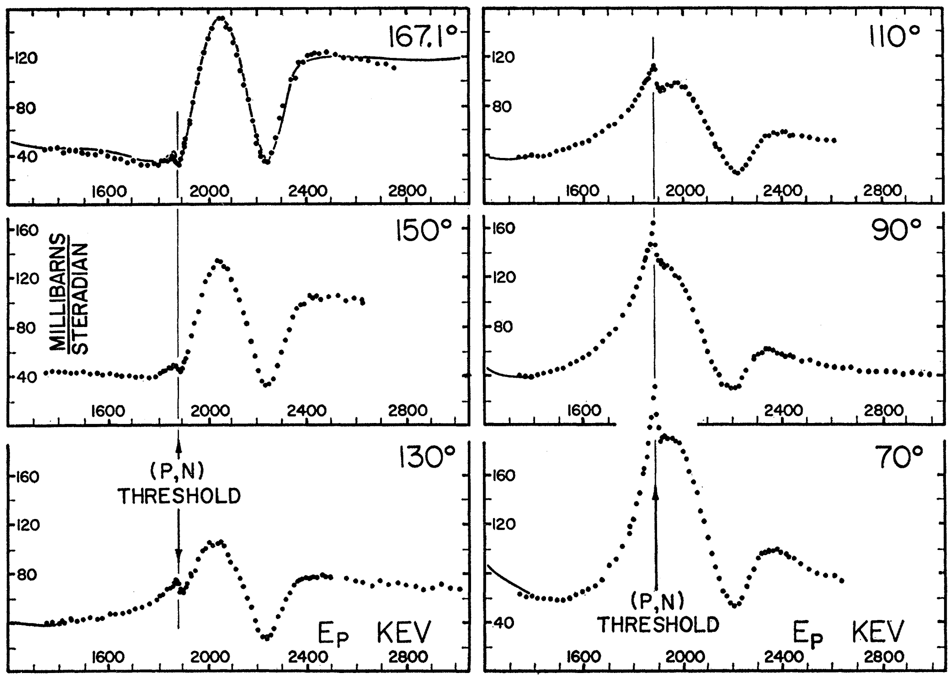

FIG. 3. Differential cross section *vs* proton energy for the elastic scattering of protons from Li^7. Cross sections and angles are in the center-of-mass system. The short solid lines on the 130°, 110°, 90°, and 70° graphs give the data of Warters, Fowler, and Lauritsen (the 90° graph shows their 89.2° data). The solid line on the 167.1° graph gives the data at 166.3° by Bashkin and Richards.

butions were made and are considered accurate only to a factor of 2. However, these corrections to the data amounted to 4% or less except at 70° where they were somewhat higher. Counting statistics varied between 1 and 3%. The accuracy of the Warters, Fowler, and Lauritsen data[1] is quoted as 5%. Based upon this, it is believed that the present absolute cross section measurements are accurate to 10% except at 70° where a somewhat larger uncertainty might be assigned. The relative cross sections which do not depend upon an absolute calibration have an internal consistency better than 10%.

Absolute values for the cross sections were also measured by Bashkin and Richards[2] with a stated uncertainty of at least 20%. These data for the slightly different angle of 166.3° are given on the 167.1° graph as the solid line. The good agreement between these and the present data may be noted.

IV. DISCUSSION

All of the $Li^7(p,p)Li^7$ differential cross section curves exhibit an anomaly near the $Li^7(p,n)Be^7$ threshold. This anomaly is seen to change in shape as the angle of detection changes. Also, the position of the peak of this anomaly is lower than the threshold energy for 167.1°, but moves up to this energy at the smaller angles. An examination in greater detail of the anomaly for 90° is seen in Fig. 4. While gathering the proton data, neutrons from the $Li^7(p,n)Be^7$ reaction were detected with a shielded long counter at 0° to the incident beam. Figure 4 also shows this neutron curve. The threshold is indicated by the point at which the neutron counting rate starts rising; the target thickness is given by the energy interval of rising portion. This interval is seen to coincide with the energy interval of the approximately flat portion of the proton peak. This behavior is consistent with a spike just at threshold which is rounded off by the target thickness. Such a spike or cusp in a reaction or scattering has been predicted by Wigner[3] to occur at the thresholds for competing reactions. Newson[4] and Haeberli[5] have made the suggestion (which is equivalent above threshold to Wigner's prediction) that the anomaly might be explained on the basis of the effects of the rapidly changing total width of a resonance when the neutron partial width becomes a possible contribution.

From the curves at the six angles, it would appear that they may represent the result of two interfering resonances. A dip occurs in all of the curves near the energy of 2240 kev at which the companion $Li^7(p,n)Be^7$ reaction indicates a resonance. It is possible, therefore, that this is one of the resonances. The other resonance would be lower in energy and several hundred kev wide and perhaps would correspond to a resonance indicated by the yield of hard gamma rays[6,7] from the $Li^7(p,\gamma)Be^8$ reaction. Although other combinations of two or more levels might be possible, the discussion to follow is based on the assumption of these particular two levels.

