the effect is proportional to $Z_1^{\frac{1}{2}}m^{\frac{1}{2}}$ and has an enhanced importance for *heavy ion bombardment.* A change from protons to N^{14} in the bombardment of heavy nuclei increases the redistribution effects by a factor ~ 10, while a change from protons to alpha particles gives a factor $2\sqrt{2}=2.8$.

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Electric-Monopole Transitions in Atomic Nuclei*

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LOW-ENERGY electric-monopole, or $E0$ transitions, $(\Delta\mathbf{I}=0, \text{no})$, proceed solely by internal conversion, with zero units of angular momentum transferred to the ejected electron. Gamma-ray emission of this multipole order is strictly forbidden, although $E0$ pair production is possible for transition energies $>2mc^2$. It has not been previously emphasized that (1) the electric-monopole mode of de-excitation is available between any two equal-parity states of the same spin, zero or otherwise, (2) that in such cases, $E0$ internal conversion may compete favorably with the paralleling $M1$ and $E2$ radiative transitions in heavy nuclei, and (3) that monopole matrix elements, $\langle i|\sum r_p^2|f\rangle$, may be particularly useful in the study of nuclear structure.

Figure 1 presents the transition probability for monopole conversion in the K shell for various atomic numbers.[1,2] The "strength parameter," ρ, is defined by

$$\langle i|\sum r_p^2|f\rangle=\rho R_0^2, \tag{1}$$

where $R_0=1.20\times10^{-13}A^{\frac{1}{3}}$ cm. In the naive approximation of complete overlap of the initial and final nuclear wave functions (the "Weisskopf" approximation for higher multipoles[3]), ρ would be of the order of unity. Conversion in the L shell occurs almost entirely in the L_{I} subshell, with K/L conversion ratios falling between 6 and 4 for $Z=70-100$. The results in Fig. 1 have been computed in the "point-nucleus" approximation for the electron wave functions.[4] Calculations of the effects of the finite nuclear size and atomic screening indicate that these results should be increased by 10 to 20% for $Z=85$. The data in Fig. 1, therefore, correspond to conservative lower limits for the total $E0$ conversion probability.

Transitions between two equal-parity states of the same spin are usually thought of as proceeding only by $M1$ and $E2$ transitions. $E0$ conversion is also possible in such cases. A convenient measure of the monopole contribution is ϵ^2, the ratio of the rate of $E0$ *conversion* to the rate of $E2$ *gamma-ray* emission, defined in analogy with δ^2, the ratio of the rates of $M1$ to $E2$ gamma-ray emission. Empirical lifetimes of $E2$ ground-state transitions in even-even nuclei with $Z=60-80$ are between 10 and 100 times shorter than the corresponding Weisskopf estimates.[5] Even with this enhancement of the quadrupole component, Fig. 1 indicates that values of ϵ^2 of the order of unity are to be expected for energies $\sim 1mc^2$, if ρ is set equal to one. Since δ^2 is frequently of the order of unity or less in these nuclei, the $E0$ mode of decay may be of considerable importance. This situation is illustrated in Fig. 2.

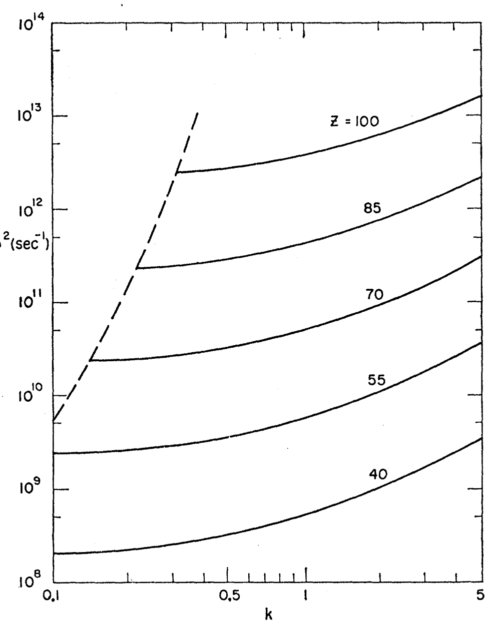

FIG. 1. Transition probability for electric-monopole conversion in the K shell divided by ρ^2, as computed in the "point-nucleus" approximation. Results are given for various atomic numbers as functions of the nuclear transition energy, k, in units of mc^2. The strength parameter, ρ, defined by Eq. (1), is of the order of unity in the "Weisskopf" approximation.

