R and R_H caused by fluctuations in carrier density, n, it may be shown that on the basis of a simple, single-carrier model:

$$\langle \Delta V_I^2 \rangle = V_I^2(\langle \Delta n^2 \rangle / n^2) \quad \text{and} \quad \langle \Delta V_H^2 \rangle = V_H^2(\langle \Delta n^2 \rangle / n^2), \quad (2)$$

where V_I is the dc potential drop between the probes and V_H is the usual Hall voltage. It is convenient to treat the data in this manner, since all potentials are directly measurable and the results are independent of sample dimensions and calibration of the magnetic field. The quantities V_I and V_H were determined for the sample by standard techniques.

Carrier fluctuations calculated from Eqs. (2) are shown in Fig. 2. The agreement between the results is within the experimental errors of the present data. In particular, the carrier density fluctuations as directly measured by the Hall effect have the $1/f$ spectrum and the proper order of magnitude to account for excess noise. We believe that these results constitute the first proof that excess noise is a result of carrier density fluctuation.

[1] H. C. Montgomery, Bell System Tech. J. **31**, 950 (1952).

Scattering of Neutrons by Phonons in an Aluminum Single Crystal

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IT has long been realized[1] that the normal vibrational modes of a crystal could be determined from measurements of the energy distribution of coherently scattered neutrons. The basic theory of inelastic coherent scattering is largely due to Weinstock.[2] The neutron gains or loses energy corresponding to the annihilation or production of one or more phonons in the crystal. The zero-phonon process represents Bragg scattering. If this process is avoided by selecting the angular setting of the crystal, the one-phonon process can be dominant. The partial differential cross section for production of one phonon of the fth normal mode in a monoatomic crystal is[3]

$$G_1^f = \frac{\pi^2 \hbar \sigma_{\text{coh}}}{2MBN} \cdot \frac{[(\mathbf{k}-\mathbf{k}')\cdot\boldsymbol{\alpha}_f]^2}{\omega_f(1-x_f)} \frac{k'}{k} \times \exp(-2W_f^\tau) \cdot \delta(\mathbf{k}-\mathbf{k}'-2\pi\boldsymbol{\tau}+\mathbf{q}), \quad (1)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the initial and final neutron propagation vectors, ω_f is the phonon angular frequency, $\boldsymbol{\alpha}_f$ is the phonon unit polarization vector, $\boldsymbol{\tau}$ is a vector of the reciprocal lattice, $\mathbf{q}$ is the phonon wave vector, $x_f = \exp(-\hbar\omega_f/k_BT)$, N is the number of atoms in the crystal, B is the volume per atom, and M is the mass of the atom, and $\exp(-2W_f^\tau)$ is the Debye-Waller factor. The δ function in this expression represents the momentum conservation between neutron, phonon and crystal. In addition, the neutron-phonon system must conserve energy so that[4]

$$E' = \frac{\hbar^2 k'^2}{2m} = E - \hbar\omega_f = \frac{\hbar^2 k^2}{2m} - \hbar\omega_f. \quad (2)$$

When these energy and momentum conditions are simultaneously satisfied, neutrons are scattered with an intensity given by (1).

Since $\mathbf{q}$ is restricted to the zone around each reciprocal lattice point $\boldsymbol{\tau}$, this latter can be unambiguously

