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Faraday Effect in Germanium at Room Temperature*†

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When a plane polarized electromagnetic wave passes through a semiconductor and a static magnetic field is applied along the direction of propagation, there occurs a rotation of the plane of polarization and the transmitted radiation is found to be elliptically polarized. This effect is due to the influence of the free charge carriers in the semiconductor and has been analyzed using the Drude-Zener model. For small losses, weak magnetic fields, and small values of $\omega\tau$ (assuming the relaxation time τ to be energy independent) the angle of rotation of the plane of polarization can be expressed to a first order of approximation by the simple formula:

$$\theta=\tfrac{1}{2}(\mu_0/\epsilon_0)^{\frac{1}{2}}(\sigma_0\mu B/\sqrt{K'})t,$$

where μ is the Hall mobility, σ_0 is the dc conductivity, B is the magnetic field, t is the thickness of sample traversed, K' is the dielectric constant of the material at the frequency employed in the experiments, and ϵ_0 and μ_0 are the dielectric constant and the permeability of free space respectively. For spherical energy surfaces, the degree of ellipticity, which is a second-order effect, can be expressed by the relation $\mathcal{E}=(\mu_0/\epsilon_0)^{\frac{1}{2}}[\sigma_0(\mu B)(\omega\tau)/\sqrt{K'}]t$ where $\omega/2\pi$ is the frequency and τ is the relaxation time. Thus, for small losses the ellipticity is proportional to the latter quantity. For the case of low frequencies, the effect can be explained by the introduction of a Hall-effect type field into Maxwell's equations. In general, the angle of rotation and the ellipticity may depend on the direction of the applied magnetic field because of the non-spherical nature of the energy surfaces.

Room-temperature measurements of the angle of rotation at microwave frequencies on both n- and p-type samples of germanium gave values of 3780 cm^2/volt-sec and 3300 cm^2/volt-sec for the electron and hole mobilities respectively. The method should be applicable to the determination of mobilities in powdered samples without using electrodes, if the field inside the powdered particles is determined by a Clausius-Mosotti type approximation.

I. INTRODUCTION

THE rotation of the plane of polarization of a plane polarized electromagnetic wave of optical frequencies passing through a substance under the influence of a static magnetic field along the direction of propagation is known as the Faraday effect and has been extensively investigated, both experimentally and theoretically.[1] The effect can be explained by considering the influence of the magnetic field on the equations of motion of the bound electrons which gives rise to different velocities of propagation for the right- and left-handed circularly polarized components of the plane polarized wave. The effect has also been observed in ferromagnetic materials at microwave frequencies.[2] In this case the mechanism is the influence of the precession of the electron spin. Ferromagnetic materials with low loss (ferrites) have found use in microwave circuits.[3]

When a plane-polarized electromagnetic wave passes through a semiconductor and a static magnetic field is applied along the direction of propagation, the plane of polarization is found to rotate and the transmitted radiation becomes elliptically polarized. In this case the effect is caused by the free charge carriers and the mechanism is similar to that producing the Hall effect. Faraday rotation in artificial dielectrics has been predicted by Wicher[4] from an analysis in terms of the Hall field. Using large magnetic fields (~16 kilo-oersted) the Faraday rotation in germanium has been investigated at microwave frequencies by Suhl and Pearson.[5] Their work was done at low temperatures (77°K) where the collision frequency of the free charge carriers is small compared to the signal frequency. Estimates of the effective masses of these carriers could be computed from their measurements.

The present work is concerned with the investigation of the Faraday effect in semiconductors at room temperature (where the collision frequency is greater than the signal frequency) with relatively small magnetic fields, in order to establish what information can be gained from such a simple experiment. The angle of rotation of the plane of polarization of microwaves traversing thin (*ca* 4 mm) samples of n- and p-type germanium inserted in a circular guide (TE_{11} mode) has been measured. For magnetic fields of 1400 gauss the angle of rotation was about 3 degrees. A simple analysis in terms of a dielectric medium having a concentration of free charge carriers characterized by a mobility and an isotropic effective mass (Drude-Zener model) shows, that the angle of rotation depends on the dc conductivity and the mobility of the charge carriers, as well as on the static dielectric constant, the magnetic field and the sample thickness. Using known values for the static dielectric constant and the dc conductivity, the mobility of the charge carriers can be calculated from our experiments. The relaxation time or the effective mass of the charge carriers does not affect the angle of rotation to a first order of approximation. A knowledge of an accurate value of the relaxation time is, therefore, unnecessary in order to calculate the mobility to great accuracy. If the approximation of

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[1] See for instance: W. Schütz, *Handbuch der Experimentalphysik* (Akademische Verlagsgeschellschaft MBH, Leipzig, 1936), pp. 1–199.

[2] C. Hogan, Revs. Modern Phys. **25**, 253 (1953).

[3] C. Hogan, Bell System Tech. J. **31**, 1 (1952).

[4] E. Wicher, J. Appl. Phys. **22**, 1327 (1951).

[5] H. Suhl and G. L. Pearson, Phys. Rev. **92**, 858 (1953).

low losses is made, the above analysis predicts a degree of ellipticity of the transmitted radiation which depends linearly on the relaxation time. Both effects are complicated by the anisotropy of the effective masses of the charge carriers, especially when the applied magnetic field is large.

II. ANALYSIS OF THE MECHANISM BY THE DRUDE-ZENER THEORY[6,7]

The Faraday effect in semiconductors can be readily analyzed in terms of the classical Drude-Zener model. The equations of motion of the electrons or holes under the influence of the static magnetic field and the microwave electromagnetic field are identical in this analysis to those of importance in the classical theory of cyclotron resonance in semiconductors, as developed, for instance, in the publications of Dresselhaus, Kip, and Kittel[8] and Lax, Zeiger, and Dexter.[9]

The effect will first be analyzed for a model of simple spherical energy surfaces. Consider the propagation of plane electromagnetic waves in a medium of dielectric constant ϵ_{st}'(real) containing N free charge carriers per unit volume, each with a charge q, a single effective mass m, and a relaxation time τ[10] subjected to a static magnetic field B_z in the direction of propagation. The charge carriers obey the following equation of motion:

$$m\frac{d\mathbf{v}}{dt}+\frac{m}{\tau}\mathbf{v}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B}), \tag{1}$$

where $\mathbf{v}$ is the drift velocity of the charge carriers.

If the motion of the charge carriers, as well as that of the electromagnetic field is confined to the (xy) plane (the static magnetic field being applied in the z direction) we have, assuming isotropic scattering:

$$m\dot{v}_\pm+\frac{m}{\tau}v_\pm=qE_\pm\mp iqB_z, \tag{2}$$

where

$$\begin{aligned} v_\pm&=v_x\pm iv_y,\\ E_\pm&=E_x\pm iE_y. \end{aligned} \tag{3}$$

For steady-state solutions under harmonic electric fields the total current density including contributions from these charge carriers and the displacement current is

$$J_{\text{total}\pm}=\left[\epsilon_{st}'-\frac{Nq^2/\omega(m\omega\pm qB_z)}{(m/\tau)^2+(m\omega\pm qB_z)^2}\right]\dot{E}_\pm+\frac{(Nq^2m/\tau)E_\pm}{(m/\tau)^2+(m\omega\pm qB_z)^2}, \tag{4}$$

where ω is the angular frequency of the impressed radiation. Thus, corresponding to the two values E_+ and E_- for the electric field, there are two values for the effective dielectric constant and conductivity respectively:

$$\begin{aligned} \epsilon_\pm'&=\epsilon_{st}'-\frac{\sigma_0/\omega(\omega\tau\pm\mu B_z)}{1+(\omega\tau\pm\mu B_z)^2},\\ \sigma_\pm&=\frac{\sigma_0}{1+(\omega\tau\pm\mu B_z)^2}, \end{aligned} \tag{5}$$

