Polarization of Protons Elastically Scattered from Nuclei*

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The polarization of 130-Mev protons elastically scattered from Be, C, Al, and Fe has been calculated by means of the optical model of the nucleus, including a spin-orbit coupling term. The results are in reasonable agreement with the measurements of Dickson, Rose, and Salter. Calculations were also carried out for proton energies of 50 Mev and 90 Mev, in order to explain the rapid decrease of the polarization as the energy is decreased below 130 Mev. By using a central potential whose real part decreases with increasing proton energy between 50 Mev and 130 Mev, reasonably good agreement was obtained for the dependence of the polarization on energy for Be and C.

I. INTRODUCTION

SEVERAL experiments[1] have shown that high-energy protons elastically scattered from nuclei have a large spin polarization. Both the magnitude and the direction[2] of the observed polarization can be accounted for by the spin-orbit coupling introduced by Mayer and Jensen for the nuclear shell model.[3] Dickson, Rose, and Salter[4] have recently measured the polarization of 130-Mev protons elastically scattered from various nuclei. These authors[4] have also shown that the polarization P for Be and C at scattering angles of 20° and 30° decreases rapidly as the proton energy is decreased below 130 Mev. The purpose of this paper is to present calculations of P for energies of 50–130 Mev, using the optical model of the nucleus.[5] It was suggested by Rose[6] that the rapid decrease of P as the energy T is decreased may be due to the increase of the magnitude of the real potential V_r with decreasing T, which was found by Taylor[7] and by Mandl and Skyrme.[8] This expectation was verified in the present calculations in which reasonable agreement has been obtained for the variation of P with energy, by assuming values for the parameters of the optical model which reproduce approximately the measured total cross sections as a function of energy. The phase shifts were obtained by means of the WKB approximation, using a Woods-Saxon[9] type of radial dependence for the central potential V, i.e., a uniform potential well near the center of the nucleus, which goes over into an exponential tail at large distances. The spin-orbit coupling U was taken proportional to $(1/r)(dV/dr)$.

II. RESULTS OF THE CALCULATIONS

The calculations of P were carried out by the same procedure as in the earlier work[10] on the polarization of 300-Mev protons[11] scattered from Al. The central potential was taken as[9]

$$V=\frac{V_r+iV_i}{1+\exp[(r-r_0)/a]},\tag{1}$$

where V_r and V_i determine the real and imaginary parts of V, respectively; r_0 is the nuclear radius and a determines the length of the exponential tail. In most of the calculations, a was taken as 0.49×10^{-13} cm, which is the value used by Woods and Saxon[9] for the scattering of 22-Mev protons by Pt. In the work of I, r_0 was taken as $1.07\times10^{-13}A^{\frac{1}{3}}$ cm. Preliminary calculations for energies of 50–130 Mev were carried out with this r_0, but it was found that the calculated total neutron cross sections[12] and absorption cross sections[13] are appreciably smaller than the experimental values for any reasonable choice of V_r and V_i. For this reason, r_0 was increased to $1.23\times10^{-13}A^{\frac{1}{3}}$ cm. It should be noted that the calculated polarization is not sensitive to the choice of r_0. With the present choice of r_0 and a, $\langle r^2\rangle^{\frac{1}{2}}$ as given by Eq. (25) of I is 2.83, 3.39, and 4.07×10^{-13} cm for C, Al, and Fe, respectively. These values are somewhat higher than $\langle r^2\rangle^{\frac{1}{2}}$

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[1] Oxley, Cartwright, and Rouvina, Phys. Rev. **93**, 806 (1954); Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **93**, 1430 (1954); Marshall, Marshall, and de Carvalho, Phys. Rev. **93**, 1431 (1954); J. M. Dickson and D. C. Salter, Nature **173**, 946 (1954); de Carvalho, Marshall, and Marshall, Phys. Rev. **96**, 1081 (1954).