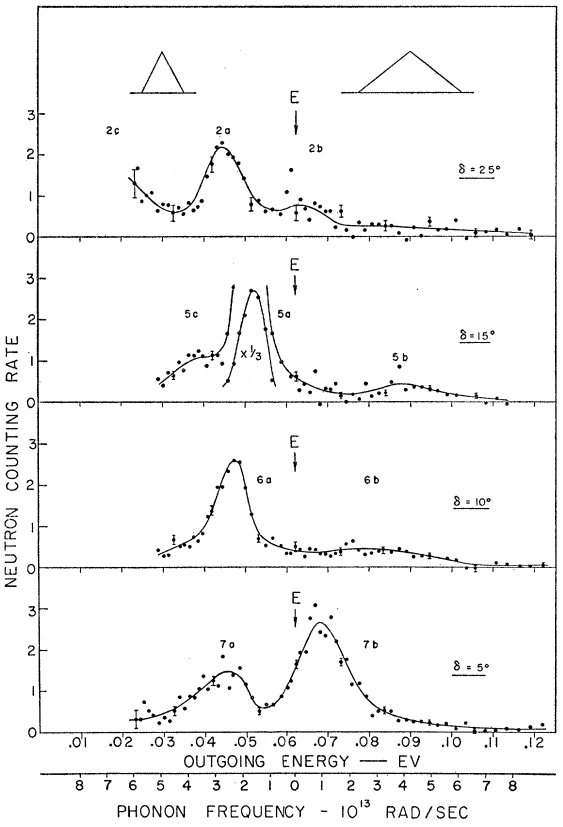

FIG. 1. Typical energy distributions of neutrons inelastically scattered by an aluminum single crystal and approximate resolution functions. The incident neutron energy E is indicated by the arrows.

assigned. Then the two conservation equations are sufficient to determine the angular frequency ω in terms of the phonon wave vector $\mathbf{q}$. For a monoatomic lattice $\omega(q)$ is expected to separate into three branches corresponding in the low-frequency limit to the longitudinal and two transverse polarizations of sound waves. In principle the directions of the polarization vectors, $\boldsymbol{\alpha}$, can be obtained by repeated observation of the same point in a zone of reciprocal space using different initial and final neutron momenta, and comparison of the intensities of the observed neutron groups with Eq. (1).

Monoenergetic neutrons provided by a crystal spectrometer (wavelength 1.14_8 A, energy 0.062_2 ev) were scattered at an angle of 95.1° by an aluminum single crystal oriented so that its (1,1,1) axis was in the plane of the spectrometer and its $(0,1,\bar{1})$ axis vertical. The energy distribution of the scattered neutrons was obtained by examination with another crystal spectrometer.[5] The angle and wavelength used satisfy the Bragg condition for the (3,3,3) reflection when the crystal is oriented with its $(\bar{1},\bar{1},\bar{1})$ direction bisecting the angle of scattering. Measurements were made with the crystal rotated about the vertical $(0,1,\bar{1})$ direction by an angle δ from the (3,3,3) Bragg scattering position, keeping the angle of scattering constant. Energy distributions have been obtained for eleven angular settings of the crystal, all at room temperature, of which a selection is shown in Fig. 1. Neutron groups are seen which represent both energy losses and gains, corresponding to one-phonon production and annihilation. The estimated resolution function is shown for two energies.

Figure 2 shows the $(0,1,\bar{1})$ plane of the reciprocal lattice of aluminum, each reciprocal lattice point surrounded by its zone. The heavy lines in the figure are used to illustrate the experiment. The initial neutron propagation vector $\mathbf{k}$ is defined in magnitude and direction by the incident energy and the orientation of the specimen crystal. The direction of the outgoing neutron of propagation vector $\mathbf{k}'$ is then determined by the angle of scattering, fixed in these experiments at 95.1°. The

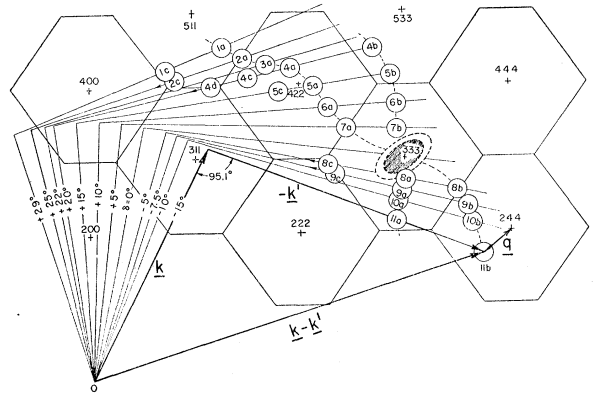

FIG. 2. Reciprocal lattice diagram of aluminum showing positions of scattered neutron groups, and approximate resolution function in reciprocal space. The position of 2*c*, 4*d*, and existence of 4*b*, 5*b*, 8*c*, and 9*c* is somewhat doubtful. One group (2*b*) is believed to be spurious and has not been entered.

