Case (b): g factors have the Dirac-limit or "quenched" values:

$$g_p = 1.143, \quad g_n = 0;$$

Case (c): g factors are those empirically determined by the adjacent Co^{59} and Ni^{61} nuclear moment values:

$$g_p = 1.325, \quad g_n \cong -0.2/\tfrac{3}{2} \cong -0.13.$$

The results are tabulated in Table I.

There is excellent agreement in magnitude between the measured μ value and that calculated for the Schmidt-limit case. This may be taken as an indication that the magnetic moment is positive.

TABLE I. Comparison of $\mu(Co^{60})$ values.

	Calculated		
Measured	(a) Schmidt-limit	(b) Dirac-limit	(c) Empirical
± 3.800 nm	$+3.878$ nm	$+4.00$ nm	$\sim +4.4$ nm

ACKNOWLEDGMENTS

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Emission of Prompt Neutrons from Fission*

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An analysis of the total energies of the fragment pairs from fission is used with the mass equation of fission to estimate the distributions in the excitation energy of the fragments from spontaneous and neutron-induced fission of several nuclides. These excitations are used with simple neutron boil-off considerations to calculate the probabilities of emission of 0, 1, 2, 3··· prompt neutrons. The calculated results are in good agreement with recent measurements.

The same excitation energy distributions and neutron boil-off considerations are used with an assumption of an isotropic angular relation between the fragments and the emitted neutrons to calculate the energy spectrum of neutrons from thermal and 3-Mev neutron-induced fission of U^{235}. For thermal-neutron fission, the calculated spectrum is in fair agreement with recent measurements. The calculations indicate little change in the spectrum for 3-Mev fission. The average energy of the prompt gamma rays is 3.8 Mev from this analysis.

INTRODUCTION

THE fission of a heavy nucleus is a process which divides a very complex nucleus usually into two similarly complex nuclei. In this process, many modes of fission into different divisions of both neutron and proton numbers are observed. Further, different divisions of the available total energy of fission into the energy of repulsion of the fragments and the energy of excitation of the fragments are observed. As yet, no full explanation of these many complexities of fission has been published.

In recent years, great improvements in the empirical data related to fission have been published. These data make possible a better understanding of the division of the energies released in the fission process and of the subsequent emission of the excitation energy as the prompt neutrons and prompt gamma rays of fission. In the present analysis of these processes, data of atomic masses are first used to calculate the total energy of fission. Then, empirical data of the distribution of the fragment kinetic energy part of this total energy are used to calculate the distribution in the excitation energy of the fragments. Next, neutron boil-off considerations are applied to determine the probabilities of emission of 0, 1, 2, 3··· prompt fission neutrons and their energies. The energies of these neutrons are then transformed to the laboratory system by means of an assumed fragment-neutron angular relation. Finally, the residual excitation energy, which appears as the prompt gamma rays of fission, is determined.

For this analysis, the statistical properties of the nuclei of the unstable, neutron-rich products of fission are used. In the absence of complete information on these nuclear properties and for simplicity, assumptions of the atomic and nuclear systematics are made. In particular, the excited fission fragments are considered as having a continuum of energy levels, with expressions for the densities of these levels the same as those of stable nuclides. Also, the atomic masses of the fission fragments are determined from mass-spectroscopic measurements of their stable isobars by means of assumed extensions of the mass systematics of isobars. These assumed systematics of the atomic masses of the fission fragments are also used to calculate neutron

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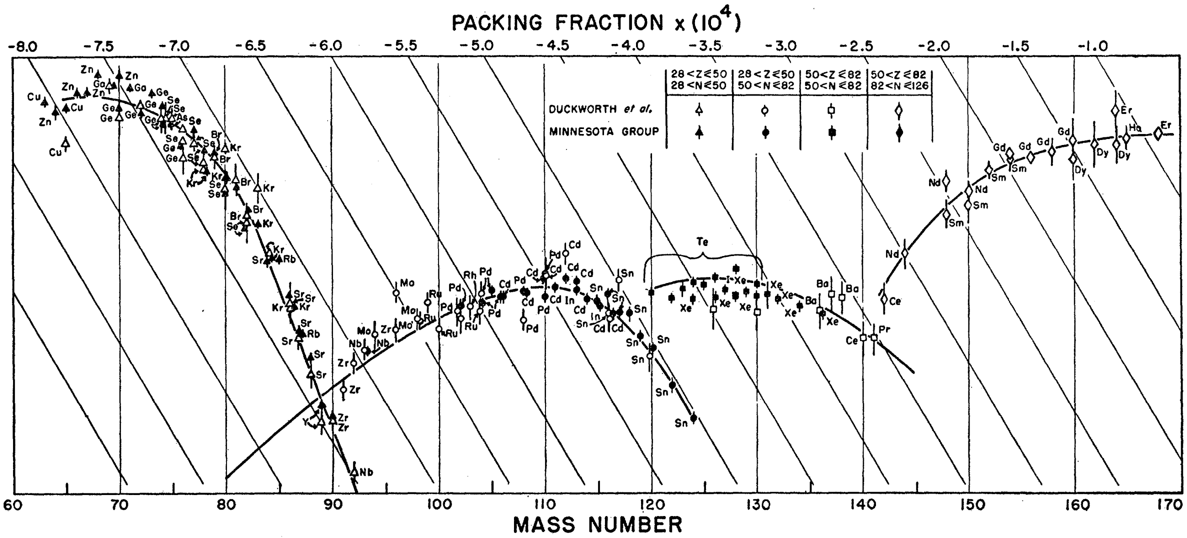

