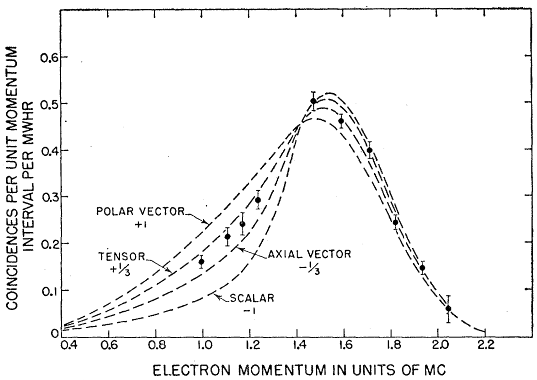

FIG. 3. The momentum spectrum of the electrons. The points represent the experimental data with standard deviations, and the dashed curves are the theoretical spectrum shapes for the pure interactions normalized by least squares.

The momentum spectrum of the electrons was obtained by taking a series of such runs. At each spectrometer setting runs were taken with different electron multiplier bias voltages so that an extrapolation to zero bias gave the number of coincidences per unit neutron intensity independent of slight variations in the gain of the electron multiplier. All runs were normalized by a neutron beam monitor. The result of all runs taken from September, 1954 to June, 1955 is shown in Fig. 3. In this figure the dashed curves are the calculated spectrum shapes for the pure beta-decay interactions using the Monte Carlo results for the detection efficiency of the geometry used in the experiment. They have been normalized by least-squares fitting to the experimental points. The best least-squares fit is obtained with the electron-neutrino angular correlation coefficient equal to $+0.089$ with a standard deviation of ± 0.108. This result is consistent with a beta-decay interaction of the form ST with $g_T^2/g_S^2 = 1.49_{-0.56}^{+1.44}$.

I wish to thank Dr. J. M. Kennedy and the computing staff of the Theoretical Physics Branch for arranging the Monte Carlo calculations and for calculating the theoretical spectrum shapes.

Cross Sections and the Complex Potential*

C. E. PORTER
Brookhaven National Laboratory, Upton, Long Island, New York
(Received August 30, 1955)

A NUMBER of calculations[1–4] have shown the importance of the edge-diffuseness of a complex potential in scattering problems. In the course of studying the characterization, especially at low energies, of such a potential in more detail a rather simple relation has been obtained between cross-section *ratios* and the imaginary part of the potential. For brevity we restrict our discussion to S-waves.

If we write the radial wave function as u/r, the differential equation for u is

$$\frac{d^2u}{dr^2}+\left[k^2+\frac{2M}{\hbar^2}(V+iW)\right]u=0, \tag{1}$$

with u subject to the boundary and normalization conditions:

$$u(0)=0, \quad u(r)\underset{r\to\infty}{\longrightarrow}v(r). \tag{2}$$

One can show that the wave number times the ratio of the reaction cross section to the elastic cross section is given by

$$k\sigma_r/\sigma_{el}=-\mathrm{Im}(k\cot\delta), \tag{3}$$

where δ is the complex phase shift. By exploring the consequences of Eqs. (1) to (3) it is possible to obtain the *exact* result

$$|v(0)|^2\left(\frac{k\sigma_r}{\sigma_{el}}\right)=\frac{2M}{\hbar^2}\int_0^\infty dr\, u^*(r)W(r)u(r). \tag{4}$$

A similarly simple result involving the ratio of the total cross section to the elastic cross section follows from Eq. (4) if $|v(0)|^2k$ is added to both sides. It is also possible to relate the real part of $k\cot\delta$ to the real part of the potential as is done in formulating variational principles with real potentials,[5] but the connection with the cross sections is not as direct. Expressions related to Eq. (4) have been given in a paper[6] on barrier penetration effects in light nuclei; however, the natural appearance of partial cross-section ratios and the application to low-energy neutron measurements do not seem to have been emphasized.

The form for $v(r)$ that is commonly used in discussions of effective range theory[7] is

$$v(r)=\sin(kr+\delta)/\sin\delta, \quad |v(0)|^2=1. \tag{5}$$

If $v(r)$ is so chosen that $u(r)$ is the S-wave solution for an incident plane wave of unit amplitude, then

$$v(r)=\frac{(4\pi)^{\frac{1}{2}}}{k}e^{i\delta}\sin(kr+\delta), \quad |v(0)|^2=\sigma_{el}. \tag{6}$$

In the case of low-energy neutron measurements,[8–10] the reaction cross section in Eq. (4) should be replaced by the cross section for compound nucleus formation, and the elastic cross section should be replaced by the shape-elastic cross section.[11] Using the normalization of Eq. (6), the known connection between the cross section for compound nucleus formation and the ratio $\Gamma_n^{(0)}/D$ of the average reduced neutron width to the average level spacing, and the usual expression for complex scattering length,[12] one obtains from Eq. (4) that the strength function[13] $s(E)=\langle\gamma^2\rangle/D$ can be expressed at zero energy as

$$s(0)=-\frac{1}{\pi}\,\mathrm{Im}\frac{a}{R}=\frac{1}{2}\frac{\lambda\!\!\!^{-(0)}}{R}\frac{\Gamma_n{}^{(0)}}{D},$$
$$=\frac{M}{2\pi^2\hbar^2R}\int_0^\infty dr\,|u_0(r)|^2W(r), \tag{7}$$

where R is the nuclear radius, a is the complex scattering length, $\lambda\!\!\!^{-(0)}$ is the wavelength divided by 2π to which the neutron width is reduced, and $u_0(r)$ is the gross neutron wave function at zero energy.

