the various processes are compared with those calculated on the basis of the statistical model from the formula of Lepore and Stuart.[5] The experimental value of σ_1/σ_2, where σ_1 and σ_2 are the cross sections for single and double π production, respectively, is compared with that predicted by the statistical model and the excited-state model. A similar comparison is made for the experimental value of $\sigma_1(\pi^+)/\sigma_1(\pi^0)$; both results are compatible with the two theories.

Figures 1 and 2 show the c.m. angular distribution for single π production of π^+ and π^0 mesons with respect to the nucleon of lower energy. The evidence of angular correlation is not reconcilable with the present formulation of the statistical theory and suggests a production process characterized by the emission of the π meson by a single nucleon.

In Fig. 3 are plotted the experimental values calculated for the decay of an excited nucleon into a nucleon and a π^+. There is some evidence of a peak around 150 Mev as suggested by the Peaslee model. However, this model gives fractional weights of $\frac{3}{4}$ and $\frac{1}{4}$ for the intermediate compound states $(T_z', T_z) = (3/2, -1/2)$

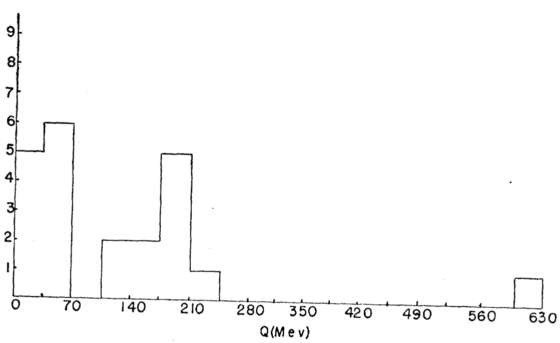

FIG. 3. Q values calculated for the decay of an excited intermediate state into a nucleon and π^+, for single π^+ emission.

and (1/2,1/2), respectively, where T_z is the z component of isotopic spin and the prime refers to the excited state. Experimentally this ratio is found to be 1/6 which is incompatible with the present formulation of the excited-state model.

Details of the analysis and additional data will be presented in a forthcoming paper. We wish to thank Mrs. V. Miller and Mrs. J. Milks for their assistance in scanning and to express our appreciation to the Brookhaven Cosmotron staff for their assistance in obtaining the exposure.

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Effects of Finite Amplitude in Coulomb Excitation*

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THE standard theory of Coulomb excitation[1,2] makes use exclusively of first-order effects of the field of the incident particle. Estimates described below show, however, that the probability amplitudes in close collisions are large enough to be considered, especially in connection with angular distribution effects.[2] On the semiclassical theory the excitation of sublevels of $J=2$ with magnetic quantum numbers μ from an initial level $J=0$ has the probability

$$\sum_\mu |c_\mu|^2 = \frac{49\pi}{100\hbar^2}(Z_1e^2Q)^2 \sum_\mu \left| \int_{-\infty}^{t} e^{i\omega t} Y_{2\mu} \frac{dt}{r^3}\right|^2,$$

where $Y_{2\mu}$ is the spherical harmonic normalized to unity for integration over solid angles, r the interparticle separation, Z_1 the projectile's charge, and Q the nuclear quadrupole moment. At the perihelion ($t=0$) of close collisions,

$$\left(\sum_\mu |c_\mu|^2\right)_{t=0} = 0.068 (Z_1e^2/\hbar v)^2 (Q/a'^2)^2,$$

where $2a'$ is the distance of closest approach and v is the relative velocity. Taking nominally the nuclear charge $Z_2=78$, $Z_1=1$, $Q=7\times10^{-24}$ cm^2, one obtains $|c_\mu| = (0.065, 0.053, 0.065)$ for $\mu=(2, 0, -2)$ and $\sum_\mu |c_\mu|^2 = 0.011$. On its way out, the projectile causes transitions from sublevels μ' of the excited state or from other states. There is, in addition, the effect of the first order c_μ giving an effect on c_μ itself. The change in a c_μ is thus of the order of 6 percent of a c_μ, corresponding to 12 percent in gamma intensity from μ, provided phase relations are disregarded. For $J=2\rightarrow J=2$ transitions, one finds a factor $(2\sqrt{5})/7=0.64$ relative to $J=0\rightarrow J=2$ but the $\mu=2$ to $\mu=0$, $\mu=-2$ to $\mu=0$ transitions double the effect on $|c_0|$. While the estimate of 12 percent does not apply to gamma angular correlation coefficients directly, a literal application of first-order theory is seen to be questionable.

The finite-amplitude effects need consideration in connection with gamma angular distribution measurements[3] and inelastic scattering studies. In principle they are capable of supplying information on *static quadrupole moments* of excited states as well as transition moments between excited states. The effects would stand out better in coincidence experiments giving the final projectile direction and the associated gamma distribution. The essential amplitude parameter is

$$Z_1e^2Q/\hbar v a'^2 \propto m^{\frac{1}{2}}E^{\frac{3}{2}}/Z_1Z_2^2,$$

where m is the reduced mass. For fixed a', conditions for compound nucleus formation are roughly similar;

the effect is proportional to $Z_1^{\frac{1}{2}}m^{\frac{1}{2}}$ and has an enhanced importance for *heavy ion bombardment*. A change from protons to N^{14} in the bombardment of heavy nuclei increases the redistribution effects by a factor ~10, while a change from protons to alpha particles gives a factor $2\sqrt{2}=2.8$.

