trend is consistent with a straight line, defining $(Z^2/A)_c = 40.2\pm 0.7$. The equation of the line leads to the semiempirical formula,

$$M_2 - M_1 = 0.090(40.2\pm 0.7 - Z^2/A)^{\frac{1}{2}}A. \qquad (4)$$

One may combine Eq. (4) with the relation:

$$M_2 + M_1 = A - \nu$$

(ν = number of neutrons emitted in fission),

to predict the positions of the peaks in the yield curves of elements that have not yet been investigated. If an average value $\bar{\nu} = 2.8$ is used, one finds

$$M_2 = \tfrac{1}{2}A - 1.4 + 0.045(40.2\pm 0.7 - Z^2/A)^{\frac{1}{2}}A, \qquad (5)$$

$$M_1 = \tfrac{1}{2}A - 1.4 - 0.045(40.2\pm 0.7 - Z^2/A)^{\frac{1}{2}}A. \qquad (6)$$

The present analysis provides a reason for the empirical observation that in the fission of different elements the position of the heavy peak remains approximately constant. If the degree of asymmetry remained unchanged from nucleus to nucleus, both peaks would move towards higher masses with increasing A. In fact, there is superimposed on this shift a coming together of the peaks with increasing Z^2/A. Since the over-all trend of Z^2/A is to increase with A, the result is that for the light peak the two shifts add up whereas for the heavy peak they partly cancel. This is illustrated in Table I, where M_2 and M_1, calculated according to (5) and (6), are compared with the observed values.

TABLE I. Positions of the peaks in the fission yield curves.

Compound nucleus	Position of peaks: Observed[a] M_2	Observed[a] M_1	Formulas (5), (6) M_2	Formulas (5), (6) M_1	Remarks	Reference
Th^{233}	140	91	139.1	91.1	Low-energy neutron fission	b
U^{239}	140	98	141.1	95.1		c
U^{236}	138.5	95	138.2	95.0		d
U^{234}	137	93	136.2	95.0		b, e
Pu^{240}	138	99	137.9	99.3		c
U^{238}	140	96	140.2	95.0	Spontaneous fission	f, g
Cm^{242}	136	103	134.7	104.5		g
Cf^{252}	139	108	140.2	109.0		h

[a] The uncertainty in the observed values of M_2 and M_1 is of the order of ±1 or ±2 mass units. (It is more in the cases of U^{239} and U^{238}.) No systematic attempt has been made to adjust $A - M_2 - M_1$ to agree with available information on the number of emitted neutrons.
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Further measurements of fission asymmetries would be interesting, especially in the region of Z^2/A close to the critical value, where the present considerations suggest a rapid decrease of $M_2 - M_1$.

It is a pleasure to acknowledge stimulating discussions with Professor S. G. Thompson, Dr. A. C. Pappas, and Dr. T. Maris.

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Systematics of Spontaneous Fission Half-Lives

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(Received July 18, 1955)

SEVERAL authors have noted the over-all trend of spontaneous fission half-lives to decrease with increasing Z^2/A as well as the considerable deviations (by several powers of 10) from any smooth dependence on this parameter.[1] We should like to discuss the close correlation which seems to exist between the half-lives and the finer details in the systematics of the ground-state masses of nuclei.[2]

A simple way of exhibiting this correlation is to plot the deviation $\delta\tau$ from a straight line in a plot of $\tau[\tau = \log_{10}(\text{half-life})]$ *vs* Z^2/A, against deviations (δM) of the masses M of the nuclei from a smooth reference surface $M_{\text{ref}}(A,Z)$. We made such a plot, with M_{ref} taken to be the semiempirical mass surface of Green[3] (based on the liquid drop model):

$$\delta M = M - M_{\text{ref}},$$

$$M_{\text{ref}} = 1000A - 8.3557A + 19.120A^{\frac{2}{3}} + 0.76278Z^2/A^{\frac{1}{3}} + 25.444(N-Z)^2/A + 0.420(N-Z) \text{ millimass units.} \qquad (1)$$

The experimental masses M were taken from Glass *et al.*[4]

In the case of even-even nuclei the plot of $\delta\tau$ *vs* δM suggested a series of straight lines, one for each Z, indicating that for the isotopes of one element special stability of a nucleus (small δM) is invariably associated with a longer lifetime (large $\delta\tau$). The lines had approximately the same slope, thus defining a spontaneous-fission hindrance factor which corresponds to about 10^5 times longer lifetime for each millimass unit of extra stability. This suggested that if the observed lifetimes were corrected for the variations in stability of the ground states, a more regular dependence of τ on Z^2/A might be discernible.

Figure 1 shows the effect on the plot of τ *vs* Z^2/A of adding to the observed τ_{exp} an empirical correction $k\delta M$ ($k\sim 5$ if δM in mMU). For even-even nuclei the values of $\tau_{\text{exp}} + k\delta M$ define a fairly smooth curve, with indications of a similar curve for odd-A nuclei. [In a preliminary plot the hindrance factor k was taken to be 5. A small but significant further smoothing of the points resulted from making k vary with Z^2/A according to $k=5-(Z^2/A-37.5)$. This is the case shown in Fig. 1.]

