Diffusion Chamber Study of Very Slow Mesons. III. Scattering of Negative Pions in Hydrogen*

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The elastic scattering in hydrogen of negative pions near 15 Mev is studied. In a path length represented by 8300 π-μ decays, twenty-six events were observed. At these low energies, the S-wave dominates the scattering. Treating the small P-wave and large Coulomb contributions as known, the data is found to be well represented by an S-wave phase shift proportional to momentum. Under this assumption, the phase shift parameters are given by the relation: $\alpha_3+2\alpha_1=(0.25\pm0.05)p/\mu c$ in radians.

A. INTRODUCTION

THE experimental arrangement previously described[1,2] has been employed in a search for pion-hydrogen scatterings. This investigation was motivated by the promising simplification that, at sufficiently low energy, only S-wave and Coulomb effects determine the scattering. Interest in the S-wave has been generated by ambiguities in the sign of the phase shift, by the apparently anomalous energy dependence[3] and the theoretical speculations thereby inspired,[4–6] and by the fact that the same low-energy S-wave parameters are contained in other phenomena: The Panofsky effect and the pi-mesonic x-rays.[7]

B. EXPERIMENTAL PROCEDURE

The 60-Mev π^- beam of the Nevis Cyclotron was moderated and allowed to spiral into the high-pressure hydrogen gas (density$=0.002$ g/cm^3) of the diffusion cloud chamber. The pion spectrum obtained (Table I) is a compromise between pion flux per photograph and background. The problem of obtaining a reasonable flux of very low-energy pions is complicated by the low stopping power of the gas, the enormous straggling and scattering of the moderated beam and by the geometry and magnetic field of the cloud chamber.

TABLE I. Flux determination.

Interval (Mev, lab)	2.5–5	5–10	10–15	15–20	20–25	25–30
Uncorrected number decays counted	1290	2080	2000	1470	880	580
Coulomb scattering correction	−11%	−4%	−2%	−1.1%	−8%	−0.9%
Fraction of scatterings counted using 5° cutoff	0.98	0.96	0.95	0.94	0.93	0.92
5° cutoff flux correction[a]	6.5%	8.5%	10.5%	13%	15%	16.5%
Corrected number decays counted[b]	1230	2190	2160	1620	970	640
Pion path length kilometers	2.16	5.47	7.09	6.36	4.36	3.22

[a] The number of π-μ decays which project to less than 5° is deduced from the decay kinematics.
[b] An additional very small geometric correction is included here. Also considered is the small effect of scatterings which are above the cut-off angle but which project to less than 5°.

Scanning involved the counting of $\pi\rightarrow\mu$ decays in flight, these being characterized by a deflection of $\geq 5°$ in either of the stereo views. This data together with the known lifetime and decay kinematics leads to a knowledge of the total pion path length as a function of momentum. The gas density is determined from continuous monitoring of pressure and temperature distribution in the gas. The latter data is obtained from thermistors which are embedded in the stainless walls of the chamber. Wall and gas temperatures at the same height were correlated by a run in which horizontal thermometers were suspended in the gas. The temperature at the center of the sensitive layer was -40°C. Corrections to the flux count are detailed in Table I.

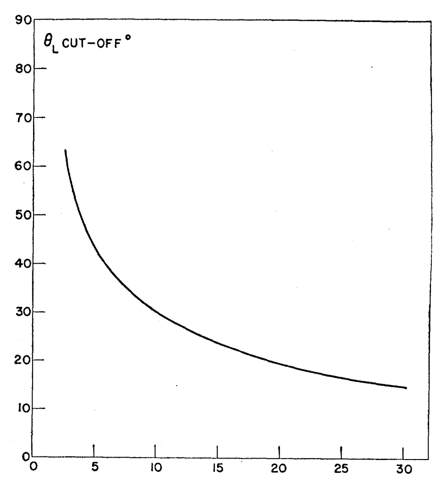

FIG. 1. Minimum acceptable scattering angle as determined from criterion that recoil proton have a range ≥ 1 mm.