[2] L. Marshall and J. Marshall, Nature **174**, 1184 (1954); Phys. Rev. **98**, 1398 (1955).

[3] E. Fermi, Nuovo cimento **11**, 407 (1954); W. Heckrotte and J. V. Lepore, Phys. Rev. **94**, 500 (1954); **95**, 1109 (1954); Snow, Sternheimer, and Yang, Phys. Rev. **94**, 1073 (1954); B. J. Malenka, Phys. Rev. **95**, 522 (1954); R. M. Sternheimer, Phys. Rev. **95**, 587 (1954).

[4] Dickson, Rose, and Salter, Proc. Phys. Soc. (London) **68**, 361 (1955).

[5] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[6] B. Rose (private communication). I would like to thank Dr. Rose for this suggestion and for showing me some data in advance of publication.

[7] T. B. Taylor, Phys. Rev. **92**, 831 (1953), and thesis, Cornell University, 1954 (unpublished).

[8] F. Mandl and T. H. R. Skyrme, Phil. Mag. **44**, 1028 (1953).

[9] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).

[10] R. M. Sternheimer, Phys. Rev. **97**, 1314 (1955). This paper will be referred to as I. In Eq. (4) of I, the contribution of $U\mathbf{l}\cdot\boldsymbol{\sigma}$ is correctly given for the plus sign ($\boldsymbol{\sigma}$ parallel to $\mathbf{l}$), but for the minus sign, $-kbl/(2T)$ should be replaced by $-kb(l+1)/(2T)$. The omission of the term 1 has a negligible effect on the results of I since the important phase shifts have l values of order 10.

[11] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **95**, 1105 (1954).

[12] A. E. Taylor and E. Wood, Phil. Mag. **44**, 95 (1953).

[13] A survey of absorption cross sections for protons of various energies is given by Millburn, Birnbaum, Crandall, and Schecter, Phys. Rev. **95**, 1268 (1954).

for the nuclear charge distribution[14,15] which is $\approx 0.95 \times 10^{-13} A^{\frac{1}{3}}$ cm. The difference can be accounted for by the finite range of the nuclear forces. The potential (1) used in the present work corresponds to an effective square well of radius $\sim 1.4 \times 10^{-13} A^{\frac{1}{3}}$ cm. This value is close to the nuclear radius used in the calculations of Taylor[7] and of Mandl and Skyrme.[8]

The calculations were carried out for Be, C, Al, and Fe. The total absorption cross section σ_a is given by Eq. (24) of I. The total neutron cross section σ_t (absorption + diffraction scattering) is obtained from the imaginary part of the forward scattering amplitude. One finds

$$\sigma_t = \frac{2\pi}{k^2} \sum_{l=0}^{l_m} \left\{ (l+1)\left[1-\exp\left(-KA\left(\frac{l+\frac{1}{2}}{k}\right)\right) \times \cos 2\delta_{l,n,r}^{+}\right] + l\left[1-\exp\left(-KA\left(\frac{l+\frac{1}{2}}{k}\right)\right) \times \cos 2\delta_{l,n,r}^{-}\right]\right\}, \quad (2)$$

where k = wave number of incident proton, K = reciprocal absorption mean free path at the center of the nucleus, A is the integral over V defined by Eq. (3) of I; $\delta_{l,n,r}^{\pm}$ is the real part of the nuclear phase shift $\delta_{l,n}^{\pm}$ for the angular momentum l; l_m is the maximum l which contributes. $\delta_{l,n}^{\pm}$ is given by Eq. (5) of I; the optical parameter k_1 for the center of the nucleus is obtained from $|V_r|$ by means of[5]

