the parameter $\zeta=0.05$ fit the data as well as the results for $\zeta=0.03$. Because of the large experimental error quoted by Stelson and Campbell, for a particular ζ we varied the X_0^2 so that the results would bracket the experimental data. $\zeta=0.05$ fits the data for a slightly smaller well depth V_0 better than does $\zeta=0.03$.

With the square-well model, the most recent total cross-section data indicate that for lead the smaller radius of $R=(1.27A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm is to be preferred to $R=1.45A^{\frac{1}{3}}\times10^{-13}$ cm. For the calculation of the inelastic scattering cross section into the isomeric level of ${}_{82}\text{Pb}^{207}$, the opposite is true, i.e., the larger radius seems to be preferred as can be seen in Fig. 6. In Fig. 6, we kept X_0^2 constant. Had we kept V_0 constant, the difference between the results would have been greater, with the larger radius even more favored. The strong-interaction model gives the same result so far as the size of the radius is concerned.

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Nuclear Levels and Transitions in Lu^{175} According to the Unified Model*

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The scheme for the decay of Yb^{175} and Hf^{175} to Lu^{175} is analyzed theoretically on the basis of the Bohr-Mottelson strong-coupling unified model. A set of spins and parities for all the levels involved is found to be uniquely consistent with the available experimental data and the level-structure predictions of the model. The anomalously large ratio of $M2$ to $E1$ radiation observed in two of the gamma transitions is accounted for as a consequence of configuration forbiddenness. Parallel remarks are made concerning the spectrum of Hf^{177}.

1. INTRODUCTION

RECENTLY, a rather detailed experimental investigation of the decay of Yb^{175} and Hf^{175} to the low-lying levels of Lu^{175} has been performed by Mize, Bunker, and Starner.[1] The Yb decay has been studied also by de Waard,[2] Akerlind, Hartmann, and Wiedling,[2] and Marty[3] and the Hf decay by Burford, Perkins, and Haynes.[4] Since these nuclei lie in a region of large deformation, the strong-coupling unified model[5] may be expected to provide useful guidance in the interpretation of the level structure and characteristic features of the decay scheme. Conversely, the example furnishes an opportunity to subject the model to further experimental test.

Relevant essentials of the strong-coupling unified model are here briefly recalled. For axially symmetric nuclei the component K of the total nuclear angular momentum I along the symmetry axis is supposed to be an approximately good quantum number. For an odd-A nucleus the rotational band based on a particular intrinsic structure is constituted of levels with spin sequence $I=K,\ K+1,\ K+2,\ \cdots$, all of the same parity as the intrinsic structure. The rotational energies are given by

$$W_{\text{rot}}=(\hbar^2/2\mathcal{I})[I(I+1)-K(K+1)], \qquad (1)$$

except in the special case $K=\Omega=1/2$, $\mathcal{I}$ being the moment of inertia. In the low-lying levels encountered here, there is no vibrational excitation, whence $K=\Omega$, where Ω is the sum of the components of the angular momenta of unpaired nucleons along the nuclear symmetry axis; also, no more than a single nucleon is excited, whence Ω is equal to the contribution of the odd nucleon only. States of an odd nucleon are conveniently identified (in the independent-particle approximation) by Ω, l, and j, the last two being good quantum numbers only in the limit of zero deformation.

Calculations by Nilsson[6] of independent-particle energy levels and wave functions for a spheroidal well with spin-orbit coupling have made possible a more detailed and unambiguous application of the strong-coupling model.[7] His energy levels as functions of deformation are shown in Fig. 2.

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] Mize, Bunker, and Starner, Phys. Rev. **100**, 1390 (1955); hereafter referred to as MBS.

[2] H. de Waard, Phil. Mag. **46**, 445 (1955); Akerlind, Hartmann, and Wiedling, Phil. Mag. **46**, 448 (1955).

[3] N. Marty, Compt. rend. **240**, 963 (1955).

[4] Burford, Perkins, and Haynes, Phys. Rev. **99**, 3 (1955).

[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

[6] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

[7] A survey of ground and low excited states of deformed nuclei based on these calculations has been made by B. R. Mottelson and S. G. Nilsson [Phys. Rev. **99**, 1615 (1955)].

