

Paramagnetic Resonance Hyperfine Structure of Co⁶⁰

W. DOBROWOLSKI, R. V. JONES, AND C. D. JEFFRIES

Department of Physics, University of California, Berkeley, California

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The microwave paramagnetic resonance hyperfine structure of Co⁶⁰ has been observed in a magnetically dilute single crystal of cobalt ammonium sulfate at 20°K. From the number of hfs components and from their spacing relative to those of Co⁵⁹, the spin $I(\text{Co}^{60})=5$ and the magnetic moment $|\mu(\text{Co}^{60})|=3.800 \pm 0.007$ nm are directly determined. These results are discussed in relation to those of β , γ spectroscopy, nuclear alignment experiments, and the nuclear shell model.

I. INTRODUCTION

THIS paper describes the first of a series of experiments under way here to determine the nuclear moments of radioactive nuclei by the observation of the hyperfine structure in the microwave paramagnetic resonance spectra¹ of ions of the transition group elements. Since the first observation of the hfs by Penrose,² this method has been used by Bleaney and Griffiths and others to investigate a number of nuclei.¹ The sensitivity of the method is sufficient to allow its use with radioactive nuclei available in millicurie quantities with half-lives of a month or longer.

In the experiment reported here, we have observed simultaneously the hfs of the paramagnetic resonance of 5.3-year Co⁶⁰ and stable Co⁵⁹, both isotopes being contained in the same single crystal of cobalt ammonium sulfate. Comparison of the spectra yield for the first time exact values of the nuclear spin I and the magnetic moment μ of Co⁶⁰. These are of particular interest in connection with the previous investigations of Co⁶⁰ by β , γ spectroscopy and nuclear alignment experiments.

II. THEORY

The paramagnetism of the Co⁺⁺ ion arises from the unpaired d electrons. The ground-state configuration of the free ion is $3d^7, ^4F$. A crystalline electric field of predominantly cubic symmetry, such as is provided by the octahedron of water molecules in the crystal used, partially removes the sevenfold orbital degeneracy, leaving an orbital triplet lowest in energy. This triplet with its fourfold spin degeneracy is split into six Kramer's doublets by the combined action of spin-orbit coupling and a small axial component of the crystalline field due to a distortion of the water octahedron.³ The states of the lowest doublet, corresponding to $m_s = \pm \frac{1}{2}$, are the only states appreciably populated at low temperatures and the application of an external magnetic field of a thousand gauss splits these states by a microwave quanta. Under these conditions, the microwave para-

magnetic resonance spectrum may be observed and would consist in this case of a single line, corresponding to an effective spin $S = \frac{1}{2}$. In addition to the external magnetic field there is an internal field at each ion due to the nuclear magnetic moment which splits the paramagnetic resonance line into $2I+1$ hfs components.

Bleaney and Ingram⁴ have investigated in detail the paramagnetic resonance hfs of Co⁵⁹ in magnetically dilute Tutton salts of the type we have used: $\text{Co}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{D}_2\text{O}$. For this case, a Hamiltonian given by Abragam and Pryce^{3,5} is suitable:

$$\mathcal{H} = \beta [g_{11} H_z S_z + g_{\perp} (H_x S_x + H_y S_y)] + A' I_z S_z + B' [I_x S_x + I_y S_y], \quad (1)$$

where x, y, z are the principle axes of the crystalline electric field, z being the axis of axial symmetry; H is the applied magnetic field; β is the Bohr magneton; $g_{11} = 6.45$ and $g_{\perp} = 3.06$ are the spectroscopic splitting factors with $z \parallel H$ and $z \perp H$, respectively; A' and B' are the magnetic hfs interaction constants; S and I are the electronic and nuclear spin quantum number, respectively. Both the g factor and the hfs show a considerable anisotropy, i.e., a dependence on the orientation of the crystal in the H field. The energy levels may be obtained by a second-order perturbation calculation. From this result may be obtained the resonance value of the field H for magnetic dipole transitions $\Delta m_s = \pm 1$, $\Delta m_I = 0$ which occur for a constant applied microwave frequency $\nu = g\beta H_0/h$. For the particular case of $H \parallel z$, $g_{11} = g$, the result is⁶