$$k_1 = k[(1+|V_r|/T)^{\frac{1}{2}} - 1]. \quad (3)$$

At $T=130$ Mev, Eq. (3) is practically equivalent to the relation $k_1 = k|V_r|/(2T)$ used in I. We note that σ_t depends on the spin-orbit coupling $U(r)\mathbf{l}\cdot\boldsymbol{\sigma}$ since it involves separately the $\delta_{l,n}^{+}$ and $\delta_{l,n}^{-}$. As explained below, $U(r)$ was taken such that its maximum value $U_{\max}$ (occurring near $r=r_0$) is 1 Mev for each element and independent of energy. σ_t and σ_a were calculated for various values of V_r and K. It was found that agreement within $\sim$20 percent for σ_t and σ_a can be obtained by assuming that $|V_r|$ decreases from 55 Mev at $T=50$ Mev to 35 Mev at $T=90$ Mev and 15 Mev at $T=130$ Mev. As previously shown by Taylor[7] and by Mandl and Skyrme,[8] this decrease is necessary in order to account for the decrease of σ_t with T. K was assumed to decrease from 4.5×10^{12} cm^{-1} at $T=50$ Mev to 4.0×10^{12} cm^{-1} at $T=90$ Mev and 3.5×10^{12} cm^{-1} at $T=130$ Mev.[16] As an indication of the agreement obtained, we note that for Al, the calculated σ_t is 730, 1090 and 1310 mb at $T=130$, 90, and 50 Mev, respectively, as compared to the experimental values[12] 770, 1090, and 1630 mb. The calculated σ_a is 450, 480, and 500 mb at $T=130$, 90, and 50 Mev, respectively, as compared to the measured values[13] 430, 470, and 500 mb. For the other elements the agreement is similar to that obtained for Al. For C, σ_t agrees with experiment within 10 percent, but the calculated σ_a is $\sim$20 percent too high. For Fe, the calculated σ_t is $\sim$10 percent too low at 130 Mev, and $\sim$20 percent too low at 50 Mev; the calculated σ_a is correct within 10 percent. We note that a small change of V_r and K would not affect significantly the behavior of the polarization.

The values of K given above may be compared with the corresponding values of $\eta\rho(0)\bar{\sigma}$, where $\rho(0)$ is the density of nucleons at the center of the nucleus, $\bar{\sigma}$ is the average of the p-p and n-p cross sections at the energy $T_n \equiv T+|V_r|$ of the proton inside the nucleus; η is a factor giving the reduction of the cross sections in the nucleus due to the Pauli principle. Following Goldberger[17] we used $\eta = 1-(7/5)(E_F/T_n)$, where E_F is the Fermi energy which was taken as 22 Mev. Assuming that the nucleon density is proportional to $1/\{1+\exp[(r-r_0)/a]\}$, one finds $\rho(0)=1.09\times10^{38}$ cm^{-3} for Al. Using the measured values of[18] $\bar{\sigma}$, one obtains $\eta\rho(0)\bar{\sigma}=4.0$, 3.8, and 3.5×10^{12} cm^{-1} for $T=50$, 90, and 130 Mev, respectively. These values are quite close to the constants K used in the calculations.

Concerning the dependence of $|V_r|$ on T, we note that at $T=0$, V_r can be determined from the requirement that the neutron scattering length be zero for lithium ($A=7$) and for vanadium ($A=50$).[19] Using the potential (1), s wave functions were obtained for various values of V_r (with $V_i=0$, $a=0.49\times10^{-13}$ cm) for both Li ($r_0=2.35\times10^{-13}$ cm) and V ($r_0=4.53\times10^{-13}$ cm). V_r was varied until an s function was found which gives zero scattering length. This wave function has one node for Li and two nodes for V. The appropriate values of V_r are -53.3 Mev for Li and -51.3 Mev for V. This result together with the choice $V_r=-55$ Mev at $T=50$ Mev indicates that $|V_r|$ may remain approximately constant for $T\leqq 50$ Mev before decreasing to the small value of $\sim$15 Mev at 130 Mev.[7,8,20]

With the above values of $|V_r|$, Eq. (3) gives $k_1=1.4\times10^{12}$ cm^{-1} at $T=130$ Mev, 3.8×10^{12} cm^{-1} at $T=90$ Mev, and 7.1×10^{12} cm^{-1} at $T=50$ Mev. These constants, together with the values of K given above, were used in evaluating P by means of Eqs. (3)–(19) of I. It may be noted that the integrals A and B of I were evaluated numerically for each nucleus, i.e., for each set of values r_0 and a. In these calculations, the Coulomb scattering was taken as that of a uniform charge distribution of radius r_0.