The decay scheme for the present case as found by MBS[1] is shown in Fig. 1. Spins and parities have been assigned on the basis of the following considerations.

2. CONSIDERATIONS OF LEVEL STRUCTURE

The measured spin 7/2 of the ground state of Lu^{175} agrees with that predicted from Fig. 2 for a reasonable prolate deformation, as noted by Nilsson and Mottelson[7] (who give the deformation $\delta \simeq 0.28$); the state of the odd proton is designated $g_{7/2}(\Omega=7/2)$, and the entire extra-closed-shell proton configuration is $(1g_{7/2})^7 \times (2d_{5/2})^4 (1h_{11/2})^8 (2d_{3/2})^2$. Evidence that the 114- and 251-kev levels observed by MBS are the 9/2+ and 11/2+ rotational excitations of the ground state is discussed by the experimenters; the energy ratios are in good agreement with the theoretical prediction of 20/9. The assignment 5/2+ to the 343-kev level[1,4] accords well with the prediction from Fig. 2 of a low excited $d_{5/2}(\Omega=5/2)$ state. The 9/2− level at 396 kev[1,2] is identified as the $h_{11/2}(\Omega=9/2)$ particle excitation.[7]

It has been observed by Burford *et al.*[4] that the level at 432 kev lies within one kev of the position of the expected first rotational excitation (7/2+) of the 343-kev (5/2+) particle state (assuming the same moment of inertia as for the ground state). Assignment of 7/2+ to this level accords with an assignment of 5/2− to the ground state of Hf^{175}, whose electron capture to that level is apparently first-forbidden. A 5/2− ground state for Hf^{175} is also predicted from Fig. 2, deriving from an $f_{7/2}(\Omega=5/2)$ odd-neutron state.[8] An alternative interpretation has been suggested by MBS. Since they observe no electron capture to the ground state of Lu^{175}, they propose the assignment 3/2− to the Hf^{175} ground state and therefore 5/2+ to the 432-kev level of Lu^{175}. From the point of view of the unified model, however, all available evidence except the absence of electron capture to the ground state lends support to the first interpretation. Furthermore, a plausible resolution of the apparent electron capture difficulty on the basis of this model has been given by Alaga.[9] Additional points pertaining purely to level structure which favor the present interpretation are the measured 5/2 spin of Yb^{173}, which like Hf^{175} has 103 neutrons, and the unavailability of any state with spin 3/2 according to Fig. 2. It would likewise be difficult to account for a second 5/2+ state in Lu^{175}.

The assignment of 7/2− to the ground state of Yb^{175}, inferred by MBS, again is in agreement with the prediction of Fig. 2.[7,10,11]

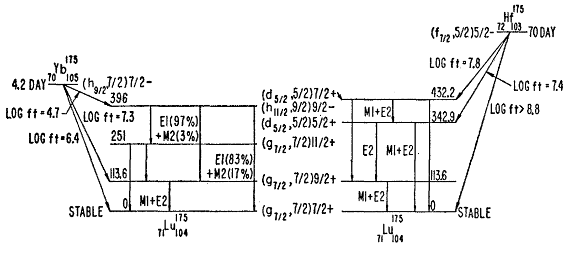

FIG. 1. Decay scheme for Yb^{175} and Hf^{175} to Lu^{175}. Levels are labeled by $(l_j,\Omega)I\pi$, where l and j refer to the principal component of the odd-nucleon state as obtained from Nilsson's calculations,[6] Ω is the component of angular momentum along the symmetry axis, I is the nuclear spin, and π is the parity. The experimental scheme is that given by Mize, Bunker, and Starner (reference 1). Energies are in kev.