$$H = H_0 - A m_I - (B^2/2H_0)[I(I+1) - m_I^2]. \quad (2)$$

For Co⁵⁹, $A = A'/g_{11}\beta = 81$ gauss, $B = B'/g_{11}\beta = 6.6$ gauss. The last term in Eq. (2) is, therefore, negligible and the spectrum consists of $2I+1$ lines uniformly spaced about H_0 as a center:

$$H = H_0 - AI, H_0 - A(I-1), \dots, H_0 + AI. \quad (3)$$

Apart from constant factors, the hfs interaction constant A is determined by the nuclear g factor and the

¹ See, for example, B. Bleaney and K. W. H. Stevens, Repts. Progr. Phys. **16**, 108 (1953); K. D. Bowers and J. Owen, Repts. Progr. Phys. **18**, 305 (1955).

² R. P. Penrose, Nature **163**, 992 (1949).

³ M. H. L. Pryce, Nature **164**, 116 (1949); A. Abragam and M. H. L. Pryce, Proc. Roy. Soc. (London) **A206**, 173 (1952).

⁴ B. Bleaney and D. J. E. Ingram, Nature **164**, 116 (1949); B. Bleaney and D. J. E. Ingram, Proc. Roy. Soc. (London) **A208**, 143 (1951).

⁵ A. Abragam and M. H. L. Pryce, Proc. Roy. Soc. (London) **A205**, 135 (1951).

⁶ See, for example, B. Bleaney, Phil. Mag. **42**, 441 (1951).

electronic wave function. The contribution of the latter may be assumed to be very nearly the same for Co^{59} and Co^{60} , so we may take $A \propto \mu_I/I$. From Eq. (2), the total field splitting between the two extreme hfs lines is $\Delta H = 2AI \propto \mu_I$. Thus, the ratio of the nuclear magnetic moments of Co^{59} and Co^{60} are given by the ratio of the total splitting ΔH for the two isotopes, independently of the spins. The spin of Co^{60} is simply determined by the number of hfs lines.

In order for the lines to be well resolved, it is necessary that (a) the thermal relaxation time τ of the Co^{++} ions be made sufficiently long by working at low temperatures (20°K), and (b) the magnetic dipole-dipole interaction between adjacent Co^{++} ions be greatly reduced by replacing most of the Co^{++} ions by diamagnetic Zn^{++} ions ($\text{Co}:\text{Zn} \cong 1:1000$). In a sufficiently dilute crystal at low temperatures, the residual line width is due to the proton nuclear magnetic moments in the octahedron of water molecules about the Co^{++} ion; further reduction in line width can be obtained by replacing the protons with deuterons.⁴

III. EXPERIMENTAL PROCEDURES

A sample of 5.3-year Co^{60} produced by the $\text{Co}^{59}(n,\gamma)\text{Co}^{60}$ reaction with pile neutrons was obtained from Oak Ridge with an initial specific activity of 20.61 curies/gram Co, corresponding to the isotopic ratio $\text{Co}^{59}:\text{Co}^{60} = 55:1$. The sample was converted to CoSO_4 and added to a heavy water solution of $\text{Zn}(\text{NH}_4)_2(\text{SO}_4)_2$ from which a single 80-mg crystal of the Tutton salt $(\frac{\text{Co}}{\text{Zn}})(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{D}_2\text{O}$ was grown, containing 1.2×10^{20} Zn atoms, 1.5×10^{17} Co^{59} atoms and 3×10^{15} Co^{60} atoms, with an activity of approximately 300 microcuries. The crystallographic axes were clearly recognizable.