FIG. 1. The valley of the mass surface. The data shown are the results of conversions of the mass-spectrographic values to those of odd mass number and of the noninteger stable charge of the isobar.

binding energies of the fragments. As an additional assumption, the excitation energy distributions of the two fragments from binary fission are assumed to be the same.

ENERGY OF FISSION

The total excitation energy E_X of the fragment pairs from binary fission is determined from the total kinetic energy E_K of the fragment pairs by the simple expression of the mass balance of binary fission given in (1).

$$M(A,\delta,Z)+E_n+B=M(A^L,\delta^L,Z^L)+M(A^H,\delta^H,Z^H)+E_K+E_X. \quad (1)$$

Here, the atomic masses M are functions of the atomic number A, the even-odd parameter δ, and the nuclear charge Z. Here and throughout this paper, the superscripts L and H respectively refer to the light and heavy fragments. The relations $A=A^L+A^H$ and $Z=Z^L+Z^H$ for conservation of nucleons apply to (1). For neutron-induced fission the energy E_n of the incident neutron and its binding energy B to the target nucleus are included, but for spontaneous fission these terms are omitted. An atomic mass unit equal to 931.15 Mev is used for the mass-energy conversion in (1).

Of the atomic masses required in (1), only the masses $M(A,\delta,Z)$ of the atoms undergoing fission have had reasonably direct determinations. Compilations[1] have been made of these masses and the binding energies B derived from them. In contrast, the ground state masses $M(A^L,\delta^L,Z^L)$ and $M(A^H,\delta^H,Z^H)$ of the fragments are determined from extensions of the semiempirical relation of the mass surface to these nuclides, for which no measurements have been reported.

The valley of the mass surface used in these mass determinations is shown in Fig. 1, which is based on the mass-spectrographic measurements by Duckworth *et al.*[2] and by the Minnesota group.[3] The data shown in Fig. 1 are the results of conversions of the measured masses to masses of the nonintegral stable charge of each isobar by the constants of parabolic mass surfaces given by Coryell.[4] In addition, the data shown result from a conversion to odd mass numbers by the even-odd parameters of Fermi.[5] The extensions of the isobaric mass surfaces to the fragment nuclides are also considered as parabolic with the same constants given by Coryell. The positions A^L, Z^L, and A^H, Z^H of the fragments on these surfaces are given in A^L and A^H by the mass ratio $R_A=A^H/A^L$ being considered and in Z^L and Z^H by the observed charge displacements of fragments from the nonintegral stable charges. However, the nuclear charges observed by Glendenin *et al.*[6] and by

[1] J. R. Huizenga and L. B. Magnusson, Argonne National Laboratory Report ANL-5158, 1953 (unpublished) and R. A. Glass, University of California Radiation Laboratory Report UCRL-2560, 1954 (unpublished). In the present calculations, values from the latter were renormalized to the U^{235} value of the former for consistency.

[2] C. L. Kegley and H. E. Duckworth, Phys. Rev. **83**, 229 (1951); and E. M. Pennington and H. E. Duckworth, Can. J. Phys. **32**, 808 (1954); and others listed by Duckworth, Hogg, and Pennington, Revs. Modern Phys. **26**, 463 (1954).

[3] Collins, Nier, and Johnson, Phys. Rev. **86**, 408 (1952); Collins, Johnson, and Nier, Phys. Rev. **94**, 398 (1954); and R. E. Halsted, Phys. Rev. **88**, 666 (1952).

[4] C. D. Coryell, Ann. Rev. Nuclear Sci. **2**, 305 (1953). Recent adjustments of the semiempirical mass surface of C. F. von Weizsäcker, Z. Physik **96**, 431 (1935) to empirical data have been made by A. E. S. Green and N. A. Engler, Phys. Rev. **91**, 40 (1953) and A. E. S. Green and D. F. Edwards, Phys. Rev. **91**, 46 (1953).