[14] Hofstadter, Fechter, and McIntyre, Phys. Rev. **92**, 978 (1953); D. G. Ravenhall and D. R. Yennie, Phys. Rev. **96**, 239 (1954).

[15] V. L. Fitch and J. Rainwater, Phys. Rev. **92**, 789 (1953); L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).

[16] This value of K corresponds to an imaginary potential $V_i=-18$ Mev.

[17] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).

[18] B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952), p. 347.

[19] R. K. Adair, Phys. Rev. **94**, 737 (1954); Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[20] R. Jastrow, Phys. Rev. **82**, 261 (1951).

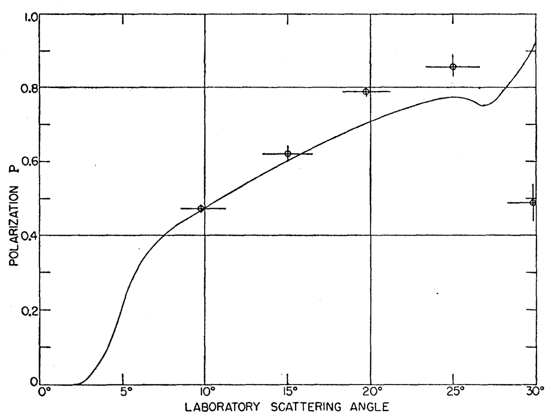

FIG. 1. Polarization P for 130-Mev protons scattered from C, calculated for the Woods-Saxon potential with $a=0.49\times10^{-13}$ cm. The circles represent the experimental values of Dickson, Rose, and Salter.[4]

The spin-orbit coupling $U(r)$ was taken proportional to $(1/r)(dV/dr)$ and is given by

$$U(r)=-\frac{b\exp[(r-r_0)/a]}{r\{1+\exp[(r-r_0)/a]\}^2}, \tag{4}$$

where b=constant. The maximum of U occurs near $r=r_0$[21]; its magnitude will be called $U_{\max}$. The parameter b was so adjusted that $U_{\max}$ is the same for each element. This value of $U_{\max}$ was obtained from the calculations of P for C at 130 Mev. Upon taking $r_0=2.81\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm, $V_r=-15$ Mev, and $K=3.5\times10^{12}$ cm^{-1}, it was found that for $U_{\max}=1.0$ Mev, one obtains reasonable agreement with the experimental points,[4] as shown in Fig. 1. The corresponding curve of P for Be at 130 Mev is not shown; the agreement obtained is about the same as for C.