magnitude of $\mathbf{k}'$ is found from the measured energy of the neutron group. The phonon vector $\mathbf{q}$ is then determined by completing $\mathbf{k}-\mathbf{k}'$ to the nearest reciprocal lattice point. The angular frequency of the phonon is obtained by Eq. (2), an auxiliary frequency scale being given in Fig. 1. An estimate of the resolution in reciprocal space is shown as the shaded area in Fig. 2. Labels have been attached to reciprocal space points corresponding to the neutron groups in Fig. 1.

In Fig. 3. the results have been plotted to show the relation between the angular frequency and wave vector of the observed phonons. This relation is a sinusoidal function for a one-dimensional lattice and

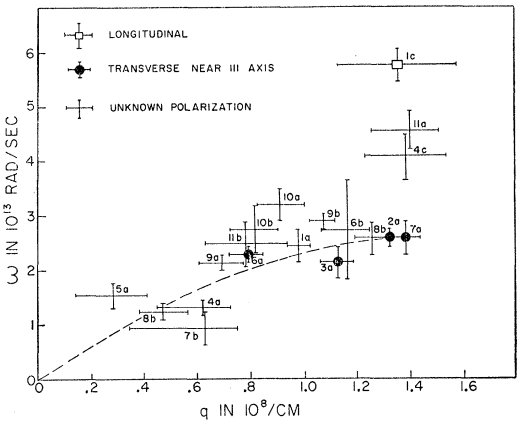

FIG. 3. Relation between ω and q for observed phonon groups, with estimated errors.

also for the transverse branches in the {1,1,1} directions of a face-centered cubic lattice.[6] The dashed curve is the sine function which has a slope at low frequencies corresponding to a velocity of sound of 3080 m/sec for transverse modes. Of the observed phonons, those which may be thought of as "transverse"[7] and having directions near the {1,1,1} axes are indicated by the black circles. Other phonons seem to be of higher frequencies. This is in accord with the results of Rosenstock[8] who showed that the (1,1,1) transverse waves have the lowest limiting frequency and that waves with other polarization or direction have limiting frequencies ranging up to about twice this value.

The experiments will be continued using this method as well as a complementary time-of-flight[9] technique.

We wish to thank Mr. G. R. DeMille for his assistance in analyzing the data and Dr. D. G. Hurst for helpful discussions.

[1] J. M. Cassels, *Progress in Nuclear Physics* (Academic Press, Inc., New York, 1950), p. 214; D. A. Kleinman, Phys. Rev. **86**, 622 (1952). The problem has been discussed in some detail by G. Placzek and L. Van Hove, Phys. Rev. **93**, 1207 (1954). An experiment of the type described below was attempted unsuccessfully by one of us (B. N. B.) in 1952.

[2] R. Weinstock, Phys. Rev. **65**, 1 (1944).

[3] See reference 2, Eq. (36).

[4] See reference 2, Eq. (39).

[5] B. N. Brockhouse, Phys. Rev. **99**, 601 (1955).

[6] G. L. Squires (private communication). This relation is valid for a crystal model with first and second nearest neighbor harmonic forces.

[7] In the low-frequency limit the polarization of the phonon can be assigned as longitudinal or transverse if the angle between $\mathbf{q}$ and $\mathbf{k}-\mathbf{k}'$ is nearly 0° or 90° respectively.

[8] H. B. Rosenstock, Phys. Rev. **97**, 290 (1955).

[9] Similar experiments using this technique are being performed by B. Jacrot at Saclay [Compt. rend. **240**, 745 (1955)] and by Carter, Hughes, Palevsky, and Zimmerman at Brookhaven [Phys. Rev. **99**, 611(A) (1955)].