[5] E. Fermi, *Nuclear Physics*, notes compiled by Orear, Rosenfeld, and Schluter (University of Chicago Press, Chicago, 1949).

[6] Glendenin, Coryell, and Edwards, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York, 1951), Paper No. 52, National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Div. IV.

Pappas[7] are of fragment isobars resulting from neutron emission. The small corrections used to obtain the original mass numbers corresponding to these observed nuclear charges were on the basis of the relative number of neutrons from the light and heavy fragments observed by Fraser.[8]

EXCITATION ENERGY DISTRIBUTION

When the identity A, Z of the nucleus undergoing fission and the energy E_n+B inducing fission are specified, the excitation energy E_X of the fission fragments is simply related to the kinetic energy E_K by (1). To simplify the analysis, only three mass ratios $R_A = A^H/A^L$ to specify regions of A^L and A^H of the fission products are considered for each nucleus undergoing fission. For fission of U^{235} by neutrons, these are 133/103, 141/95, and 149/87. Also, only the most probable, noninteger Z^L and Z^H values for each A^L and A^H, respectively, are used. Thus, the nuclear charges Z^L and Z^H are also specified by R_A. Although a distribution in the fragment nuclear charge is experimentally observed,[6] the variation in E_K+E_X resulting from this charge distribution can be shown to be small compared to other energy variations to be considered. As a final simplification, equal probabilities of odd and even numbers of both neutrons and protons, and thus of the δ^L and δ^H parameters, are assumed for the regions of fission modes represented by the A^L, Z^L and A^H, Z^H values of the analysis.

With these simplifications and (1), the distribution $X(\delta^H,\delta^L,E_X,R_A)$ in the total excitation E_X of the two fragments is readily found from the distribution $K(E_K,R_A)$ in the total kinetic energy E_K of the fragments. This kinetic energy distribution is determined from measurements with double, "back-to-back" ionization chambers by the convolution

$$I(E_I+\Delta, R_A)=\int_{-\infty}^{\infty} dE_K D(E_K, E_I+\Delta) K(E_K,R_A), \quad (2)$$

where the distribution $I(E_I+\Delta, R_A)$ is of the energy E_I reported from these measurements. The purpose of the convolution (2) is to remove the dispersion $D(E_K, E_I+\Delta)$ that has been found[9,10] in the ionization chamber data. The energy displacement Δ in (2) is principally to correct for the ionization defect[11] found[10,12] in these data and, to lesser extent, to correct for errors in the masses M and for other small errors in the analysis. The energy Δ is adjusted to make the calculated average numbers of fission neutrons $\bar{\nu}$ agree with measurements.

The ionization chamber data of E_I used in the analysis were those of Brunton and Hanna[13] for both U^{233} and U^{235} and of Brunton and Thompson[14] for Pu^{239}. On the basis of similar measurements[15,16] which show that E_I does not change appreciably with the neutron energy E_n, the thermal-neutron data of E_I of Brunton *et al.* are used in these calculations for all values of E_X+B inducing fission of these compound nuclei.[17] Similar data have been obtained for Cm^{242} spontaneous fission[18] and for U^{238} neutron-induced fission,[19] but with considerably poorer statistics for both cases. No double chamber measurements of fission of other nuclides have been reported. Consequently, the above data of U^{235} have been applied to the neutron-induced fission of U^{238} and, similarly, the above data of Pu^{239} have been applied to the spontaneous fission of Cm^{242}, Cm^{244}, and Cf^{252}. In these applications of the data, the mass distribution of the heavy fragment is considered to be fixed.[20] The similarity of these U^{233}, U^{235}, U^{238}, Pu^{239}, and Cm^{242} data from double chambers indicates that little error is introduced by this application of the data to fission of other nuclides.

Since the methods by which the dispersion in these ionization data was determined were not sufficiently sensitive to determine both the shape and width, a Gaussian dispersion

$$D(E_K, E_I+\Delta) \propto \exp\left[-\left(\frac{E_I+\Delta-E_K}{u}\right)^2\right] \quad (3)$$

is used in the convolution (2) to simplify calculations. A width $u=7.2$ Mev, which is based on other determinations,[9,10] is used.

