complex scattering length,[12] one obtains from Eq. (4) that the strength function[13] $s(E)=\langle\gamma^2\rangle/D$ can be expressed at zero energy as

$$s(0)=-\frac{1}{\pi}\,\mathrm{Im}\frac{a}{R}=\frac{1}{2}\frac{\lambda\!\!\!^{-(0)}}{R}\frac{\Gamma_n{}^{(0)}}{D},$$
$$=\frac{M}{2\pi^2\hbar^2R}\int_0^\infty dr|u_0(r)|^2W(r), \tag{7}$$

where R is the nuclear radius, a is the complex scattering length, $\lambda\!\!\!^{-(0)}$ is the wavelength divided by 2π to which the neutron width is reduced, and $u_0(r)$ is the gross neutron wave function at zero energy.

Equations (4) and (7) may be of use in numerical calculations since, once $u(r)$ is known either by integrating Eq. (1) or by approximating its solution [by a square well solution, for example, if one is interested in the effect of rounding the edge of the well which leads to complicated exact forms for $u(r)$], the integral can be evaluated for a given choice of $W(r)$.

* Work performed under contract with the U. S. Atomic Energy Commission.

[1] Melkanoff, Nodvik, Saxon, and Woods (private communiaction). The writer is grateful for a prepublication copy of their work.
[2] Z. Janković, Phil. Mag. 46, 376 (1955).
[3] P. E. Nemirovski, International Conference on the Peaceful Uses of Atomic Energy, No. 654, 1955 (unpublished).
[4] Morrison, Muirhead, and Murdoch, Phil. Mag. 46, 795 (1955).
[5] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1125.
[6] B. E. Freeman and J. L. McHale, Phys. Rev. 89, 223 (1953).
[7] See, for example, H. A. Bethe, Phys. Rev. 76, 38 (1949).
[8] Harvey, Hughes, Carter, and Pilcher, Phys. Rev. 99, 10 (1955).
[9] Bollinger, Coté, and Le Blanc (private communication). The writer wishes to express his thanks for the privilege of seeing the data of these authors prior to publication.
[10] S. E. Darden, Phys. Rev. 99, 748 (1955).
[11] Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954); see also paper No. 830, International Conference on the Peaceful Uses of Atomic Energy, 1955.
[12] M. L. Goldberger and F. Seitz, Phys. Rev. 71, 294 (1947), Eq. (14).
[13] R. G. Thomas, Phys. Rev. 97, 224 (1955). The writer would like to thank Dr. Thomas for calling his attention to the connection between the strength function and the imaginary part of the complex scattering length.

Systematics of Fission Asymmetry

W. J. SWIATECKI
Institute for Mechanics and Mathematical Physics and The Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden
(Received July 11, 1955)

ACCORDING to the liquid drop model the threshold energy for fission tends to[1]

$$E_{\text{threshold}}=c_1[(Z^2/A)_0-(Z^2/A)]^3, \tag{1}$$

in the limit $[1-(Z^2/A)/(Z^2/A)_0]\ll 1$. Here c_1 is a constant and $(Z^2/A)_0\sim 50$.[2] It is easily shown[3] that the dependence on the *cube* of $[(Z^2/A)_0-(Z^2/A)]$ is much more general than the assumption of an incompressible liquid drop. [The exponent three is associated with the existence of a point of inflection—a triple zero—in a plot of potential energy against deformation in the limit $Z^2/A\to(Z^2/A)_0$.] Many generalizations of the model (such as a nonuniform density or even the inclusion of additional forces assumed to vary smoothly with Z and A) would affect only the numerical magnitudes of $(Z^2/A)_0$ and c_1.