Figure 2 shows the results of the calculations for Al at 130 Mev. Curve A was obtained for $r_0=3.68\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm, $V_r=-15$ Mev, $K=3.5\times10^{12}$ cm^{-1}, and $U_{\max}=1.0$ Mev. The agreement with experiment is reasonably good for scattering angles $\theta<18°$. At 23°, P has a minimum which occurs at the same angle as the diffraction minimum of the differential cross section $d\sigma/d\Omega$. Neither minimum is observed, although the slope of the measured $d\sigma/d\Omega$ *vs* θ curves[4] changes abruptly at this angle. This discrepancy is probably due in part to the limited angular resolution of the apparatus which was[4] $\pm1.5°$. As shown by Fig. 2, the calculated P drops below 0.6 only between $\theta=20°$ and 23°. In order to determine whether the minimum of P could be removed by assuming a more gradual drop of V at the nuclear boundary, P was recalculated with a larger value of a. The results of this calculation are shown as curve B. The constants used were: $r_0=3.68\times10^{-13}$ cm, $a=0.80\times10^{-13}$ cm, $V_r=-15$ Mev, $K=2.5\times10^{12}$ cm^{-1}, and $U_{\max}=0.45$ Mev. Because of the increase of a, it was necessary to use lower values of K and $U_{\max}$ than for curve A in order to fit the observed σ_a and P. It is seen that curve B does not have a minimum and is thus in better agreement near $\theta=20°$. We note that at large angles ($\gtrsim23°$) both curves A and B lie above the experimental points. There are two possible reasons for this discrepancy. (1) For $\theta\gtrsim20°$, the inelastic scattering in which the nucleus is excited to a low-lying level is probably comparable with the elastic scattering. The polarization for the inelastic scattering is expected to be smaller than for the elastic scattering. Hence if an appreciable number of slightly inelastic events are included in the measurements, this would decrease the observed values of P. (2) The present calculations become less accurate at large angles ($\theta\gtrsim25°$) because of the angular dependence of the nucleon-nucleon scattering amplitudes which is not taken into account in the optical model. It has been pointed out by Fernbach, Heckrotte, and Lepore[22] that inclusion of this effect would probably decrease the calculated polarization at large angles.

Figure 3 shows the results obtained for 130-Mev protons scattered from Fe. Curve A was calculated from potential (1) with $r_0=4.68\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm, $V_r=-15$ Mev, $K=3.5\times10^{12}$ cm^{-1}, and $U_{\max}=1.0$ Mev. This curve gives somewhat too much polarization inside the main diffraction peak. A decrease of $U_{\max}$ with increasing atomic number would be compatible with the spin-orbit coupling required for the nuclear shell model.[23] Therefore, P was also calculated

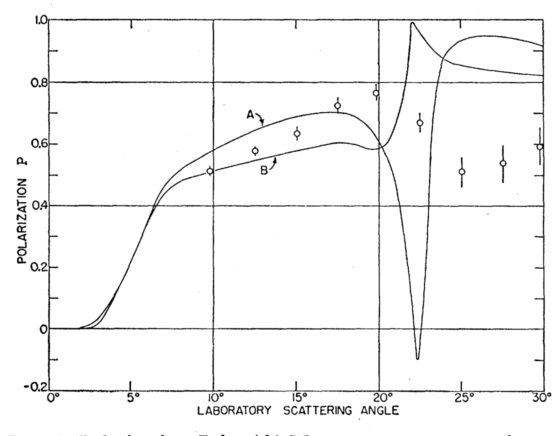

FIG. 2. Polarization P for 130-Mev protons scattered from Al, calculated for the Woods-Saxon potential. Curve A was obtained for $a=0.49\times10^{-13}$ cm. Curve B was calculated for $a=0.80\times10^{-13}$ cm. The circles represent the experimental values of Dickson, Rose, and Salter.[4]

[21] The singularity of Eq. (4) at $r=0$ due to the $1/r$ factor has no physical meaning and was effectively removed in the calculations (in the present work and in I) by assuming that U remains constant for $r\leqq0.5\times10^{-13}$ cm. This behavior arises because V does not have zero derivative at $r=0$. Due to the smallness of $\exp[(r-r_0)/a]$ in the numerator of U, the factor $1/r$ becomes effective only for $r\lesssim0.5\times10^{-13}$ cm, and in this region U was taken as constant.

[22] Fernbach, Heckrotte, and Lepore, Phys. Rev. **97**, 1059 (1955).