To compute the neutron emission probabilities, it is necessary to separate the distribution X in the total excitation energy E_X into the distributions X^L and X^H of the respective fragment excitations $E_X{}^L$ and $E_X{}^H$. Since little data are available on which to decide the method of separation, the simple separation is chosen whereby the excitation functions X^L and X^H are equal and the excitations $E_X{}^L$ and $E_X{}^H$ of each fragment are independent with only the constraint $E_X{}^L+E_X{}^H=E_X$ applying. Under these conditions, the convolution

$$X(\delta^H,\delta^L,E_X,R_A)=\int_{-\infty}^{\infty} dE_X{}^L X^L(\delta^H,\delta^L,E_X{}^L,R_A) \times X^H(\delta^H,\delta^L,E_X-E_X{}^L,R_A) \quad (4)$$

[7] A. C. Pappas, Massachusetts Institute of Technology, Laboratory for Nuclear Science Technical Report No. 63, 1953 (unpublished).
[8] J. S. Fraser, Phys. Rev. **88**, 536 (1952).
[9] R. B. Leachman, Phys. Rev. **83**, 17 (1951).
[10] R. B. Leachman, Phys. Rev. **87**, 444 (1952).
[11] J. K. Knipp and R. C. Ling, Phys. Rev. **82**, 30 (1951).
[12] R. B. Leachman and W. D. Schafer, Can. J. Phys. **33**, 357 (1955).
[13] D. C. Brunton and G. C. Hanna, Can. J. Research **A28**, 190 (1950).
[14] D. C. Brunton and W. B. Thompson, Can. J. Research **A28**, 498 (1950).
[15] J. S. Wahl, Phys. Rev. **95**, 126 (1954).
[16] E. Segrè and C. Weigand, Phys. Rev. **94**, 157 (1954).
[17] The concept of the fragment excitation energy increasing with the energy of the neutrons inducing fission was due to J. L. Fowler (private communication, 1952).
[18] R. L. Shuey, University of California Radiation Laboratory Report UCRL-959, 1950 (unpublished).
[19] W. Jentschke, Z. Physik **120**, 165 (1943).
[20] E. P. Steinberg and L. E. Glendenin, Phys. Rev. **95**, 431 (1954), and C. D. Coryell and N. Sugarman, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York, 1951), National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Div. IV, Appendix B.

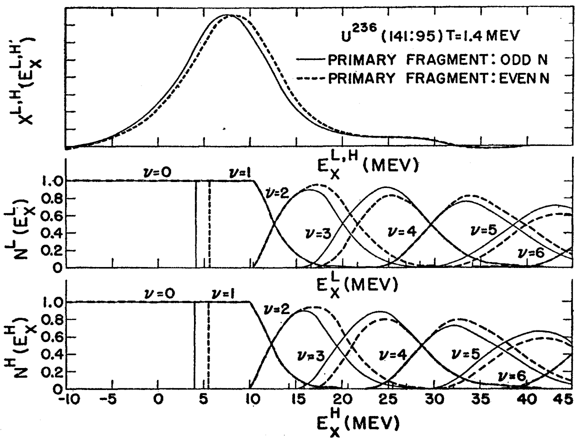

FIG. 2. Calculated excitation energy distributions, X^L and X^H, and neutron emission probabilities, N^L and N^H, for the most probable fission mode of thermal-neutron fission of U^{235} (compound nucleus U^{236}). The abscissa scales for the three sets of curves are the same.

is used to determine the functions X^L and X^H from the function X.

In practice, the empirical E_I data are difficult to carry through the convolutions (2) and (4) to obtain the excitation distributions X^L and X^H. Instead, in the present analyses the energy distributions I were fitted by a sum of 13 Gaussian expressions of equal width and various amplitudes regularly spaced along the E_I axis. It can be shown that such a function in the convolutions (2) and (4) results in a similar function of seven Gaussian terms for the X^L and X^H distribution. These fits and the convolutions were made on IBM digital computers at Los Alamos. An example of the resulting X^L and X^H distributions for thermal-neutron fission of U^{235} into $R_A=141/95$ is shown in Fig. 2. Although the negative excitation energies and negative probabilities in Fig. 2 resulting from this approximate method of analysis have no physical meaning, both have mathematical meaning and are carried in the analysis.

NEUTRON EMISSION PROBABILITIES

To convert the excitation energy probabilities into the probabilities of neutron emission, the relation[21]

$$n(\epsilon) \propto \epsilon \exp(-\epsilon/T) \tag{5}$$

for neutron boiloff from an excited nucleus is used. In (5), the emission probability $n(\epsilon)$ of neutrons with energy ϵ is characterized by a nuclear "temperature" T. The validity of expressions for neutron emission in the analysis of various experimental data has been investigated by Cohen.[22] Of interest in the present analysis is only the value of T that best describes neutron emission from nuclei that are in the mass region of fission fragments and that are excited to roughly 5 to 15 Mev. For this, a value $T=1.4$ Mev is found by fitting measured $(n,2n)$ excitation functions[23] with (5).