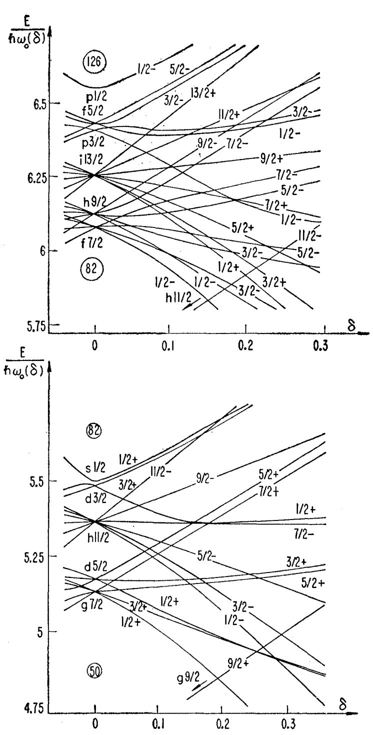

FIG. 2. Spectra for protons from $Z=50$ to $Z=82$ (below) and neutrons from $N=82$ to $N=128$ (above) as functions of nuclear deformation δ (from S. G. Nilsson, references 6 and 7). Only prolate deformations are shown. The deformation parameter is related to the intrinsic quadrupole moment Q_0 by the relation $Q_0 \simeq \frac{4}{5} Z R_0^2 \delta$, where R_0 is the nuclear charge radius. We are grateful to Dr. Ben R. Mottelson and Dr. Svend Giösta Nilsson for communicating their results to us prior to publication.

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[8] This 5/2− state in Fig. 2 is obtained continuously with increasing deformation from an $h_{9/2}$ state; it is labeled here by $f_{7/2}$ because at the large deformation in question the admixture of $f_{7/2}$ predominates. The $h_{7/2}$ and $f_{7/2}$ ($\Omega=5/2$) states exchange character due to "crossing."

[9] G. Alaga, Phys. Rev. **100**, 432 (1955).

[10] Hf^{177}, with the same number of neutrons, is also expected to have a 7/2− ground state. Recent evidence[11] tends to confirm this, in contradiction to the earlier tentative assignment of 1/2 or 3/2.[7] With the 7/2− interpretation, the decay of Lu^{177} and the consequently observed levels of Hf^{177} constitute a case with the spins of all levels equal to those of corresponding levels in the $Yb^{175} \rightarrow Lu^{175}$ case, but with parities opposite. Thus the 113- and 250-kev levels in Hf^{177} are interpreted as 9/2− and 11/2− first and second rotational excitations, the 321-kev level as a 9/2+ particle excitation (from the $i_{13/2}$ ($\Omega=9/2$) state in Fig. 2), and the Lu^{177} ground state is assigned 7/2+ in agreement with Lu^{175}.

[11] P. Marmier and F. Boehm, Phys. Rev. **97**, 103 (1955); McClelland, Mark, and Goodman, Phys. Rev. **97**, 1191 (1955); N. P. Heydenburg and G. M. Temmer, Phys. Rev. **100**, 150 (1955).

3. CONSIDERATIONS OF BETA AND GAMMA TRANSITIONS

A possible explanation for the lack of observable electron capture to the ground state of Lu^{175} (if the ground state of Hf^{175} is assumed to be $5/2-$) has recently been given by Alaga.[9] He notes that the transition to the ground state cannot occur via the principal components of the nucleonic states involved, since these differ by two in the component of orbital angular momentum along the nuclear axis. This selection rule is not operative in the transitions to the rotational sequence based on the $5/2+$ state. Hence, the ground-state transition is hindered relative to the others. It may be noted, however, that the Nilsson wave functions for the state involved in the hindered transition do contain considerable admixtures, so that some accidental cancellation may be required in order to account fully for the observed slowness.

An indication in favor of the rotational interpretation of the 432-kev level, cited by Burford *et al.*,[4] is the strength of the transition to the $5/2+$ state relative to the much more energetic transitions to the $7/2+$ and $9/2+$ states.

Ratios of reduced transition probabilities for beta or gamma transitions of given multipolarity[12] to different members of a rotational band depend only upon a geometrical factor[13]:

$$\frac{B(L, I_i \rightarrow I_f)}{B(L, I_i \rightarrow I_{f'})} = \frac{\langle I_i\, L\, K_i\, K_f - K_i | I_i\, L\, I_f\, K_f \rangle^2}{\langle I_i\, L\, K_i\, K_f - K_i | I_i\, L\, I_{f'}\, K_f \rangle^2}, \quad (2)$$