[23] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

for potential (1) with $U_{max}=0.75$ Mev (curve B) and 0.6 Mev (curve C); the constants r_0, a, V_r, and K were the same as for curve A. It is seen that B and C give lower polarization than A for $\theta<16°$. There is a deep minimum of P for curves A, B, and C at the angle of the first diffraction minimum $\theta=18°$. In view of the result obtained by increasing a for Al, calculations were carried out for $a=0.8\times10^{-13}$ cm and 1.0×10^{-13} cm. However, the resulting P still has a deep (negative) minimum, so that in contrast to Al, a moderate increase of a is not adequate. We note that the minimum of P is primarily due to the interference with the Coulomb scattering, since it was found that for neutron scattering from Fe with the same parameters as for case A, the curve of P *vs* θ has no minimum at 18° and is qualitatively similar to curve B in Fig. 2 for Al. The fact that the Coulomb scattering decreases the value of P at the diffraction minimum has been pointed out by Fernbach, Heckrotte, and Lepore.[22] This also explains why the effect is more pronounced for Fe than for Al.

Curve D of Fig. 3 was obtained for a Gaussian well $V=(V_r+iV_i)\exp(-r^2/r_g^2)$, with $r_g=3.83\times10^{-13}$ cm, $V_r=-35$ Mev, $K=4\times10^{12}$ cm^{-1} at the center of the nucleus, and $U=-2\exp(-r^2/r_g^2)$ Mev. These values of r_g, V_r, and K were chosen in order to fit σ_t and σ_a. It is seen that the minimum of P is higher than for curves A–C. Curve D has a negative region for $\theta<3°$, which is associated with the interference between Coulomb and nuclear scattering. This negative region was also found for cases A–C for Fe, and for C and Al (Figs. 1, 2), but the negative values of P for these cases are too small to be shown in the figures ($|P|<0.01$).

Concerning the discrepancy of P at the diffraction minimum, it is possible that a well shape different from the ones considered above would give better agreement with the observed polarization in this region. It should also be mentioned that the use of the WKB approximation may introduce some inaccuracies. In addition, part

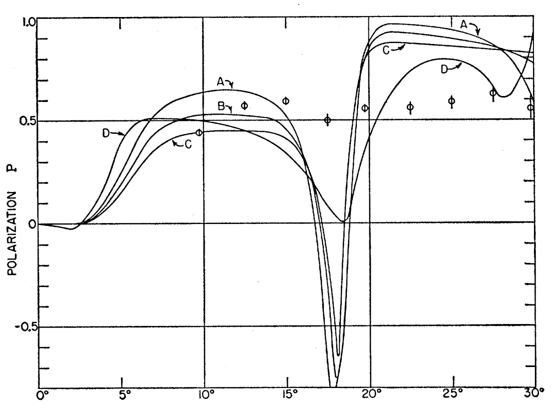

FIG. 3. Polarization P for 130-Mev protons scattered from Fe. Curves A, B, and C were calculated for $U_{max}=1$ Mev, 0.75 Mev, and 0.6 Mev, respectively, using the Woods-Saxon potential with $a=0.49\times10^{-13}$ cm. Curve D was obtained for a Gaussian well shape. The circles represent the experimental values of Dickson, Rose, and Salter.[4]

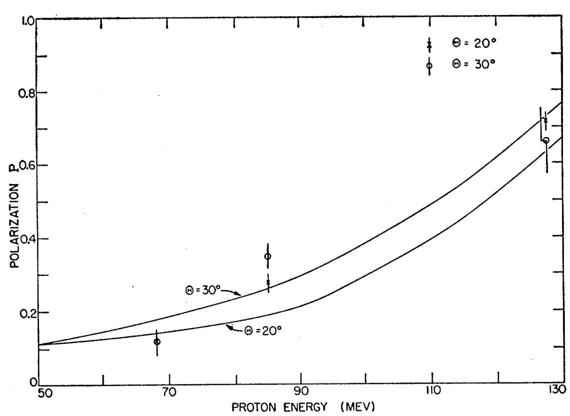

FIG. 4. Polarization P for protons scattered from Be as a function of proton energy for scattering angles $\theta=20°$ and 30°. The calculated curves were obtained for the Woods-Saxon potential with $a=0.49\times10^{-13}$ cm. The experimental values of Dickson, Rose, and Salter[4] are represented by crosses for $\theta=20°$ and by circles for $\theta=30°$.