The crystal is monoclinic ($\beta = 107^\circ$) and contains two nonequivalent Co^{++} ions per unit cell. The relationships between the crystallographic axes (a, b, c), the principal paramagnetic susceptibility axes (K_1, K_2, K_3) and the principal axes (x, y, z) of the crystalline field have been investigated by Bleaney and Ingram.⁴ The K_1K_2 plane is parallel to the ac plane, with $\Psi(c, K_1) = 130^\circ$; $b \parallel K_3$. There is a set of crystalline field axes ($x_1y_1z_1$) and ($x_2y_2z_2$) for each of the two nonequivalent magnetic ions. Each of the ions gives a separate paramagnetic resonance spectrum, but, because of the rather high anisotropy of the spectroscopic splitting factor, it is possible to separate the two spectra completely by orienting the crystal so that the applied field $H \parallel z_1$ for the first set of ions; z_1 lies in the K_1K_3 plane with $\alpha(K_1, z_1) = 33^\circ$. We have mounted the crystal in the center of a rectangular 3-cm microwave cavity (TE_{102} mode) with K_2 along the long dimension of the cavity and z_1 along the short dimension, which is oriented parallel to the field H . This corresponds to the orientation assumed in Eqs. (2) and (3). For this orientation, the hfs of the second

set of ions is not clearly resolved and we have not investigated in detail this spectra.

The microwave apparatus used to observe the paramagnetic resonance absorption is similar to that of other workers in this field; in particular, it is patterned after a design of A. F. Kip. The 3-cm cavity containing the sample is connected via a thin-walled wave guide (thermal isolation section) to one arm of a 4-arm magic T . A variable match terminates the other side arm. A klystron oscillator at 9200 Mc/sec feeds power through a gyrator and variable attenuator into the E arm of the magic T . A silicon crystal detector terminates the H arm and is used to observe the resonance absorption of power by the sample. The frequency of the klystron is automatically controlled to the cavity resonant frequency. The sample cavity is inserted into a glass Dewar containing either liquid helium or liquid hydrogen jacketed by another Dewar of liquid nitrogen. These Dewars are installed in the gap of an electromagnet whose field is highly stabilized and finely controllable. Superimposed on the dc field is an 80-cps modulating field, so that the absorption signal may be observed directly on an oscilloscope connected to the crystal detector. Alternately, the derivative of the signals may be recorded on a paper tape by means of a phase-sensitive amplifier with a narrow band width ($\sim \frac{1}{10}$ cps) to discriminate against noise.

IV. RESULTS

Preliminary observation of rather weak Co^{59} signals at 4.3°K indicated that the thermal relaxation time τ was sufficiently long that the resonance lines could be easily saturated at power levels required for optimum detector performance, thus reducing the signal intensity. Further, it can be shown⁷ that the optimum signal amplitude is proportional to $(T_2/\tau)^{\frac{1}{2}}$ where $T_2 = 1/\Delta\omega$ is the inverse line width. This is another factor which reduces the signal amplitude if $\tau \gg T_2$. Operation at 20°K shortens τ sufficiently so that the observed signal intensity is considerably enhanced. Figure 1 is a typical spectrum observed for the first group of ions for which $H \parallel z_1$. The hfs for the highly abundant Co^{59} and the less abundant Co^{60} are both clearly discernable. The full line width at half-maximum is about 6 gauss. The spectrum for the second group of ions occurring at approximately twice the field value is similar but less clearly resolved.

The eight lines $A, B, \dots H$ in Fig. 1 are the hfs of Co^{59} , are equally spaced to within experimental error, and fit exactly Eq. (3) with $g = 6.41$, $A = 84.2$ gauss and $I = 7/2$. The lines 1, 2, $\dots 11$ are due to Co^{60} , are also equally spaced and fit Eq. (3) with $g = 6.41$, $A = 48.2$ gauss and $I = 5$. Since $A \propto g_I$ the ratio of the nuclear g factors is thus determined and is $g_I(\text{Co}^{60})/g_I(\text{Co}^{59}) = 0.573$. The spacing A is given unambiguously

⁷ See for example, F. Bloch, Phys. Rev. **70**, 460 (1946).