where L is the multipolarity of the transition, f and f' denote members of the same rotational sequence ($K_f = K_{f'}$), and the $\langle | \rangle$ are Clebsch-Gordan coefficients. The Hf electron capture to the rotational band based on $d_{5/2}$ is predominantly of multipolarity $L=1$ ($\Delta I = 0, \pm 1$; yes). (The first-forbidden $L=0$ ($\Delta I=0$; yes) transition to the $5/2+$ state is inhibited because the transition involves principally a change of one in j, a violation of the particle selection rule $\Delta j = 0$, while, in general, the first-forbidden $L=2$ ($\Delta I = 0, \pm 1, \pm 2$; yes) transition is considerably slower than the first-forbidden $L=0$ or 1 transitions.) The ratio of the ft values computed from (2) for the $L=1$ electron capture (upper:lower) is 2.5, in precise agreement with the experimental value. In the case of Yb, the beta decay can proceed by $L=1$ to both the ground and first rotational states, but also by $L=0$ to the ($g_{7/2}$) ground state inasmuch as the ground state of Yb, which is primarily $h_{9/2}$, contains an admixture of $f_{7/2}$. Thus the ratio of ft values (upper:lower) computed from (2) can give only a lower bound. The theoretical estimate obtained in this way is 3.5 compared with the experimental value 7.9.

One of the distinctive features of the observed gamma transitions is the relatively large ratio of $M2$ to the $E1$ radiation in the decay of the $9/2-$ to the $7/2+$ and $9/2+$ levels. An explanation for these mixtures is obtained immediately from the present model. From the Nilsson wave functions, one may calculate the $E1$ transition rate as a function of the assumed deformation. At zero deformation the transitions in question are between pure $h_{11/2}$ and $g_{7/2}$ nucleonic states; hence $\Delta j = -2$ and the $E1$ rate vanishes. In the limit of very large deformation the components of nucleonic orbital angular momentum and spin along the nuclear axis, Λ and Σ, are individually good quantum numbers; the initial and final nucleonic states for these transitions then have opposite spin components and the $E1$ transition rate again vanishes. Even for intermediate deformations the theoretical transition rate remains quite small. In particular, for the deformation $\delta = 0.28$ the $E1$ rate for the $9/2- \rightarrow 7/2+$ transition is inhibited relative to the corresponding spherical independent-particle model result (for an $h_{9/2} \rightarrow g_{7/2}$ transition) by a factor 1.4×10^{-4}. There are a number of possible $E1$ transitions in other nuclides which should be inhibited in precisely the same way.[14]

From the observed $E1+M2$ mixtures and the relative gamma intensities[1] for the transitions from the $9/2-$ state to the $9/2+$ and $7/2+$ members of the ground-state rotational band, one can obtain the ratio of the transition rates ($9/2- \rightarrow 9/2+$ to $9/2- \rightarrow 7/2+$) for the $E1$ and $M2$ radiations separately. For the $M2$ radiation the ratio agrees within experimental error with the prediction of Eq. (2); for the $E1$ radiation, the ratio is larger than that obtained from Eq. (2) by a factor ~ 8. The discrepancy in the case of the $E1$ radiation is not surprising, since the $E1$ transitions are nearly forbidden and only small admixtures to the strong-coupling wave function due to rotational excitation are required to increase the $E1$ intensity by such a factor.[15]

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The authors wish to thank Dr. J. P. Mize for extensive discussions of the experimental material and to acknowledge helpful comments from Dr. B. R. Mottelson.

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[12] In the case of beta transitions, the term multipolarity here refers to the total angular momentum of the electron and neutrino.

[13] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

[14] Among these are the transitions in Hf^{177} from the $9/2+$ state to the rotational band based on the $7/2-$ ground state.[10] For a deformation $\delta = 0.26$ the $E1$ transition $9/2+ \rightarrow 7/2-$ is calculated to be slower than a spherical independent-particle model $i_{11/2} \rightarrow h_{9/2}$ transition by a factor probably no greater than 4×10^{-3} and perhaps much smaller, the exact value depending on the radial integrals for admixed states of different orbital angular momentum. Such an inhibition accords with the interpretation of this transition as an $E1+M2$ mixture.

[15] A similar case[10,14] may occur in Hf^{177}, for which the ratio of the $9/2+ \rightarrow 9/2-$ to the $9/2+ \rightarrow 7/2-$ $E1$ transition rate appears also to be much larger than that predicted from Eq. (2).[11]