of the discrepancy may be due to experimental difficulties such as the limited angular resolution and multiple scattering in the target. Thus if θ_s is the root-mean-square angle for multiple scattering, particles observed at an angle θ to the incident beam have undergone nuclear scattering over a range of angles extending approximately from $\theta-\theta_s$ to $\theta+\theta_s$. If θ_s is of order 2°, this effect would increase the measured values of P at the diffraction minimum.

Dickson, Rose, and Salter[4] have presented results for the differential cross sections $d\sigma^+/d\Omega$ and $d\sigma^-/d\Omega$ for scattering to the left and to the right, respectively, from the second target. Since the incident beam was scattered to the left and its polarization is 0.68, $d\sigma^\pm/d\Omega$ is given by $\langle d\sigma/d\Omega\rangle(1\pm0.68P)$ where $\langle d\sigma/d\Omega\rangle$ is the differential scattering cross section averaged over spin directions. The calculated values of $d\sigma^\pm/d\Omega$ are appreciably smaller than those given by Dickson *et al.*,[4] presumably because the latter include some inelastic scattering. In this connection, we note that the present calculations reproduce approximately the total diffraction scattering cross section for neutrons, which is given by $\sigma_t-\sigma_a$. The calculated curves of $d\sigma^\pm/d\Omega$ for the potential (1) have a noticeable diffraction minimum at an angle which is 30° for Be, 27° for C, 23° for Al, and 18° for Fe. This minimum was not observed[4] for Be and C, while for Al and Fe, there is a pronounced change of slope of the $d\sigma^\pm/d\Omega$ curves at 22° and 18°, respectively, which is due to the diffraction effect. We note that for Al and Fe the calculated angle agrees with the data. Dickson *et al.*[4] have suggested that for Be and C there is, in fact, no diffraction minimum, whereas for Al and heavier nuclei the elastic cross section has maxima and minima which are partly smeared out by the angular resolution, multiple scattering in the target, and the presence of some inelastic events. It should be noted that the depth of the calculated minimum of $d\sigma^\pm/d\Omega$ depends rather

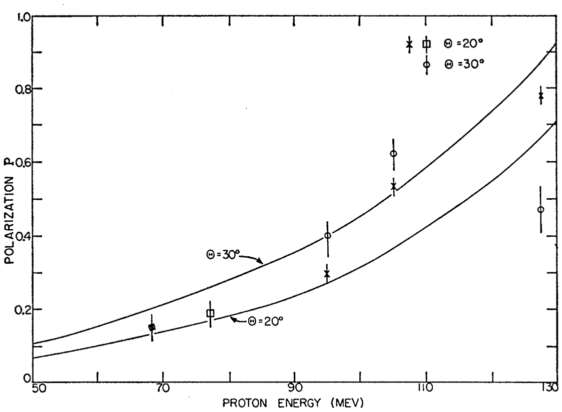

FIG. 5. Polarization P for protons scattered from C as a function of proton energy for scattering angles $\theta=20°$ and $30°$. The calculated curves were obtained for the Woods-Saxon potential with $a=0.49\times10^{-13}$ cm. The experimental values of Dickson, Rose, and Salter[4] are represented by crosses for $\theta=20°$ and by circles for $\theta=30°$. The square shows the value obtained by Strauch[25] at $\theta=20°$.

strongly on the shape of the nuclear potential well. Thus for the Gaussian well used for Fe (see curve D in Fig. 3.) the calculated curve of $d\sigma^-/d\Omega$ decreases uniformly with θ, while $d\sigma^+/d\Omega$ has only a shallow minimum at $\theta=19°$.