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[1] Sargent, Cornelius, Rinehart, Lederman, and Rogers, Phys. Rev. 98, 1349 (1955).
[2] Sargent, Rinehart, Rogers, and Lederman, Phys. Rev. 99, 885 (1955).
[3] de Hoffmann, Metropolis, Alei, and Bethe, Phys. Rev. 95, 1586 (1954).
[4] R. Marshak, Phys. Rev. 88, 1208 (1954).
[5] H. P. Noyes and A. E. Woodruff, Phys. Rev. 94, 1401 (1954).
[6] M. Ross, Phys. Rev. 95, 1687 (1955).
[7] Deser, Goldberger, Baumann, and Thirring, Phys. Rev. 96, 773 (1954); K. A. Brueckner, Phys. Rev. 98, 769 (1955).

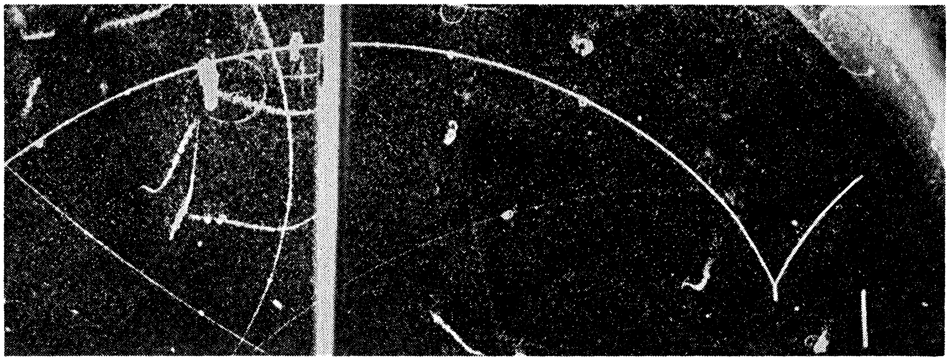

FIG. 2. Diffusion chamber photograph of a 3-Mev meson scattering in hydrogen. Both recoil proton and scattered meson come to rest in the gas.

All deflections are examined closely for correlated recoils at the vertex. Events with recoil tracks are measured and the results compared with the predictions of pion-hydrogen kinematics. The decay and scattering kinematics are not sufficiently different over the entire range of angles to distinguish a scattering event from a decay with accidental association of a blob or with a delta ray at the vertex. This circumstance forced the adoption of a minimum proton recoil length criterion of 1.0 mm. The remaining accidental coincidence rate was then negligible. Figure 1 gives the minimum scattering angle (a function of pion energy) inferred from the recoil criterion. Figure 2 illustrates a "typical" pion-hydrogen collision.

The scanning efficiency was determined by independent rescanning of the film. The scanning was considered satisfactory when the second scanning was 90% efficient in the π-μ decay count relative to the first. The greatest contribution to error in cross section arises from the counting of small-angle π-μ decays. The projected angular distribution of π-μ decay angles is a check that these errors are small compared to statistical errors.

C. EXPERIMENTAL RESULTS AND ANALYSES

The examination of approximately 45 000 photographs yielded a total pion path length of 30 km as inferred from about 8000 π-μ decays. (The total scanning time amounted to about 300 scanning days). A total of twenty-six acceptable scattering events were counted. The relevant data is presented in Table II.

The large spread in energy of the flux and of the events together with the limited statistics makes the method of maximum likelihood the appropriate tool for the most efficient utilization of the data.[8]

The π^--proton elastic scattering cross section, for sufficiently low energy, is[9]

$$\frac{d\sigma}{d\Omega}=\lambda\left[\left(\frac{e^2/hv_r}{2\sin^2(\theta/2)}+\frac{2\alpha_1+\alpha_3}{3}+\frac{2\alpha_{33}}{3}\cos\theta\right)^2+\frac{\alpha_{33}{}^2}{9}\sin^2\theta\right]. \quad (1)$$

In the maximum likelihood procedure the P-wave (α_{33}) and Coulomb contributions are treated as known functions of momentum[10] and the likelihood function is maximized to obtain the best value for the S-wave parameter, $2\alpha_1+\alpha_3$. The first assumption made is that this quantity varies linearly over the momentum interval $(0.2\leqq p/\mu c\leqq 0.6)$:

$$2\alpha_1+\alpha_3=m+q\eta \text{ (radians)}, \quad (2)$$

where $\eta=p/\mu c$.