Equations (4) and (7) may be of use in numerical calculations since, once $u(r)$ is known either by integrating Eq. (1) or by approximating its solution [by a square well solution, for example, if one is interested in the effect of rounding the edge of the well which leads to complicated exact forms for $u(r)$], the integral can be evaluated for a given choice of $W(r)$.

* Work performed under contract with the U. S. Atomic Energy Commission.

1 Melkanoff, Nodvik, Saxon, and Woods (private communication). The writer is grateful for a prepublication copy of their work.
2 Z. Janković, Phil. Mag. 46, 376 (1955).
3 P. E. Nemirovski, International Conference on the Peaceful Uses of Atomic Energy, No. 654, 1955 (unpublished).
4 Morrison, Muirhead, and Murdoch, Phil. Mag. 46, 795 (1955).
5 P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1125.
6 B. E. Freeman and J. L. McHale, Phys. Rev. 89, 223 (1953).
7 See, for example, H. A. Bethe, Phys. Rev. 76, 38 (1949).
8 Harvey, Hughes, Carter, and Pilcher, Phys. Rev. 99, 10 (1955).
9 Bollinger, Coté, and Le Blanc (private communication). The writer wishes to express his thanks for the privilege of seeing the data of these authors prior to publication.
10 S. E. Darden, Phys. Rev. 99, 748 (1955).
11 Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954); see also paper No. 830, International Conference on the Peaceful Uses of Atomic Energy, 1955.
12 M. L. Goldberger and F. Seitz, Phys. Rev. 71, 294 (1947), Eq. (14).
13 R. G. Thomas, Phys. Rev. 97, 224 (1955). The writer would like to thank Dr. Thomas for calling his attention to the connection between the strength function and the imaginary part of the complex scattering length.

Systematics of Fission Asymmetry

W. J. Swiatecki
Institute for Mechanics and Mathematical Physics and The Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden
(Received July 11, 1955)

According to the liquid drop model the threshold energy for fission tends to[1]

$$E_{\text{threshold}}=c_1[(Z^2/A)_0-(Z^2/A)]^3, \tag{1}$$

in the limit $[1-(Z^2/A)/(Z^2/A)_0]\ll 1$. Here c_1 is a constant and $(Z^2/A)_0\sim 50$.[2] It is easily shown[3] that the dependence on the *cube* of $[(Z^2/A)_0-(Z^2/A)]$ is much more general than the assumption of an incompressible liquid drop. [The exponent three is associated with the existence of a point of inflection—a triple zero—in a plot of potential energy against deformation in the limit $Z^2/A\to(Z^2/A)_0$.] Many generalizations of the model (such as a nonuniform density or even the inclusion of additional forces assumed to vary smoothly with Z and A) would affect only the numerical magnitudes of $(Z^2/A)_0$ and c_1.

To a similar degree of generality it can be shown[3] that if below a certain value of Z^2/A [$Z^2/A<(Z^2/A)_c$] the symmetrical saddle-point shape[1] becomes unstable against asymmetric distortions[4] [in the liquid drop model $(Z^2/A)_c/(Z^2/A)_0$ is somewhere around 0.5–0.7 [3]], there appear two asymmetric saddle-point shapes, whose degree of asymmetry is proportional to $[(Z^2/A)_c-(Z^2/A)]^{\frac{1}{2}}$, i.e.,

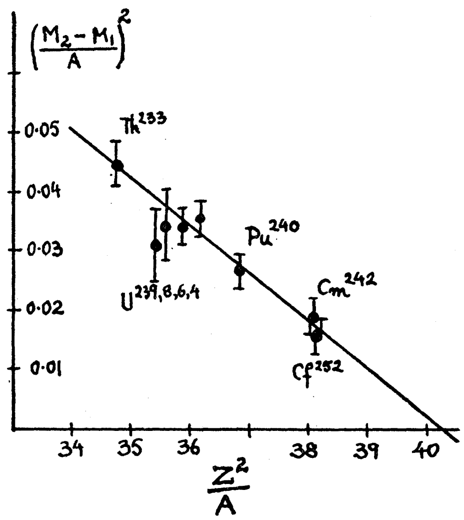

FIG. 1. The square of the relative degree of asymmetry, $(M_2-M_1)/A$, as a function of Z^2/A.

$$\text{Asymmetry}=\pm c_2[(Z^2/A)_c-(Z^2/A)]^{\frac{1}{2}}, \tag{2}$$

and whose threshold energy lies below the threshold of the symmetric saddle point shape by an amount

$$\Delta E=c_3[(Z^2/A)_c-(Z^2/A)]^2. \tag{3}$$

Equation (2) suggests that the degree of asymmetry in nuclear fission should decrease in a characteristic manner with increasing Z^2/A. As a measure of the degree of asymmetry we have taken the distance M_2-M_1 between the peaks in the double-humped fission yield curve, and in Fig. 1 we have plotted $(M_2-M_1)^2/A^2$ against Z^2/A of the target nucleus. The asymmetry is seen to decrease with Z^2/A and the