Figures 4 and 5 show the polarization as a function of proton energy T for Be and C at $\theta=20°$ and $30°$. The calculations were done for $T=50$, 90, and 130 Mev, using potential (1) and the spin-orbit coupling (4) with $U_{\max}=1.0$ Mev. For Be, we took $r_0=2.56\times10^{-13}$ cm and $a=0.49\times10^{-13}$ cm; for C: $r_0=2.81\times10^{-13}$ cm and $a=0.49\times10^{-13}$ cm. The values of V_r and K were as follows: (a) at $T=50$ Mev[24]: $V_r=-55$ Mev, $K=4.5\times10^{12}$ cm^{-1}; (b) at $T=90$ Mev: $V_r=-35$ Mev, $K=4.0\times10^{12}$ cm^{-1}; (c) at $T=130$ Mev: $V_r=-15$ Mev, $K=3.5\times10^{12}$ cm^{-1}. The experimental points are those obtained by Dickson, Rose, and Salter[4] for Be and C, and the value found by Strauch[25] for protons of average energy 77 Mev scattered from C at 20°. It is seen that the present calculations are in agreement with the rapid rise of P with increasing energy. The increase of P with T can be explained qualitatively as follows. The polarization is determined by the relative difference between the effective potentials for the two spin directions which is given by $(2l+1)U_{\max}/|V_r|$. If $U_{\max}$ remains approximately constant, while $|V_r|$ decreases, the ratio will increase, resulting in a rapid rise of the polarization.

Concerning the value $U_{\max}=1$ Mev used in the present calculations, we note that the $U_{\max}$ required for the nuclear shell model is of the same order of magnitude. An estimate of this value of $U_{\max}$ can be made from the energy levels of O^{17}. According to Adair,[26] the 5.08-Mev level of O^{17} is the $^2D_{\frac{3}{2}}$ state corresponding to the $^2D_{\frac{5}{2}}$ ground state. Although the excited state is only $\sim$50 percent pure, one obtains in this manner an estimate of 1.02 Mev for the average value of $|U(r)|$ for a $1d$ neutron in the field of O^{16}. The $1d$ wave function was calculated for the potential (1) with $V_r=-53$ Mev, $V_i=0$, $r_0=3.08\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm. By averaging $U(r)$ over the $1d$ density, one finds that $\langle|U(r)|\rangle_{1d}=1.02$ Mev corresponds to $U_{\max}=1.53$ Mev. Hence it appears that the required $U_{\max}$ for the light nuclei does not change appreciably over the range 0–130 Mev.

III. CONCLUSIONS

Optical model calculations have been carried out for the polarization of protons of energy 50–130 Mev elastically scattered from Be, C, Al, and Fe. The calculated results are in reasonable agreement with the data obtained by Dickson, Rose, and Salter[4] and by Strauch.[25] The rapid decrease of P as the proton energy is decreased below 130 Mev is attributed to the increase of the real part of the central potential with decreasing energy. The spin-orbit coupling $U(r)\mathbf{l}\cdot\boldsymbol{\sigma}$ was assumed to be concentrated on the nuclear surface. Reasonable agreement with the measured values of P inside the first diffraction peak was obtained by taking $U_{\max}=1$ Mev for the maximum U occurring on the nuclear surface. It can be concluded that a study of the polarization at low energies ($\lesssim130$ Mev) gives direct information about the real part of the central potential as well as the spin-orbit coupling.

ACKNOWLEDGMENTS

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[24] It is expected that the case of Be and C at 50 Mev is near the limit of validity of the WKB approximation. The l values which contribute extend up to $l_m=7$. However only the phase shifts for $l\leq3$ correspond to particle paths for which the closest distance from the center of the nucleus is less than r_0. Nevertheless, the present calculations probably still give reasonable results at this energy. This limitation is expected to become unimportant for $T\gtrsim90$ Mev.

[25] K. Strauch, Phys. Rev. **98**, 234 (1955).

[26] R. K. Adair, Phys. Rev. **92**, 1491 (1953).