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Electric-Monopole Transitions in Atomic Nuclei*

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Low-energy electric-monopole, or $E0$ transitions, $(\Delta\mathbf{I}=0, \text{no})$, proceed solely by internal conversion, with zero units of angular momentum transferred to the ejected electron. Gamma-ray emission of this multipole order is strictly forbidden, although $E0$ pair production is possible for transition energies $>2mc^2$. It has not been previously emphasized that (1) the electric-monopole mode of de-excitation is available between any two equal-parity states of the same spin, zero or otherwise, (2) that in such cases, $E0$ internal conversion may compete favorably with the paralleling $M1$ and $E2$ radiative transitions in heavy nuclei, and (3) that monopole matrix elements, $\langle i|\sum r_p^2|f\rangle$, may be particularly useful in the study of nuclear structure.

Figure 1 presents the transition probability for monopole conversion in the K shell for various atomic numbers.[1,2] The "strength parameter," ρ, is defined by

$$\langle i|\sum r_p^2|f\rangle=\rho R_0^2, \tag{1}$$

where $R_0=1.20\times10^{-13}A^{\frac{1}{3}}$ cm. In the naive approximation of complete overlap of the initial and final nuclear wave functions (the "Weisskopf" approximation for higher multipoles[3]), ρ would be of the order of unity. Conversion in the L shell occurs almost entirely in the L_{I} subshell, with K/L conversion ratios falling between 6 and 4 for $Z=70-100$. The results in Fig. 1 have been computed in the "point-nucleus" approximation for the electron wave functions.[4] Calculations of the effects of the finite nuclear size and atomic screening indicate that these results should be increased by 10 to 20% for $Z=85$. The data in Fig. 1, therefore, correspond to conservative lower limits for the total $E0$ conversion probability.

Transitions between two equal-parity states of the same spin are usually thought of as proceeding only by $M1$ and $E2$ transitions. $E0$ conversion is also possible in such cases. A convenient measure of the monopole contribution is ϵ^2, the ratio of the rate of $E0$ *conversion* to the rate of $E2$ *gamma-ray* emission, defined in analogy with δ^2, the ratio of the rates of $M1$ to $E2$ gamma-ray emission. Empirical lifetimes of $E2$ ground-state transitions in even-even nuclei with $Z=60-80$ are between 10 and 100 times shorter than the corresponding Weisskopf estimates.[5] Even with this enhancement of the quadrupole component, Fig. 1 indicates that values of ϵ^2 of the order of unity are to be expected for energies $\sim1mc^2$, if ρ is set equal to one. Since δ^2 is frequently of the order of unity or less in these nuclei, the $E0$ mode of decay may be of considerable importance. This situation is illustrated in Fig. 2.

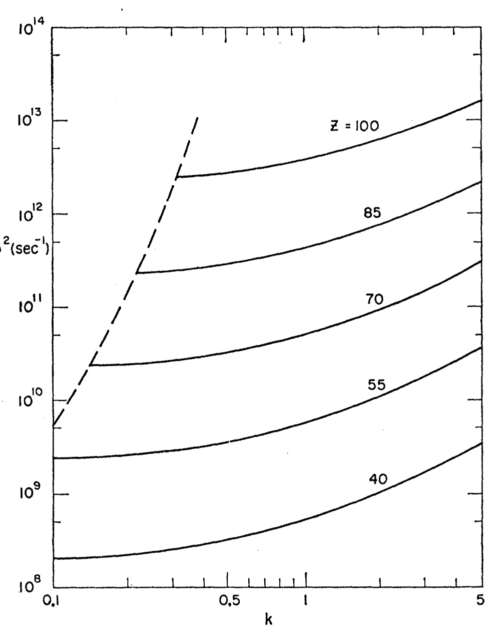

FIG. 1. Transition probability for electric-monopole conversion in the K shell divided by ρ^2, as computed in the "point-nucleus" approximation. Results are given for various atomic numbers as functions of the nuclear transition energy, k, in units of mc^2. The strength parameter, ρ, defined by Eq. (1), is of the order of unity in the "Weisskopf" approximation.

As pointed out by Scharff-Goldhaber and Weneser,[6] there exists a large and regular class of moderately heavy even-even nuclei having 2+ first- and second-excited states. Such nuclei provide a fertile field for the search for data on monopole transitions, which until now have been available only from a study of a few isolated $E0$ transitions of the $0+\rightarrow0+$ type. Sufficient experimental data are already available for the determination of upper limits for the monopole contributions in two transitions of the $2+\rightarrow2+$ type: