

0.0437, 0.0421, and 0.0425, in the (100), (111), and (110) directions respectively, the same at both 4.2°K and 1.3°K. Its line width was narrower than any previously reported,  $\Delta H/H \approx 0.04$  between half power points at 1.3°K, corresponding to  $\tau = 3 \times 10^{-10}$  sec. In one direction, (100), a weak extra line was observed at  $m^*/m_0 = 0.057 \pm 0.002$ , down a factor of twenty from the main line. Although this line did not suffer a large change in intensity relative to the main resonance as the temperature was varied from 4.2° to 1.3°, it probably is also associated with quantum effects.

The ratio of the intensity of the hole resonances to the electron resonances has been found to depend critically on the sample, the temperature, the microwave power level, and the light intensity. This suggests that temporary hole traps are probably present. It is believed that these traps may have been responsible for earlier failure<sup>2</sup> to observe hole resonances at 1.3°K.

Because of the observed electron resonances at 1.3°K in this sample have an  $\omega\tau = 80$ , almost an order larger than has been previously reported,<sup>3</sup> it seems worthwhile reporting these measurements:  $m^*/m_0 = 0.1341$  for (100); 0.0814 and 0.206 for (111); 0.0983 and 0.361 for (110). These five measurements form a consistent determination of the two constants of the energy ellipsoid:  $m_t = 0.0813 \pm 0.002$  and  $m_l = 1.600 \pm 0.008$ . These agree with previous observations within<sup>3</sup> their expressed accuracy.

We are indebted to W. Kohn and J. M. Luttinger for their continual interest and stimulating discussions during the course of these measurements, to H. W. Dail for experimental assistance, to F. J. Morin and T. H. Geballe for the loan of crystals, and to H. R. Moore and W. H. Brattain for the design and loan of the mechanical chopper and synchronous detector.

<sup>1</sup> W. Kohn and J. M. Luttinger, *Phys. Rev.* **96**, 529 (1954); J. M. Luttinger and W. Kohn, *Phys. Rev.* **97**, 869 (1955).

<sup>2</sup> Fletcher, Merritt, and Yager, *Phys. Rev.* **98**, 1560(A) (1955).

<sup>3</sup> Dresselhaus, Kip, and Kittel, *Phys. Rev.* **98**, 368 (1955); Lax, Zeiger, and Dexter, *Physica* **20**, 818 (1954).

### Cyclotron Resonance in Metals: Bismuth

J. K. GALT, W. A. YAGER, F. R. MERRITT, B. B. CETLIN,  
AND H. W. DAIL, JR.

*Bell Telephone Laboratories, Inc., Murray Hill, New Jersey*

(Received August 31, 1955)

WE have observed the cyclotron resonance<sup>1</sup> behavior of metallic bismuth at 24 000 Mc/sec under conditions such that the steady magnetic field was along the trigonal axis. So far as we are aware, this is the first time such observations have been made successfully on a metal. The (00·1) face of a single crystal of bismuth was made to act as part of one wall of a microwave cavity, and our data consist of observa-

tions of the variation of energy absorbed in the cavity at resonance as a function of the dc magnetic field applied normal to this wall. Considerable care and effort were required to obtain clean, strain-free (00·1) surfaces of bismuth.

The experimental arrangement is diagrammed in Fig. 1. A degenerate circular cavity of coin silver in the

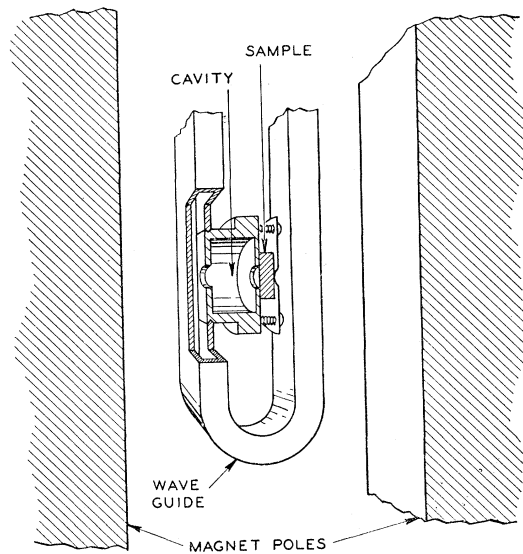


FIG. 1. Diagram of experimental geometry for cyclotron resonance experiments in bismuth. The cavity can be moved back and forth across the broad face of the wave guide so that the coupling window accepts the best approximation to circular polarization. The magnet pole faces are oriented parallel to the ends of the cavity. The sample covers a hole in one end of the cavity and thus forms part of the cavity wall at the center of this end.

shape of a pillbox was excited from one end through a coupling window in such a way as to produce a circularly polarized field near the axis of the cavity. The excitation of circular polarization was achieved by proper placement of the coupling window on the broad face of a rectangular wave guide, and by the adjustment of dielectric vanes inside the cavity. The bismuth sample covered a hole in the wall at the end opposite the coupling window, thus forming the cavity wall in an area where circularly polarized radiation impinges upon it. A sample of calcium copper acetate hexahydrate was in the cavity near the bismuth at all times. It was possible to establish the degree of circularity of our polarization by observing the ratio of the intensities of paramagnetic resonance in this salt when observed with the dc magnetic field in opposite directions. A ratio of at least 10 to 1 between the intensities of the two circular polarizations was maintained in taking our data.

The power absorption coefficient of our best sample of bismuth varied with the dc magnetic field as shown in Fig. 2 at 4.2°K. The direction of increased power absorption was established by observation of the para-

magnetic resonance in the calcium copper acetate hexahydrate sample in the cavity. The scale of  $H$  in Fig. 2 extends from  $10^4$  gauss in one direction to  $10^4$  gauss in the opposite direction. There are clearly two contributions to the absorption. One is a broad increase in absorption as the field becomes more positive, which has been dotted in the center of the figure. The other is a resonance line of more conventional shape, which however *reduces* the power absorption coefficient in the bismuth at its maximum. We associate this line with cyclotron resonance, since the increase in conductivity associated with such a resonance *increases* the  $Q$  of the cavity. We emphasize that careful determination of the shape of this line shows that it is not symmetrical about zero field. Furthermore, there is evidence that at least two lines are present (4 points of inflection). The shape of the curve in this low field region is not symmetrical, or even the sum of two simple symmetrical lines. In fact it seems to vary considerably from sample to sample, presumably because of a variation in strain.

We feel that these results can be interpreted qualitatively in terms of the plane wave analysis developed by Anderson.<sup>2</sup> The broad variation in the absorption coefficient is of the form expected from the interaction of the electromagnetic field and the majority carriers. We are able in bismuth to see an unusually large part of this curve because the majority carriers here have a rather small mass. The resonance line of more conven-

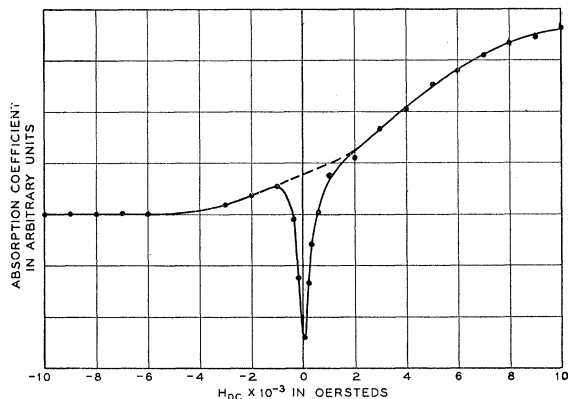


FIG. 2. Absorption coefficient *vs* dc magnetic field for circularly polarized radiation incident on the (00·1) plane of bismuth at 4.2°K. These data were taken at 24 000 Mc/sec. The vertical scale is only approximately linear. The magnetic field is normal to the (00·1) plane. Zero absorption is somewhere below the axis of abscissas.

tional shape we associate with a group of minority carriers which are screened from close coupling with the electromagnetic field by the majority carriers. Our best estimate from a comparison of our data at 4.2°K with Anderson's results is that the majority carriers are holes of mass approximately  $0.3 m_0$  and that the center of the line associated with minority carriers in Fig. 2 corresponds to holes of mass approximately  $m=0.0015$

$m_0$ , where  $m_0$  is the mass of a free electron. These masses agree rather well with two of those quoted by Shoenberg.<sup>3</sup>

The authors wish to thank R. E. Enz for aid in growing the bismuth crystals used in this research.

<sup>1</sup> J. Dorfmann, Doklady Akad. Nauk (S.S.S.R.) **81**, 765 (1951); R. B. Dingle, "Proceedings of the international conference on very low temperatures," edited by R. Bowers (Oxford, England, 1951); R. B. Dingle, Proc. Roy. Soc. (London) **A212**, 38 (1952); W. Shockley, Phys. Rev. **90**, 491 (1953); Dresselhaus, Kip, and Kittel, Phys. Rev. **92**, 827 (1953); Lax, Zeiger, Dexter, and Rosenblum, Phys. Rev. **93**, 1418 (1954); Dexter, Zeiger, and Lax, Phys. Rev. **95**, 557 (1954); Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

<sup>2</sup> P. W. Anderson, following Letter [Phys. Rev. **100**, 749 (1955)]. See also Dresselhaus, Kip, and Kittel, Phys. Rev. **100**, 618 (1955).

<sup>3</sup> D. Shoenberg, Trans. Roy. Soc. (London) **A245**, No. 891, 1 (1952).

## Electromagnetic Theory of Cyclotron Resonance in Metals

P. W. ANDERSON

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received August 31, 1955)

IN connection with experiments of Galt *et al.*<sup>1</sup> on cyclotron resonance in Bi, we have worked out the theory of the electromagnetic ("depolarizing") effects in cyclotron resonance in metals. For this purpose we confined ourselves to the simplest possible model for the electrons; that is, an isotropic effective mass and a single, classical mean free time. Under these circumstances one can define a conductivity  $\sigma$  of a group of electrons of mass  $m$  and mean free time  $\tau$ , circularly polarized with angular frequency  $\omega$  in a plane perpendicular to a steady magnetic field  $H$ :

$$\sigma = \frac{ne^2\tau}{m[1 + i(\omega_c - \omega)\tau]}, \quad (1)$$

where

$$\omega_c = -eH/mc. \quad (2)$$

We considered two possible measuring setups. First, we took the case of circularly polarized plane radiation incident on a sample which is perpendicular to the magnetic field. This we call the "transverse circular" case. In this case it can be shown to be legitimate to insert (1) into the usual skin-effect loss equation and we get

$$\text{Absorption coefficient} \propto \text{Re}\{[i + (\omega_c - \omega)\tau]^{-1}\}. \quad (3)$$

This is plotted in Fig. 1. It can be shown that in the dissipationless metal ( $\tau \rightarrow \infty$ ) the scale of Fig. 1 is so enlarged that the cyclotron resonance point is a transition point between a "stop" band and a "pass" band. This perhaps clarifies the peculiar shape of Fig. 1.

If, as seems to be often true in metals showing marked De Haas-van Alphen effects,<sup>2</sup> there is a minority group of electrons with markedly different  $m$  and  $\tau$