Certain limitations on the values of the orbital

[3] Eugene P. Wigner, Phys. Rev. **73**, 1002 (1948). See also reference 2.
[4] H. W. Newson (private communication).
[5] W. Haeberli (private communication).
[6] P. C. Price, Proc. Phys. Soc. (London) **A67**, 849 (1954).
[7] Swann, Rothman, Porter, and Mandeville, Phys. Rev. **98**, 1183(A) (1955).

angular momenta (l) of the incident protons and on the angular momenta (J) of the states of the compound Be^8 nucleus follow from the widths of the levels and the magnitudes of the resonant cross sections. Such information concerning the level at 2240 kev is obtained from the $Li^7(p,n)Be^7$ data[8] while the information about the other level is inferred from the $Li^7(p,p)Li^7$ data of the present experiment. Because of the other contributions, in particular the Coulomb, it is difficult to give precisely the resonant part of the cross section for this $Li^7(p,p)Li^7$ scattering. However, an estimate of its value may be made by integration of the differential cross section data over the six angles for which it is given.

The resonant part of the total cross section for the $Li^7(p,n)Be^7$ reaction at resonance is

$$\frac{(2J+1)}{(2I+1)(2i+1)}\pi\lambda\!\!\!^{-2}\frac{4\Gamma_p\Gamma_n}{(\Gamma_p+\Gamma_n)^2}, \tag{1}$$

and for the $Li^7(p,p)Li^7$ scattering is

$$\frac{(2J+1)}{(2I+1)(2i+1)}\pi\lambda\!\!\!^{-2}\frac{4\Gamma_p^2}{(\Gamma_p+\Gamma_n)^2}, \tag{2}$$

where I is the angular momentum of target nucleus, i is the angular momentum of bombarding particle, $2\pi\lambda\!\!\!^{-}$ is the wavelength of the incident proton, Γ_p is the partial width for proton emission from the state in Be^8, and Γ_n is the partial width for neutron emission. The total widths Γ of the levels have been assumed to be equal to the sum of only the proton and neutron widths as other competing reactions show only a small, if any, resonance cross section due to these levels. Comparison of the experimental data with cross sections calculated from (1) and (2) shows that J must be at least 2 for the $Li^7(p,n)Be^7$ resonance, and at least 1 for the other resonance.

The partial width Γ_p is equal to $2k_pv_{lp}\gamma_p^2$ where k_p is the wave number, v_{lp} is the penetration factor for protons of angular momentum l, and γ_p^2 is the reduced width for emission of a proton. A similar relation holds for neutron emission. The same l value should be involved in Γ_n as in Γ_p as the modes of decay of Be^8 into Li^7+p and into Be^7+n are similar since the ground states of Be^7 and Li^7 are mirror states. Teichman and Wigner[9] have given a theoretical upper limit (sum rule limit) of $3\hbar^2/2MR$ for the reduced widths, where M is reduced mass of separating system and R is maximum radius of nuclear interaction. Using this upper limit for computing Γ_p and Γ_n, the sum of Γ_p plus Γ_n is much smaller than the widths of the two levels for l values greater than 2. Thus these higher values of l are not permitted. Application of relations (1) and (2) in conjunction with the small maximum value of Γ_n allowed for $l=2$ eliminates $l=2$ protons for the $Li^7(p,n)Be^7$ resonance and J values larger than 3 for the other resonance.

[8] R. Taschek and A. Hemmendinger, Phys. Rev. **74**, 373 (1948).
[9] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

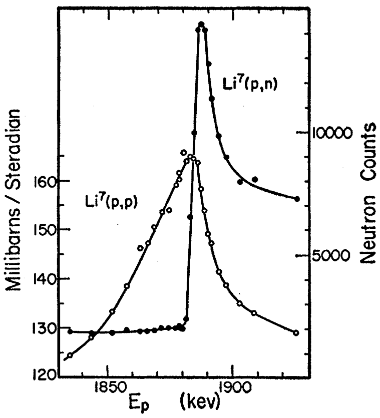

FIG. 4. Detail of the anomaly for detection angle of 90°. The $Li^7(p,p)Li^7$ data are given by the open circles with the scale given to the left. The data given by the solid circles with the scale at the right are for the forward traveling neutrons from the $Li^7(p,n)Be^7$ reaction.

The angular momenta I, i, and l combine to give the angular momentum J of the compound state. Since the parity of Li^7 is odd, the parity of a Be^8 state is $(-)^{l+1}$. As a result of the above limitations, the J and parity values of $2\pm$ or $3+$ for the $Li^7(p,n)Be^7$ resonance and of $1\pm$, $2\pm$, or $3\pm$ for the other resonance are allowed. The $2+$ value is improbable for either state unless it has total isotopic spin 1 since a resonance for the decay of the Be^8 into two alpha particles is not observed in this region.[10]

Exact fitting of parameters to these two levels has not been successful at present.