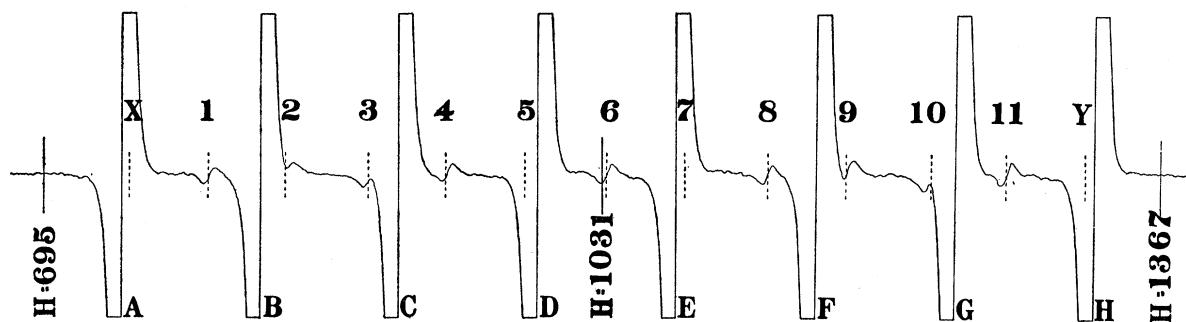


FIG. 1. Paramagnetic resonance hfs spectrum of Co⁵⁹ and Co⁶⁰ in magnetically dilute Co(NH₄)₂(SO₄)₂·6D₂O with the magnetic field H parallel to the axis of the axial crystalline electric field. The derivative of the absorption lines is displayed as a function of the field H (in gauss) at a fixed frequency of 9267 Mc/sec. The eight (off-scale) lines A, B, \dots, H are the hfs of Co⁵⁹ ($I=7/2$). The lines 1, 2, \dots , 11 are the hfs of Co⁶⁰, showing that $I=5$. Relative intensity measurements made by adjusting the amplifier gain prove that there are no Co⁶⁰ hfs lines at X, Y .

by the two successive lines 2 and 3, for example. This defines the positions of lines 5 and 7, which are evidently obscured by the superposition of the intense lines D and E of Co⁵⁹. This question naturally arises: can there also be Co⁶⁰ hfs lines at the positions X and Y which are likewise obscured by the intense lines A and H ? This would correspond to a spin $I(\text{Co}^{60})=6$. Since the relative abundance of the two isotopes is accurately known, the question can be resolved by a comparison of the line intensity of the two sets of hfs lines.

If the microwave power is small compared to the saturation value, which is the case here, the intensity of a single hfs line is given by⁶

$$P_{m_I} = k \left(\frac{N}{2I+1} \right) |M|^2, \quad (4)$$

where k is a parameter dependent only on such non-nuclear factors as the absolute temperature, the line width and the fixed microwave frequency; N is the number of nuclei, $|M|^2$ is the transition probability for $\Delta m_s = \pm 1$, $\Delta m_I = 0$ magnetic dipole transitions. In very strong magnetic fields $|M|^2$ is dependent only on the electronic quantum numbers, the spectroscopic splitting factor, and the microwave power. In intermediate fields, however, there is a slight dependence on I , m_I , and A due to the $A\mathbf{I} \cdot \mathbf{S}$ hfs coupling to states characterized by the total quantum numbers F and m_F . $|M|^2$ has been calculated in the general case by Silsbee⁸ for a Hamiltonian of the form of Eq. (1) with $H_x = H_y = 0$. Application of his results to our case shows that the transition probability is still independent of nuclear parameters to within 1 percent, and we treat it here as a constant. As expected, the Co⁵⁹ and Co⁶⁰ hfs lines are observed to have the same line width, since they are due to the same electronic transitions. Thus from Eq. (4) we conclude that

$$\frac{N(\text{Co}^{59})}{N(\text{Co}^{60})} = \left[\frac{2I(\text{Co}^{59})+1}{2I(\text{Co}^{60})+1} \right] \cdot \left[\frac{P_{m_I}(\text{Co}^{59})}{P_{m_I}(\text{Co}^{60})} \right]. \quad (5)$$

⁸ H. B. Silsbee (private communication).