The likelihood function is plotted in Fig. 3. The result of the statistical analysis is:

$$m\doteq 0 \text{ radian}, \; q=0.25 \text{ radian}. \quad (3)$$

Permissible variations may be read from Fig. 3.

The momentum interval was next split in an attempt to observe a change in slope between the 2.5–10 Mev interval and 10–30 Mev interval. The best fit in each region turned out to be identical to (3). However, within the very poor statistics which resulted, the data

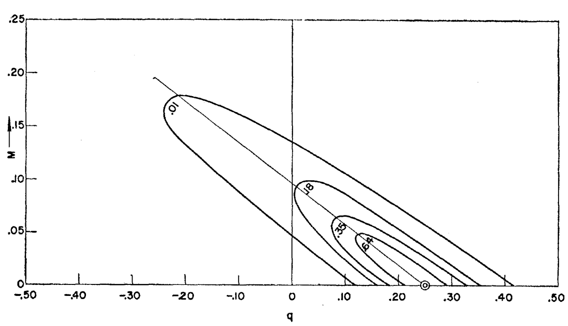

FIG. 3. Likelihood function

$$L(m,q)=\exp\left[-\int_{E_1}^{E_2}\int_{\theta_{\min}}^{\pi}N_{\theta,E}(m,q)\prod_{1}^{26}\frac{d\sigma}{d\Omega}(m,q)d\theta dE\right]$$

for S-wave scattering parameters: $\alpha_1+2\alpha_3=m+q\eta$. The likelihood is normalized to unity at the maximum value. N is the expected number of events at θ, E, using parameters m, q.

[8] F. Solmitz (unpublished mimeographed note). The authors wish to thank Dr. J. Orear for calling our attention to this note and for suggestions on the application of maximum likelihood to this problem.

[9] L. Van Hove, Phys. Rev. **88**, 1358 (1952); F. Solmitz, Phys. Rev., **94**, 1799 (1954). Here we assume that we neglect all but the largest P-wave phase shift.

[10] See reference 3 and J. Orear, Phys. Rev. **96**, 176 (1954).

could tolerate a change in slope as large from 0.30η in the high-momentum interval to 0.10η in the low-momentum interval. Under the assumption that $m=0$, a standard error may be assigned to the average best fit slope over the entire interval:

$$2\alpha_1+\alpha_3=(0.25\pm0.05)\eta, \quad (4)$$

where the error arises wholly from the statistics of 26 events. The result is not very sensitive to reasonable uncertainties in the P-wave extrapolation, assuming an initial η^3 dependence. Systematic errors due to scanning, flux count etc., are estimated to be less than 20 percent of the statistical error. The small amount of P-wave eliminates the possibility of sign ambiguity which appears at higher energies.[11]

D. DISCUSSION

In order to obtain α_1 and α_3 separately, this result must be combined with those of other experiments. These are either π^+ scattering which yields α_3 directly or π^- charge-exchange scattering which yields $\alpha_1-\alpha_3$. At present the π^+ scattering data below 30 Mev are quite preliminary.[12,13] The charge exchange scattering at 42, 30, and 20 Mev has been studied by Spry[14] and by Tinlot and Roberts.[15] We take as the best value

$$\alpha_1-\alpha_3=(0.27\pm0.03)\eta. \quad (5)$$

The combination of (4) and (5) leads to the solution:

$$\alpha_1=(+0.17\pm0.04)\eta, \quad \alpha_3=(-0.10\pm0.02)\eta \quad (6)$$

It is interesting to note that this solution is essentially that obtained by Orear[10] from an analysis which weighted data above 30 Mev most heavily. The solutions (6) for α_3 are also in agreement with the available π^+ data.[12,13] Recent experiments with pi-mesonic x-rays[16] give convincing evidence that the nuclear potential is repulsive for S-wave mesons. This has been interpreted[7] as establishing at least the sign of $(\alpha_1+2\alpha_3)$ as negative, in agreement with (6), but not in agreement with solutions which make use of these results together with the Panofsky effect, photomeson production, and detailed balancing arguments.[17]