To a similar degree of generality it can be shown[3] that if below a certain value of Z^2/A $[Z^2/A<(Z^2/A)_c]$ the symmetrical saddle-point shape[1] becomes unstable against asymmetric distortions[4] [in the liquid drop

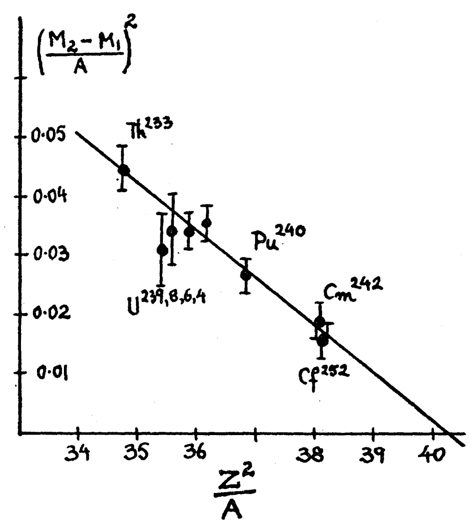

FIG. 1. The square of the relative degree of asymmetry, $(M_2-M_1)/A$, as a function of Z^2/A.

model $(Z^2/A)_c/(Z^2/A)_0$ is somewhere around 0.5–0.7 [3]], there appear two asymmetric saddle-point shapes, whose degree of asymmetry is proportional to $[(Z^2/A)_c-(Z^2/A)]^{\frac{1}{2}}$, i.e.,

$$\text{Asymmetry}=\pm c_2[(Z^2/A)_c-(Z^2/A)]^{\frac{1}{2}}, \tag{2}$$

and whose threshold energy lies below the threshold of the symmetric saddle point shape by an amount

$$\Delta E=c_3[(Z^2/A)_c-(Z^2/A)]^2. \tag{3}$$

Equation (2) suggests that the degree of asymmetry in nuclear fission should decrease in a characteristic manner with increasing Z^2/A. As a measure of the degree of asymmetry we have taken the distance M_2-M_1 between the peaks in the double-humped fission yield curve, and in Fig. 1 we have plotted $(M_2-M_1)^2/A^2$ against Z^2/A of the target nucleus. The asymmetry is seen to decrease with Z^2/A and the trend is consistent with a straight line, defining $(Z^2/A)_c = 40.2\pm0.7$. The equation of the line leads to the semiempirical formula,

$$M_2-M_1=0.090(40.2\pm0.7-Z^2/A)^{\frac{1}{2}}A. \quad (4)$$

One may combine Eq. (4) with the relation:

$$M_2+M_1=A-\nu$$

(ν=number of neutrons emitted in fission),

to predict the positions of the peaks in the yield curves of elements that have not yet been investigated. If an average value $\bar{\nu}=2.8$ is used, one finds

$$M_2=\tfrac{1}{2}A-1.4+0.045(40.2\pm0.7-Z^2/A)^{\frac{1}{2}}A, \quad (5)$$

$$M_1=\tfrac{1}{2}A-1.4-0.045(40.2\pm0.7-Z^2/A)^{\frac{1}{2}}A. \quad (6)$$

The present analysis provides a reason for the empirical observation that in the fission of different elements the position of the heavy peak remains approximately constant. If the degree of asymmetry remained unchanged from nucleus to nucleus, both peaks would move towards higher masses with increasing A. In fact, there is superimposed on this shift a coming together of the peaks with increasing Z^2/A. Since the over-all trend of Z^2/A is to increase with A, the result is that for the light peak the two shifts add up whereas for the heavy peak they partly cancel. This is illustrated in Table I, where M_2 and M_1, calculated according to (5) and (6), are compared with the observed values.

Further measurements of fission asymmetries would be interesting, especially in the region of Z^2/A close to the critical value, where the present considerations suggest a rapid decrease of M_2-M_1.

TABLE I. Positions of the peaks in the fission yield curves.