As pointed out by Scharff-Goldhaber and Weneser,[6] there exists a large and regular class of moderately heavy even-even nuclei having 2+ first- and second-excited states. Such nuclei provide a fertile field for the search for data on monopole transitions, which until now have been available only from a study of a few isolated $E0$ transitions of the $0+\rightarrow0+$ type. Sufficient experimental data are already available for the determination of upper limits for the monopole contributions in two transitions of the $2+\rightarrow2+$ type:

the 677-kev transition in Hg^{198}, and the 331-kev transition in Pt^{196}. Experimental values of the K-conversion coefficients,[7,8] α_K, and the $M1/E2$ mixing ratios,[9,10] δ^2 are available for these transitions. The latter were determined by measurement of the gamma-gamma directional correlation between the mixed transition and the subsequent $2+\rightarrow 0+$ pure $E2$ ground-state transition. The relationship between these quantities is

$$\epsilon_K{}^2=(\alpha_K-\alpha_{E2})-\delta^2(\alpha_{M1}-\alpha_K), \tag{2}$$

where α_{E2} and α_{M1} are the K-conversion coefficients of the pure $E2$ and $M1$ transitions, respectively. Substitution of the experimental values of α_K and δ^2, and the theoretical values of α_{E2} and α_{M1} from Rose *et al.*,[11] indicates that $\epsilon_K{}^2$ is zero to within the limits of experimental error. If one uses the quoted limits of experimental error and assumes a 2% interpolation uncertainty in the values of α_{E2} and α_{M1}, one obtains the upper limits $\epsilon_K{}^2(Hg^{198})\leqslant 2\times 10^{-3}$ and $\epsilon_K{}^2(Pt^{196})\leqslant 5\times 10^{-3}$. The lifetimes of the mixed transitions have not been measured in these nuclei. In the absence of such data we assume that the reduced transition probability of the $E2$ component is twice[12] that known for the ground-state transition in Hg^{198}.[5] These estimates lead to the conservative upper limits $\rho(Hg^{198})\leqslant 1/13$ and $\rho(Pt^{196})\leqslant 1/34$.

There is evidence[13,14] that the $M1$ K-conversion coefficients of Rose *et al.*[11] may be too large by $\sim$35%

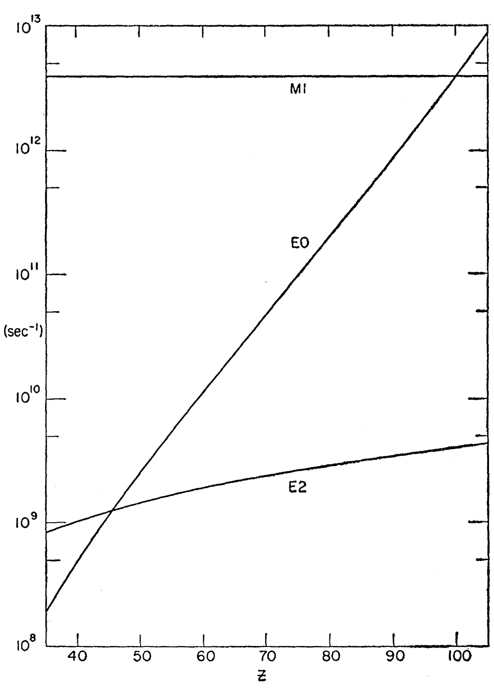

FIG. 2. Transition probability for electric-monopole conversion in the K shell as a function of atomic number for a transition energy of one mc^2. These results have been derived from Fig. 1 by assuming $\rho=1$. The analogous "Weisskopf" estimates of the $M1$ and $E2$ gamma-ray transition probabilities are included for comparison

for $Z\sim 80$. If the values of α_{M1} are lowered by this amount, the upper limits of the strength parameters become $\rho(Hg^{198})\leqslant 1/6$ and $\rho(Pt^{196})\leqslant 1/21$. Thus, at least in the case of Pt^{196}, experiment seems to indicate that the monopole matrix element is strongly attenuated. In comparison, monopole transitions of the $0+\rightarrow 0+$ type in C^{12}, O^{16}, and Ge^{72} exhibit strength parameters lying between 1/2 and 1/9.[2,15,16]

Monopole matrix elements computed for reasonable interpretations of the pure shell model and of the various collective models are found to vanish to lowest order. The experimental determination of the magnitudes of such matrix elements, therefore, would provide a sensitive probe into the finer details of nuclear models. Definite measurements of ρ in heavy nuclei might be conveniently obtained from directional-correlation experiments involving the K-conversion electrons of the mixed $E0+M1+E2$ transition since the $E0-E2$ interference term appearing in the coefficient of $P_2(\cos\theta)$ is then proportional to ϵ instead of its square.

An extensive analysis of the effects of the finite nuclear size and atomic screening on the absolute and relative conversion properties of $E0$ transitions is being carried out, in addition to the evaluation, with M. E. Rose, of the expected conversion-electron correlation functions. These results, as well as a detailed discussion of the significance of $E0$ matrix elements, are being prepared for submittal to *The Physical Review*.

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