[11] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).
[12] J. Orear, Phys. Rev. **98**, 239A (1955).
[13] S. Whetstone and D. Stork, Phys. Rev. **99**, 673(A) (1955).
[14] W. Spry, Phys. Rev. **95**, 1295 (1954).
[15] J. Tinlot and A. Roberts, Phys. Rev. **95**, 137 (1954).
[16] Stearns, Stearns, Di Benedetti, and Leipuner, Phys. Rev. **96**, 804 (1955); Camac, McGuire, Platt, and Schulte, Phys. Rev. **99**, 897 (1955).
[17] A nonlinear extrapolation to zero which would give agreement with the Panofsky effect and the condition $\alpha_1+2\alpha_3<0$ implies $d^2\alpha_1/d\eta^2>0$. For detailed objections to this behavior see H. P. Noyes, *Fifth Annual Rochester Conference on High Energy Physics* (Interscience Publishing Company, New York, 1955). A crude verification of the Panofsky ratio is obtained from reference 1 on internal pairs by assuming the validity of the theoretical branching ratios. This yields $(\pi^-+p\rightarrow\pi^0+n)/\pi^-+p\rightarrow n+\gamma)=1.0\pm0.5$, where the uncertainty is in the nature of a maximum error.

TABLE II. Pion-proton scattering events.

C.m. energy Mev	C.m. scattering angle (degrees)
2.1	81
2.1	130
3.2	79
3.3	121
4.0	72
4.8	120
5.8	52
6.4	52
6.5	41
7.4	64
7.8	42
8.1	46
8.4	95
10	131
10.5	65
11.3	57
11.5	29
11.9	94
12	39
12.7	38
13	32
13	101
18.4	109
19	43
20	24
22	124

A much more detailed study of meson scattering at even lower energies would be of great interest but will be achieved only with enormous labor, using this technique. The new liquid hydrogen bubble chamber device is an ideal instrument for this type of study and will probably account for the next advance in this domain.

ACKNOWLEDGMENTS

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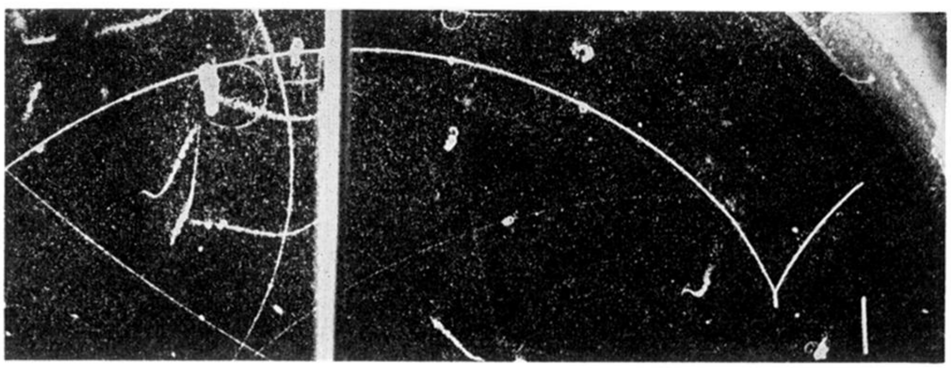
Note added in proof.—A new result for the minus-plus photo pion production ratio in deuterium was reported by G. Bernardini at the Pisa Conference on Elementary Particles, June, 1955. This raises the Panofsky effect result to $\alpha_1-\alpha_3=0.23\pm0.02\eta$ in agreement with (5).



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