The probabilities $N^L(\delta^L,E_X{}^L,\nu^L,R_A)$ and $N^H(\delta^H,E_X{}^H,\nu^H,R_A)$ of emitting ν^L and ν^H neutrons are derived from (5) with the condition that neutrons are always emitted from the excited nucleus when emission is energetically possible. For simplicity, T is considered to be constant. Examples of these calculated emission probabilities[24] are shown in Fig. 2. The binding energies of fission neutrons involved in these calculations were computed from the extensions of the mass surface to the region of fission fragments as discussed above. The calculated binding energies used for Fig. 2 are in Table I. Similar calculations of binding energies, but of stable nuclides, have been compared[25] with measured binding energies. These comparisons indicate that binding energy calculations that take into account shell effects agree with measurements.

The probabilities $P_\nu{}^L(\delta^H,\delta^L,\nu^L,R_A)$ and $P_\nu{}^H(\delta^H,\delta^L,\nu^H,R_A)$ of, respectively, ν^L and ν^H neutrons emitted from the fragments are obtained by combining the respective excitation and emission probabilities in

TABLE I. Neutron binding energies calculated from the semi-empirical mass surface. Values are for the fission products of U^{235} when $R_A=141/95$. The even-odd term δ_1 represents primary fission products with an odd number of neutrons; δ_2 represents an even number of neutrons.

ν	$B^L(\delta_1)$	$B^L(\delta_2)$	$B^H(\delta_1)$	$B^H(\delta_2)$
1	4.17	5.71	4.05	5.63
2	10.26	10.26	9.93	9.93
3	15.02	16.56	14.30	15.88
4	21.60	21.60	20.63	20.63
5	26.91	28.45	25.46	27.04

$$P_\nu{}^L(\delta^H,\delta^L,\nu^L,R_A)=\int_{-\infty}^{\infty} dE_X{}^L X^L(\delta^H,\delta^L,E_X{}^L,R_A) \times N^L(\delta^L,E_X{}^L,\nu^L,R_A), \tag{6}$$

and the corresponding equation for the heavy fragments. These integrations were made on IBM 701 digital computers to the cases of emission of nine neutrons. After the results for the various δ^L and δ^H conditions are combined, the probability $P_\nu(\nu,R_A)$ of emitting a total number ν of neutrons from both fragments is obtained from

$$P_\nu(\nu,R_A)=\sum_{\eta=0}^{\nu} P_\eta{}^L(\nu^L,R_A)P_{\nu-\eta}{}^H(\nu^H,R_A), \tag{7}$$

[21] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

[22] B. L. Cohen, Phys. Rev. **92**, 1245 (1953). *Note added in proof.*—See also D. L. Livesey, Can. J. Phys. **33**, 391 (1955).

[23] H. C. Martin and R. F. Taschek, Phys. Rev. **89**, 1302 (1953); H. C. Martin and B. C. Diven, Phys. Rev. **86**, 565 (1952); and Brolley, Fowler, and Schlacks, Phys. Rev. **88**, 618 (1952).

[24] Similar calculations of emission probabilities have been made for fragments by D. L. Hill, dissertation, Princeton University, 1951 (unpublished), and for (p,xn) reactions by J. D. Jackson, Phys. Rev. **95**, 651 (1954).

[25] J. A. Harvey, Phys. Rev. **81**, 353 (1951), and Sher, Halpern, and Mann, Phys. Rev. **84**, 387 (1951).

where η is a summation parameter analogous to ν. The probabilities from the different mass ratios R_A are then combined with their respective weightings. In the present calculations, not all the distributions were normalized and so the emission probabilities were finally normalized by

$$\sum_{\nu=0}^{9} P_\nu(\nu)=1.$$

The results of these calculations of P_ν for neutron-induced fission are compared in Fig. 3 with the experimental measurements by Diven *et al.*[26] with a large tank of liquid scintillator. The energy displacements Δ appearing in (2) and (3) of these calculations for U^{233}, U^{235}, and Pu^{239} were determined by normalizations of the calculations to the 2.54 ± 0.04, 2.46 ± 0.03 and 2.88 ± 0.04 values, respectively, of the average number

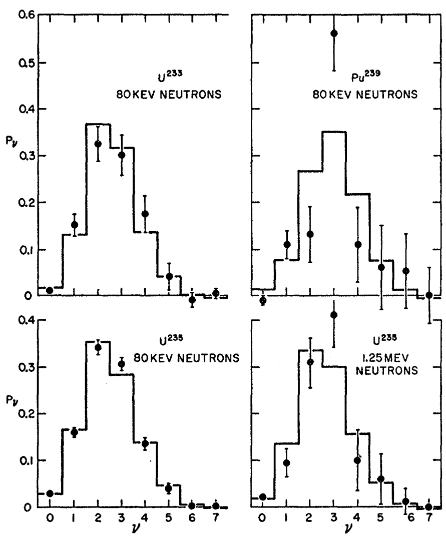

FIG. 3. Neutron emission probabilities for neutron-induced fission. Calculated results are given by lines. Circles are the data of Diven *et al.*