On the basis of both experimental evidence and theoretical prediction, Lane[11] states that it is probable that levels formed in the $1p$ nuclear shell by P-wave protons will have reduced widths small compared to the theoretical limit, whereas states formed by S-wave and D-wave protons will have reduced widths comparable to this limit. If this is the case, P-wave and D-wave states can form distinguishable resonances in this region, but the maximum limit for Γ_p for S-wave states is so large that these states would be spread out to form a background. The presence of an appreciable S-wave background has been implied by other experiments.

Adair[12] has analyzed the neutron data from the $Li^7(p,n)Be^7$ reaction in the region of the 2240-kev resonance and has shown that they are consistent with a $3+$ state formed by $l=1$ protons. The 90° $Li^7(p,p)Li^7$ curve indicates interference effects at this resonance. This state can interfere only with odd-l states since at

[10] Heydenburg, Hudson, Inglis, and Whitehead, Phys. Rev. **74**, 405 (1948). These $Li^7(p,\alpha)\alpha$ yield data are presented on a relative scale. During the course of the present experiment, the differential cross section for this reaction at one energy and angle was determined for the laboratory system. The result was 0.97 millibarn /steradian±20% at 132° for 1550 kev.
[11] A. M. Lane, Atomic Energy Research Establishment (Harwell) Report T/R 1289, 1954 (unpublished).
[12] R. K. Adair, Phys. Rev. **96**, 709 (1954).

90° the interference with even-l states and with the potential and Coulomb scattering vanishes. This would imply that the other resonance is also formed by $l=1$ protons as an interference effect of such magnitude due to more remote resonances is unlikely. Interference between these two levels might be expected to occur in the $Li^7(p,n)Be^7$ reaction also. However, the predominant interference as indicated by the angular distribution of the neutrons[8] is between states of opposite parity.

V. ACKNOWLEDGMENTS

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Dynamics of Neutron Scattering by Molecules*

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The scattering of low-energy neutrons by molecules is investigated by a method which facilitates the treatment of inelastic processes. Dynamical systems characterized by the degrees of freedom of gas molecules are examined in detail. The formalism is applied first to the derivation of the differential cross section for scattering by an ensemble of coupled harmonically oscillating nuclei in thermal equilibrium. Rotator scattering is investigated in the limits of high and low energy, and inelastic corrections to the static approximation are also calculated. The cross section of a thermally excited monatomic gas is presented in a closed form which shows its dependence on the ratio of the mean velocity of thermal motion to the neutron velocity. Results appropriate to large and to small values of the velocity ratio are derived. Quantitative estimates for the validity of the various approximations are given.

1. INTRODUCTION

In recent years, the diffraction of slow neutrons by gases has grown in importance as a research tool, providing a useful complement to other techniques for the determination of nuclear scattering lengths and various molecular properties. But precise theoretical formulas for the differential cross sections have been exhibited, heretofore, only under the restrictive assumption of elastic scattering, or more accurately, the assumption that the energy transferred in a neutron-molecule collision is negligible in comparison with the incident neutron energy. In practice, molecules in thermally excited quantum states may impart to the neutron, and molecules of small inertia may absorb, energy in amounts which bear a significant ratio to the neutron energy. For these reasons, in many cases of experimental interest, the explicit consideration of inelasticity is necessary for a quantitative understanding of the neutron cross sections.

In principle, the cross sections can be computed by an explicit summation over contributions from all the scattering processes that occur. But the large number of molecular states encountered in a gas at thermal equilibrium and the large number of energetically permissible quantum transitions of the gas molecules will, as a rule, render such a procedure impractical. Recourse to more general and implicit methods of performing the required summations must then be sought.

A convenient step in achieving this is the use of operator representations in the description of the internal and external coordinates of the scattering molecules. Expressions for the cross section may then be developed which implicitly sum, in closed form, the contributions of all transitions permitted by the conservation laws. This device is the central one of the present work. We return to it repeatedly, considering in turn interactions with the various molecular degrees of freedom.

The operator techniques to be used are formulated in Sec. 2. In the three succeeding sections, they are illustrated by separate applications to problems involving vibrational, rotational, and translational degrees of freedom. The assembly and generalization of these results for application to the full problem of diffraction by gases is given in the paper which follows.

2. GENERAL FORMULATION

The scattering of neutrons by the nuclei of chemically bound atoms may be accurately treated by means of the familiar Fermi pseudopotential approximation.[1] The short-range potentials acting between neutrons and

* Preliminary accounts of this work were presented at the New York and Washington meetings of the American Physical Society, 1954, Phys. Rev. 94, 790(A) (1954) and Phys. Rev. 95, 605(A) (1954).

† National Science Foundation Predoctoral Fellow.

[1] E. Fermi, Ricerca sci. 7, 13 (1936).