The observed spectrum of Fig. 1 excludes all spin values of Co⁶⁰ except $I=5$ and $I=6$; to distinguish between these two choices it is necessary to observe relative line intensities with an accuracy of roughly 10 percent. We have done this by reducing the gain of the signal amplifier during the observation of the intense lines A, B, \dots, H , by means of a calibrated attenuator. The Co⁵⁹ and Co⁶⁰ hfs lines then appear undistorted on the same paper trace with approximately equal amplitudes and a measurement of the relative amplitudes is limited chiefly by the signal-to-noise ratio of the Co⁶⁰ lines, which is approximately 12:1. The observed hfs lines of both isotopes show a slight and similar decrease in amplitude with decreasing magnetic field due to a slight increase in line width, the area under the absorption line being conserved. This effect is too small to be visible in Fig. 1. A similar behavior has been observed by Owen⁹ for Ni²⁺ ions, and is due to the circumstance that the fundamental frequency line width $\Delta\omega$ when transformed to an equivalent magnetic line width ΔH yields slightly different values depending upon the slope of the energy levels in the Breit-Rabi diagram. Since for our purpose we have only to compare hfs lines of equal line width, this small effect does not affect the validity of our spin determination, which is made in the following manner: For a series of three independent runs we have measured the relative intensity of line 1 to the average of lines A and B , line 4 to the average of lines C and D , line 6 to the average of lines D and E , line 8 to the average of lines E and F , line 11 to the average of lines G and H . The results are, using Eq. (5),

$$N(\text{Co}^{59})/N(\text{Co}^{60}) = 50.3 \pm 3.5, \text{ assuming } I(\text{Co}^{60}) = 5,$$

and

$$N(\text{Co}^{59})/N(\text{Co}^{60}) = 42.4 \pm 3, \text{ assuming } I(\text{Co}^{60}) = 6.$$

The isotopic abundance ratio as determined by specific activity measurements is

$$N(\text{Co}^{59})/N(\text{Co}^{60}) = 55.4 \pm 1.7,$$

⁹ J. Owen, thesis, Oxford University, 1953 (unpublished).

which within experimental error agrees with $I=5$ and excludes $I=6$. Since all other spin values are excluded by the spectrum of Fig. 1, we conclude that

$$I(\text{Co}^{60})=5. \quad (6)$$

In order to determine precisely the nuclear magnetic moment, we have carefully compared the total splitting ΔH between lines A and H , and between lines 1 and 11. For a series of four independent runs, we find:

$$\frac{\Delta H(\text{Co}^{60})}{\Delta H(\text{Co}^{59})} = \frac{\mu(\text{Co}^{60})}{\mu(\text{Co}^{59})} = 0.8191 \pm 0.0016. \quad (7)$$

Taking $\mu(\text{Co}^{59})=4.6399$, without diamagnetic correction, as given by Proctor and Yu,¹⁰ we obtain finally:

$$|\mu(\text{Co}^{60})| = 3.800 \pm 0.007 \text{ nm}. \quad (7)$$

The sign of the magnetic moment is not determined by this experiment, as is easily seen from the form of Eq. (3): the observed hfs spectrum will be identical upon a reversal of the sign of A . As discussed below, there is good evidence from the nuclear shell model that the sign is positive.

V. DISCUSSION

Co^{60} has been extensively investigated by β , γ spectroscopy and by nuclear alignment experiments. It decays principally¹¹ by a 0.32-Mev β particle to the second excited state of Ni^{60} ; $\log(ft) \cong 7.5$, and the Fermi-Kurie plot^{12,13} is linear down to about 60 keV, indicating a first forbidden or possibly allowed β transition. Angular correlation measurements¹⁴ on the γ , γ cascade of Ni^{60} have definitely established the assignment of the spins $I=4$ (even parity) and $I=2$ (even parity) to the second and first excited states, respectively. Thus, the spin of 5.3-year Co^{60} may be either 4 or 5, and the decay scheme originally proposed by Deutsch and Goldhaber¹⁵ suggested $I=5$ as most likely. More recently Keister and Schmidt¹⁶ have proposed $I=4$ on the basis of a weak forbidden β transition to the first excited state of Ni^{60} which corresponds only to $\Delta J=2$. Our direct measurement of the spin $I=5$ from hfs indicates that this interpretation is incorrect.¹⁷

The low-temperature nuclear alignment experiments

¹⁰ W. G. Proctor and F. C. Yu, *Phys. Rev.* **81**, 20 (1951).