$\bar{\nu}$ of neutrons from thermal-neutron fission.[27] In Fig. 4 are similar comparisons with the same type of experimental measurements for spontaneous fission. For Cm^{242}, normalization of the calculations was to the $\bar{\nu}=2.57\pm0.07$ value of Hicks *et al.*[26] The values of Diven *et al.*[26] of $\bar{\nu}=2.81\pm0.06$ for Cm^{244} and $\bar{\nu}=3.87\pm0.08$ for Cf^{252}, both of which are in good agreement with the results of Hicks *et al.*,[26] similarly were used for normalization. The normalization required for

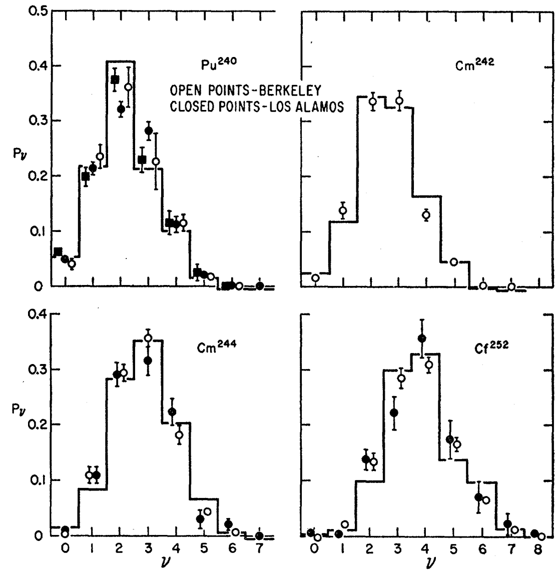

FIG. 4. Neutron emission probabilities for spontaneous fission. Calculated results are given by lines. Data are: open circles, Hicks *et al.*; closed circles, Diven *et al.*; and squares, Hammel and Kephart.

thermal-neutron fission of Pu^{239} was used for the spontaneous Pu^{240} calculations. In both Figs. 3 and 4, the agreement between all data is seen to be good.

In Table II, the average values $\bar{E}_K$ of the kinetic energies resulting from the above calculations are compared with average energies $\bar{E}_I$ from recent measurements with ionization chambers.[13–15,18,28,29] The $\bar{\nu}=2.58\pm0.09$ value for 1.5-Mev neutron fission of U^{238} used to derive this $\bar{E}_K$ value in Table II is from Diven *et al.*[26] The computed $\bar{E}_K$ values of this table contain uncertainties in M and T which are estimated to result in $\bar{E}_K$ uncertainties of roughly 5 Mev. Therefore, the $\Delta=\bar{E}_K-\bar{E}_I$ values in the table are in reasonable agreement with the $\Delta=12.4$ Mev value from other determinations.[10,12]

TABLE II. Average total energies of fission fragments. The energies $\bar{E}_I$ from ionization chamber measurements include source and collimator corrections when reported. The indicated uncertainties in the energies $\bar{E}_K$ from $\bar{\nu}$ calculations are from the uncertainties in $\bar{\nu}$ only. The letter S signifies spontaneous fission.

Fission case	$\bar{E}_K$ (Mev)	$\bar{E}_I$ (Mev)	Δ (Mev)	E_I Reference
U^{233} ($E_n=0$)	167.4±0.3	149.6	17.8	13
U^{235} ($E_n=0$)	169.4±0.2	154.7	14.7	13
U^{238}	168.7±0.7	149.0	19.7[a]	15
Pu^{239} ($E_n=0$)	177.0±0.3	159.8	17.2	14
Cm^{242} (S)	184.9±0.6	173	11.9	18
		160	24.9	28
Cm^{244} (S)	182.4±0.3			
Cf^{252} (S)	187.2±0.5	170	17.2	29

[a] $\bar{E}_I$ measurement for $E_n=2.5$ Mev; $\bar{\nu}$ measurement for $E_n=1.5$ Mev. However, $\bar{E}_I$ is assumed to be independent of E_n.

[26] R. B. Leachman, Proceedings of the International Conference on Peaceful Uses of Atomic Energy, Geneva (1955). Included in this review are the scintillator tank measurements of the following groups: Diven, Martin, Taschek, and Terrell, following paper [Phys. Rev. **101**, 1012 (1956)]; Hicks, Ise, and Pyle, accompanying paper [Phys. Rev. **101**, 1016 (1956)]; and J. E. Hammel and J. F. Kephart, Phys. Rev. (to be published).

[27] J. A. Harvey and D. J. Hughes, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

[28] Hanna, Harvey, Moss, and Tunnicliffe, Phys. Rev. **81**, 466 (1951).

[29] H. R. Bowman (private communication, 1955).

TABLE III. Multiplicities of neutrons from the spontaneous fission of U^{238}.

Multiplicity	Calculations Normalized to $\bar{\nu}=2.2$	Calculations Normalized to $\bar{\nu}=2.5$	Geiger and Rose measurements
$\langle\nu^2\rangle_{Av}/\bar{\nu}$	2.70	2.96	3.26
$\langle\nu^3\rangle_{Av}/\bar{\nu}$	8.02	9.58	12.73

The calculated binding energies of neutrons for the different modes of fission, together with the assumption of the relative excitation energies of the light and heavy fragment, determine $\bar{\nu}^L/\bar{\nu}^H$ in the calculations. The assumption of the same distributions of excitation energies used in the calculations results in $\bar{\nu}^L/\bar{\nu}^H=1.05$ for U^{235} fission. The $\bar{\nu}^L/\bar{\nu}^H=1.30$ measurement by Fraser[8] indicates the excitation energies of the light fragment should be, instead, 2 Mev greater than those of the heavy fragment.

A measurement of the dispersion in ν for U^{235} fission was made by Feynman *et al.*[30] from the statistical fluctuations in the Los Alamos Homogeneous Reactor. This measurement of $\langle\nu^2\rangle_{Av}-\bar{\nu}=5.2$ is to be compared with a calculated value of 4.76 for $E_n=0$. A measurement of the same quantity was made by Bonner *et al.*[31] with a two-counter technique[32] of neutron detection. They found $\langle\nu^2\rangle_{Av}-\bar{\nu}=4.1$.

Calculations for the spontaneous fission of U^{238} were made with normalizations to both the measurement[33] of 2.2 and the measurement[34] of 2.5 reported for this nuclide. For both normalizations, the calculated multiplicities shown in Table III are less than the measurements of Geiger and Rose.[35]

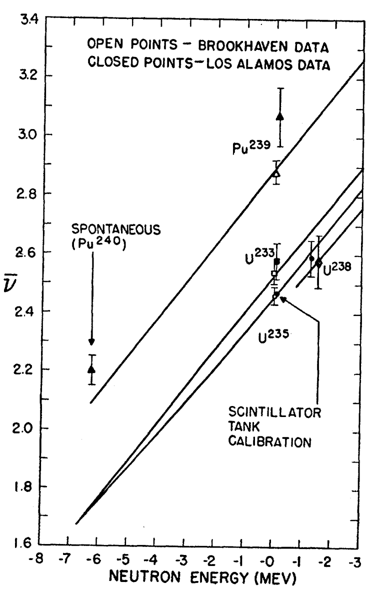

FIG. 5. Variation of $\bar{\nu}$ with the energy of neutron inducing fission. Lines are calculations based on the method of Fowler and normalized to $E_n=0$ measured values. Los Alamos data are of Diven *et al.*; Brookhaven data are from BNL-325.

With the method of Fowler used to increase E_X with E_n, the variations of $\bar{\nu}$ with E_n as shown in Fig. 5 are calculated. Previous to the scintillator tank measurements, $\bar{\nu}$ had been measured for $E_n=0.7$ Mev by Terrell[26] and for $E_n=1.0$ Mev by Fowler[26] by different methods, but with results in agreement with Fig. 5.

The sensitivity of the calculated results to extremes of T and u is seen by Table IV. These results show the slope $d\bar{\nu}/dE_n$ is principally dependent on T, while the multiplicity $\langle\nu^2\rangle_{Av}$ is dependent on both T and u. All the calculated slopes $d\bar{\nu}/dE_n$ are in reasonable agreement with the results in Fig. 5. Diven *et al.*[26] measured $\langle\nu^2\rangle_{Av}=7.32\pm0.19$ for 80-kev fission of U^{235}. On the basis of the calculated $d\langle\nu^2\rangle_{Av}/dE_n=0.63$ Mev^{-1} for this region of neutron energy inducing fission, $\langle\nu^2\rangle_{Av}$ for thermal-neutron fission is 7.27 ± 0.19. All the calculated $\langle\nu^2\rangle_{Av}$ values are in reasonable agreement with this value, with the best agreement obtained by the use of the previously selected values $T=1.4$ Mev and $u=7.2$ Mev.

TABLE IV. Results of calculations of neutron emission from thermal-neutron fission of U^{235}. Calculations have been normalized to $\bar{\nu}=2.46$ by adjusting Δ for each "temperature" T and dispersion u used. The slope $d\bar{\nu}/dE_n$ is from additional calculations for E_n between 0 and 3 Mev.

T (Mev)	u (Mev)	Δ (Mev)	$d\bar{\nu}/dE_n$ (Mev^{-1})	$\langle\bar{\nu}^2\rangle_{Av}$
1.0	7.2	15.7	0.137	7.36
1.0	8.5	15.7	0.137	7.18
1.4	5.9	14.7	0.124	7.36
1.4[a]	7.2[a]	14.7	0.124	7.21
1.4	8.5	14.7	0.125	7.07
1.8	7.2	13.7	0.115	7.14

[a] Values used for other calculations in this paper.

ENERGIES OF FISSION NEUTRONS AND GAMMA RAYS

In the analysis above, only the emission energy ϵ of a fission neutron in the frame of reference of the fragment is considered. A transformation of this neutron energy into the laboratory energy E involves the angular distribution between the neutrons and fragments. As part of the present investigation, Monte Carlo calculations of the energy spectrum of the fission neutrons have been made with the usual assumption[36] of an isotropic emission of neutrons from the moving fragments.

These Monte Carlo calculations followed the same procedures as the above integral calculations, but, in addition, included the energy transformations to the laboratory system and an averaging of the residual energy. This residual energy, which appears as prompt gamma rays, is the excitation energy remaining after all the neutrons that are energetically possible are emitted. For the energy transformation, the velocity of

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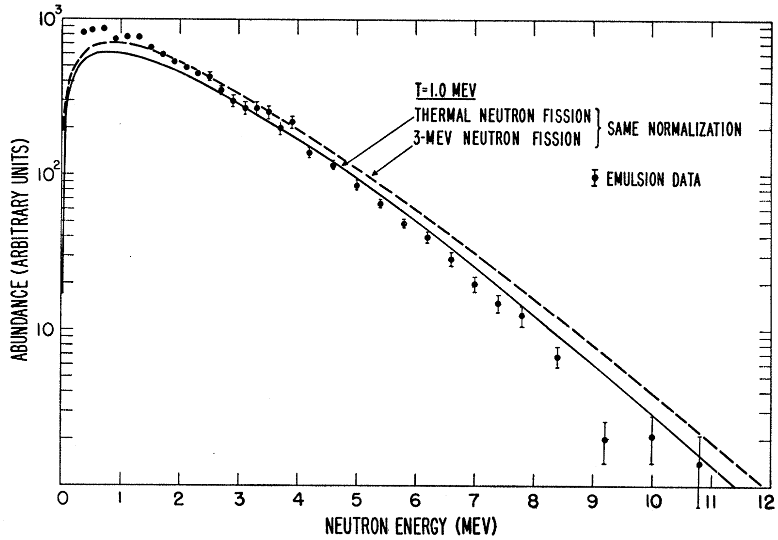

FIG. 6. Calculations of the energy spectra of fission neutrons for $T=1.0$ Mev. Data are of Frye and Rosen.

each fragment is determined from (1) by the δ^H, δ^L, E_X, and R_A condition of each fission.

In Fig. 6 are shown the resulting spectra for fission of U^{235} by thermal and 3-Mev neutrons. Each curve is the result of $1.3(10^5)$ fissions in Monte Carlo calculations. Just as in the case of neutron emission calculations, the excitation energies for 3-Mev fission are increased according to the method of Fowler. In these calculations of Fig. 6, $T=1.0$ Mev and $u=7.2$ Mev were used. Use of $T=1.4$ Mev in the calculations gives a poorer fit to the measurements of Frye and Rosen,[26] but $T=1.0$ and $u=8.5$ Mev results in a considerably better fit than the results of Fig. 5.

These calculations confirm the expectation that the high-energy end of the neutron spectrum, which is largely determined by the high-energy tail of the distributions in the excitation energies, is strongly influenced by the dispersion u used in (3). On the other hand, the low-energy end of the spectrum is strongly influenced by the neutron-fragment angular distribution. On the basis of the collective model of fission,[37] the neutrons are expected to be emitted preferentially along the line of the fragment directions, instead of isotropically as assumed in the calculations. For these reasons, little importance is attached to the relatively poor fit of the calculated and experimental energy spectra of fission neutrons. However, the similarity of the calculated spectra in Fig. 6 for 0-Mev and 3-Mev fission is considered significant.

The calculated residual excitation energy appearing as prompt gamma rays is 3.8 Mev for thermal-neutron fission and 4.1 Mev for 3-Mev fission. The calculations indicate this energy is shared approximately equally by the heavy and light fragments. The calculated gamma-ray energy is found to be relatively insensitive to the T and u values used, with variations of 0.4 Mev in T resulting in only 0.1-Mev variations in the calculated gamma-ray energy. These calculated gamma-ray energies are in reasonable agreement with the earlier 5.1 ± 1-Mev measurement[38] and 4.6 ± 1-Mev measurement,[39] but are in disagreement with a recent 7.5-Mev measurement.[40]

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