The result can be stated in the form of an empirical formula for half-lives; e.g., for even-even nuclei,

$$\tau_{ee}=f(Z^2/A)-k\delta M, \qquad (2)$$

where f is the curve defined by the even-even points in Fig. 1. The relation of the points for odd-A nuclei to the curve obtained from (2) by a shift upwards of 6.6 units is also shown in Fig. 1. The lifetime of the odd-odd nucleus E^{254} (einsteinium, $Z=99$) is consistent with a further shift of 4.9 units. The curve $f(Z^2/A)$ can be represented for example by a cubic, which leads to the following formulas for the lifetimes:

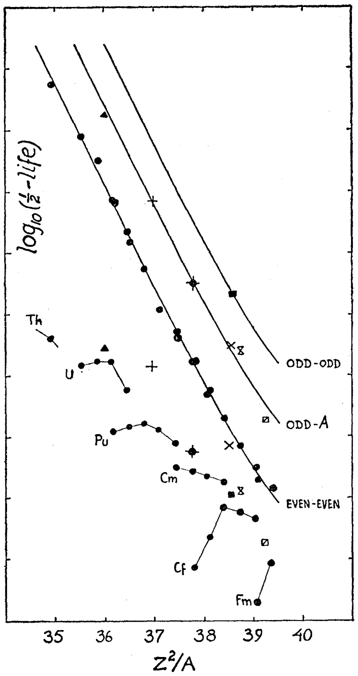

FIG. 1. Plot of spontaneous fission half-lives against Z^2/A. The observed lifetimes τ_{exp} occupy the bottom left-hand part of the figure; the "corrected" values $\tau_{\text{exp}}+k\delta M$ group themselves around the three curves. Experimental points for even-even nuclei are joined by straight lines. Odd-A nuclei are designated by special symbols which, reading from left to right along the odd-A curve, refer to U^{235}, Pu^{239}, Bk^{249}, Cf^{249}, E^{253} (einsteinium, $Z=99$), and Fm^{255} (fermium, $Z=100$). The odd-odd nucleus E^{254} is marked by a square.

$$\left.\begin{aligned}\tau_{ee}&=18.2\\ \tau_{\text{odd }A}&=24.8\\ \tau_{oo}&=29.7\end{aligned}\right\}-7.8\theta+0.35\theta^2+0.073\theta^3-(5-\theta)\delta M, \qquad (3)$$

where $\theta=(Z^2/A)-37.5$, and δM is the deviation in mMU of the experimental mass from the surface (1). Table I compares the observed half-lives with the values calculated by means of (3). The remarkable degree of smoothing achieved by means of the unsophisticated correction $k\delta M$ is illustrated by the fact that the deviations from (3) rarely exceed 0.5. (Note that a shift in τ of this amount would be produced by an error of 0.1 mMU in δM.)

TABLE I. Values of $\log_{10}$(half-life).

Nucleus	Experimental[a]	Formula (3)
Even-even nuclei		
Th 230	$\geqslant 7.18$	19.39
232	18.15	18.84
U 232	13.90	13.56
234	16.30	15.98
236	16.30	15.21
238	15.90	15.52
Pu 236	9.54	9.66
238	10.69	11.57
240	11.08	11.09
242	10.86	11.22
244	10.40	10.13
Cm 240	6.28	6.27
242	6.86	7.27
244	7.15	7.09
246	7.48	7.88
Cf 246	3.32	3.27
248	3.85	3.92
250	4.18	4.24
252	1.82	1.60
254	−0.70	−1.02
Fm 254	−0.30	−0.85
256	−3.52	−3.02
Odd-A nuclei		
U 235	17.26?	18.02
Pu 239	15.74	15.42
Bk 249	8.78	8.67
Cf 249	9.18	8.65
E 253	5.48	4.38
Fm 255	1.30	2.79
Odd-odd nuclei		
E 254	5.18	5.17

[a] The experimental values are from a summary by A. Ghiorso, kindly lent to me by Professor S. G. Thompson.

The importance of shell structure in the fission process is suggested by the fact that, according to the present considerations, the oscillations of the masses (associated with individual particle structure) in the range $\delta M=1-3$ mMU shorten the lifetimes by factors of 10^5 to 10^{15}. On the other hand the irregularities in the original plot of τ_{exp} against Z^2/A are seen to be largely due to irregularities in the ground-state masses, associated with *shell structure in the ground-state configuration.* The smoothness of the points $\tau_{\text{exp}}+k\delta M$ suggests that, after correcting for shell structure in the ground-state configuration, the description of the fission process in terms of a model in which single-particle features are treated in an average way may be useful. Qualitative reasons for the greater validity of such an averaged description for the more strongly deformed nuclear shapes occurring in fission may be found in the disappearance for such shapes of degeneracies in the energy spectrum associated with the proximity to a spherically symmetric configuration.

It is a pleasure to acknowledge discussions with Professor S. G. Thompson and Dr. A. C. Pappas and stimulating contacts with Dr. Aage Bohr and Dr. B. R. Mottelson and members of the C.E.R.N. Theoretical Study Group in Copenhagen.

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[1] See for example J. R. Huizenga, Phys. Rev. **94**, 158 (1954).

[2] The existence of correlations between nuclear masses, fission thresholds, and half-lives has been considered by Professor D. Frisch, to whom I am greatly indebted for stimulating discussions.

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