¹¹ Deutsch, Elliot, and Roberts, *Phys. Rev.* **68**, 193 (1945).

¹² C. Y. Fan, *Phys. Rev.* **87**, 252 (1952).

¹³ Bolla, Terrani, and Zappa, *Nuovo cimento* **12**, 875 (1954).

¹⁴ See, e.g., H. Frauenfelder, in *β and γ Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. 19.

¹⁵ M. Deutsch and G. Scharff-Goldhaber, *Phys. Rev.* **83**, 1059 (1945).

¹⁶ G. L. Keister and F. H. Schmidt, *Phys. Rev.* **93**, 140 (1954).

¹⁷ A recent private communication from F. H. Schmidt indicates that perhaps the weak forbidden β spectrum was distorted by scattering such that it was fitted by a $\Delta J=2$ (No) adjustable shape when actually the spectrum should exhibit a $\Delta J=3$ (No) unique shape. The ft value does not exclude this possibility.

with Co^{60} of the Oxford¹⁸⁻²² and Leiden²³⁻²⁶ groups have previously been interpreted on the basis of $I=4$ or 5. The experimental data have not been precise enough to distinguish between the two possible spin choices. The magnetic moment values as determined by the variation with temperature of the anisotropy of the γ radiation from the aligned nuclei are:

Oxford group²¹: $\mu=3.5 \pm 0.5$ nm, assuming $I=5$,

$$\mu=3.2 \pm 0.5 \text{ nm, assuming } I=4; \quad (9)$$

Leiden group²⁶: $\mu=4.3 \pm 0.2$ nm, assuming $I=5$.

Co^{60} is the first case for which both alignment experiments and direct spin and moment measurements (from hfs) have been performed. Although no comparison of spin values is possible in the present case, the reasonable agreement between the magnetic moment values of Eq. (9) and our precise value, Eq. (8), indicates that the interpretation of the complex alignment experiments is essentially correct.

Our results may also be compared with the expectations of the nuclear shell model of Mayer and Jensen.²⁷ For an odd-odd nucleus, Nordheim²⁸ has pointed out that in most cases the configuration is obtained from those of the corresponding odd-even and even-odd nuclei. Co^{60} has 27 protons and 33 neutrons; the shell model assigns the odd proton an $f_{7/2}$ orbit and the odd neutron a $p_{3/2}$ orbit.²⁹ These can couple to give a maximum spin $I=5$, as observed. If we assume the odd proton and neutron are jj -coupled to a resultant spin I , the resultant nuclear g factor is

$$g_I = \frac{1}{2} \left[(g_p + g_n) + (g_p - g_n) \frac{j_p(j_p+1) - j_n(j_n+1)}{I(I+1)} \right],$$

where g_p , j_p , g_n , j_n are the g factors and angular momentum quantum numbers of the proton and neutron, respectively. Taking an ($f_{7/2}$, $p_{3/2}$) configuration, we calculate g_I for the following simple assumptions:

Case (a): g factors have the Schmidt-limit values:

$$g_p = 1.656, \quad g_n = -1.277;$$

¹⁸ Daniels, Grace, and Robinson, *Nature* **168**, 780 (1951).

¹⁹ Bleaney, Daniels, Grace, Halban, Kurti, and Robinson, *Phys. Rev.* **85**, 688 (1952).

²⁰ M. A. Grace and H. Halban, *Physica* **18**, 1227 (1954).

²¹ Bleaney, Daniels, Grace, Halban, Kurti, Robinson, and Simon, *Proc. Roy. Soc. (London)* **221A**, 170 (1954).

²² Ambler, Grace, Halban, Kurti, Durand, Johnson, and Lemmer, *Phil. Mag.* **44** (216) 1953.

²³ Gorter, Poppema, Steenland, and Beun, *Physica* **17**, 1050 (1951).

²⁴ Gorter, Tolhoek, Poppema, Steenland, and Beun, *Physica* **18**, 155 (1952).

²⁵ Poppema, Beun, Steenland, and Gorter, *Physica* **18**, 1235 (1952).