Compound nucleus	Position of peaks Observed[a] M_2	Observed[a] M_1	Formulas (5), (6) M_2	Formulas (5), (6) M_1	Remarks	Reference
Th^{233}	140	91	139.1	91.1	Low-energy neutron fission	b
U^{239}	140	98	141.1	95.1		c
U^{236}	138.5	95	138.2	95.0		d
U^{234}	137	93	136.2	95.0		b, e
Pu^{240}	138	99	137.9	99.3		c
U^{238}	140	96	140.2	95.0	Spontaneous fission	f, g
Cm^{242}	136	103	134.7	104.5		g
Cf^{252}	139	108	140.2	109.0		h

[a] The uncertainty in the observed values of M_2 and M_1 is of the order of ±1 or ±2 mass units. (It is more in the cases of U^{239} and U^{238}.) No systematic attempt has been made to adjust $A-M_2-M_1$ to agree with available information on the number of emitted neutrons.

[b] A. Turkevich and J. B. Niday, Phys. Rev. **84**, 52 (1951).

[c] E. B. Steinberg and M. S. Freedman, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York, 1951). Paper No. 219, National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Div. IV, Part V.

[d] Glendenin, Steinberg, Inghram, and Hess, Phys. Rev. **84**, 860 (1951).

[e] Steinberg, Glendenin, Inghram, and Hayden, Phys. Rev. **95**, 867 (1954).

[f] G. W. Wetherill, Phys. Rev. **92**, 907 (1953).

[g] E. P. Steinberg and L. E. Glendenin, Phys. Rev. **95**, 431 (1954).

[h] E. P. Steinberg and L. E. Glendenin, J. Inorg. Nuc. Chem. **1**, 45 (1955).

It is a pleasure to acknowledge stimulating discussions with Professor S. G. Thompson, Dr. A. C. Pappas, and Dr. T. Maris.

[1] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[2] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).
[3] W. J. Swiatecki (to be published).
[4] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).

Systematics of Spontaneous Fission Half-Lives

W. J. SWIATECKI

Institute for Mechanics and Mathematical Physics and The Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden

(Received July 18, 1955)

SEVERAL authors have noted the over-all trend of spontaneous fission half-lives to decrease with increasing Z^2/A as well as the considerable deviations (by several powers of 10) from any smooth dependence on this parameter.[1] We should like to discuss the close correlation which seems to exist between the half-lives and the finer details in the systematics of the ground-state masses of nuclei.[2]

A simple way of exhibiting this correlation is to plot the deviation $\delta\tau$ from a straight line in a plot of $\tau[\tau=\log_{10}(\text{half-life})]$ *vs* Z^2/A, against deviations (δM) of the masses M of the nuclei from a smooth reference surface $M_{\text{ref}}(A,Z)$. We made such a plot, with M_{ref} taken to be the semiempirical mass surface of Green[3] (based on the liquid drop model):

$$\delta M=M-M_{\text{ref}},$$

$$M_{\text{ref}}=1000A-8.3557A+19.120A^{\frac{2}{3}} +0.76278Z^2/A^{\frac{1}{3}}+25.444(N-Z)^2/A +0.420(N-Z) \text{ millimass units.} \quad (1)$$

The experimental masses M were taken from Glass *et al.*[4]

In the case of even-even nuclei the plot of $\delta\tau$ *vs* δM suggested a series of straight lines, one for each Z, indicating that for the isotopes of one element special stability of a nucleus (small δM) is invariably associated with a longer lifetime (large $\delta\tau$). The lines had approximately the same slope, thus defining a spontaneous-fission hindrance factor which corresponds to about 10^5 times longer lifetime for each millimass unit of extra stability. This suggested that if the observed lifetimes were corrected for the variations in stability of the ground states, a more regular dependence of τ on Z^2/A might be discernible.

Figure 1 shows the effect on the plot of τ *vs* Z^2/A of adding to the observed τ_{exp} an empirical correction $k\delta M$ ($k\sim5$ if δM in mMU). For even-even nuclei the values of $\tau_{\text{exp}}+k\delta M$ define a fairly smooth curve, with indications of a similar curve for odd-A nuclei. [In a