²⁶ Poppema, Steenland, Beun, and Gorter, *Physica* **21**, 233 (1954).

²⁷ M. Mayer and J. Jensen, *The Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, Inc., New York, 1955).

²⁸ L. W. Nordheim, *Revs. Modern Phys.* **23**, 322 (1951).

²⁹ An $f_{5/2}$ neutron orbit is also possible, but this yields a calculated magnetic moment of $\mu \cong 6.1$ nm, much greater than the observed value.

Case (b): g factors have the Dirac-limit or "quenched" values:

$$g_p = 1.143, \quad g_n = 0;$$

Case (c): g factors are those empirically determined by the adjacent Co⁵⁹ and Ni⁶¹ nuclear moment values:

$$g_p = 1.325, \quad g_n \cong -0.2/\frac{3}{2} \cong -0.13.$$

The results are tabulated in Table I.

There is excellent agreement in magnitude between the measured μ value and that calculated for the Schmidt-limit case. This may be taken as an indication that the magnetic moment is positive.

TABLE I. Comparison of $\mu(\text{Co}^{60})$ values.

Measured	(a) Schmidt-limit	Calculated (b) Dirac-limit	(c) Empirical
$\pm 3.800 \text{ nm}$	$+3.878 \text{ nm}$	$+4.00 \text{ nm}$	$\sim +4.4 \text{ nm}$

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Emission of Prompt Neutrons from Fission*

R. B. LEACHMAN†

Los Alamos Scientific Laboratory, Los Alamos, New Mexico

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An analysis of the total energies of the fragment pairs from fission is used with the mass equation of fission to estimate the distributions in the excitation energy of the fragments from spontaneous and neutron-induced fission of several nuclides. These excitations are used with simple neutron boil-off considerations to calculate the probabilities of emission of 0, 1, 2, 3... prompt neutrons. The calculated results are in good agreement with recent measurements.

The same excitation energy distributions and neutron boil-off considerations are used with an assumption of an isotropic angular relation between the fragments and the emitted neutrons to calculate the energy spectrum of neutrons from thermal and 3-Mev neutron-induced fission of U²³⁵. For thermal-neutron fission, the calculated spectrum is in fair agreement with recent measurements. The calculations indicate little change in the spectrum for 3-Mev fission. The average energy of the prompt gamma rays is 3.8 Mev from this analysis.

INTRODUCTION

THE fission of a heavy nucleus is a process which divides a very complex nucleus usually into two similarly complex nuclei. In this process, many modes of fission into different divisions of both neutron and proton numbers are observed. Further, different divisions of the available total energy of fission into the energy of repulsion of the fragments and the energy of excitation of the fragments are observed. As yet, no full explanation of these many complexities of fission has been published.

In recent years, great improvements in the empirical data related to fission have been published. These data make possible a better understanding of the division of the energies released in the fission process and of the subsequent emission of the excitation energy as the prompt neutrons and prompt gamma rays of fission. In the present analysis of these processes, data of atomic masses are first used to calculate the total energy of fission. Then, empirical data of the distribution of

the fragment kinetic energy part of this total energy are used to calculate the distribution in the excitation energy of the fragments. Next, neutron boil-off considerations are applied to determine the probabilities of emission of 0, 1, 2, 3... prompt fission neutrons and their energies. The energies of these neutrons are then transformed to the laboratory system by means of an assumed fragment-neutron angular relation. Finally, the residual excitation energy, which appears as the prompt gamma rays of fission, is determined.

For this analysis, the statistical properties of the nuclei of the unstable, neutron-rich products of fission are used. In the absence of complete information on these nuclear properties and for simplicity, assumptions of the atomic and nuclear systematics are made. In particular, the excited fission fragments are considered as having a continuum of energy levels, with expressions for the densities of these levels the same as those of stable nuclides. Also, the atomic masses of the fission fragments are determined from mass-spectroscopic measurements of their stable isobars by means of assumed extensions of the mass systematics of isobars. These assumed systematics of the atomic masses of the fission fragments are also used to calculate neutron

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† Present address: Nobel Institute of Physics, Stockholm 50, Sweden.