where $\sigma_0=Nq^2\tau/m$ is the dc conductivity and $\mu=q\tau/m$ is the mobility. The fields E_+ and E_- are solutions of Maxwell's equations in a nonmagnetic medium and for plane wave solutions propagating in the $+z$ direction, we have

$$k_\pm^2=\mu_0(\epsilon_\pm'\omega^2-\sigma_\pm\omega). \tag{6}$$

Here μ_0 is the permeability of free space and $k_\pm=\alpha_\pm+i\beta_\pm$ are the complex propagation constants corresponding to right- and left-handed circularly polarized light respectively. The difference between these propagation constants results in the production of elliptically polarized waves from plane polarized waves with the major axis of the ellipse rotated with respect to the original plane of polarization. It can be seen that for either a reversal of the direction of the magnetic field or a reversal of the sign of the carriers, the two propagation vectors are interchanged.

The angle of rotation θ and the degree of ellipticity $\mathcal{E}$ of the transmitted radiation can be expressed in terms of the real and imaginary parts of the propagation constants.[11] For small thicknesses t,

$$\theta=\tfrac{1}{2}(\alpha_--\alpha_+)t, \tag{7}$$

and

$$\mathcal{E}=\tfrac{1}{2}(\beta_+-\beta_-)t. \tag{8}$$

Hence, the angle of rotation and the degree of ellipticity are very nearly proportional to the sample thickness t (for small t) and the phenomenon can, therefore, be described as a type of "Faraday effect."

It is shown in treatises on electromagnetic theory[12] that

$$\alpha_\pm=\omega(\epsilon_0\mu_0)^{\frac{1}{2}}\left\{\frac{1}{2}\frac{\epsilon_\pm'}{\epsilon_0}\left[\left(1+\frac{\epsilon_\pm''^2}{\epsilon_\pm'^2}\right)^{\frac{1}{2}}+1\right]\right\}^{\frac{1}{2}} \tag{9}$$

and

$$-\beta_\pm=\omega(\epsilon_0\mu_0)^{\frac{1}{2}}\left\{\frac{1}{2}\frac{\epsilon_\pm'}{\epsilon_0}\left[\left(1+\frac{\epsilon_\pm''^2}{\epsilon_\pm'^2}\right)^{\frac{1}{2}}-1\right]\right\}^{\frac{1}{2}}, \tag{10}$$

where $\epsilon_\pm''=\sigma_\pm/\omega$.

From these two equations the angle of rotation and the ellipticity can be calculated using Eqs. (7) and (8) and values of $\epsilon_\pm'$ and $\epsilon_\pm''$ from Eq. (5). Although the mobility and the relaxation time appear explicitly in

[6] Some the the equations derived in this section are in essence identical to those obtained independently by Suhl and Pearson.[7] The authors are very grateful to Dr. Pearson for submitting to them a copy of his paper delivered before the meeting of the American Physical Society at Rochester, New York, June, 1953.

[7] H. Suhl and G. L. Pearson (private communication).

[8] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

[9] Lax, Zeiger, and Dexter, Physica **20**, 818 (1954).

[10] T. S. Benedict and W. Shockley, Phys. Rev. **89**, 1152 (1953); Phys. Rev. **91**, 1565 (1953).

[11] See for instance: K. Försterling, *Lehrbuch der Optik* (S. Hirzel, Leipzig, 1928), p. 44.

[12] See for instance: A. von Hippel, *Dielectrics and Waves* (John Wiley and Sons, Inc., New York, 1954), p. 28.

Eq. (5) it is possible to show that to a first approximation, the angle of rotation is independent of the relaxation time and hence a value for the mobility can be obtained from a measurement of the angle of rotation without an accurate knowledge of the relaxation time.

As a first approximation, we assume that the conductivity is independent of the frequency so that

$$\epsilon_\pm''=\sigma_0/\omega.$$

Expressing the dielectric constant in the form

$$\epsilon_\pm'=\epsilon_{st}'-(\Delta\epsilon)_\pm,$$

where

$$(\Delta\epsilon)_\pm=\frac{\sigma_0/\omega(\omega\tau\pm\mu B_z)}{1+(\omega\tau\pm\mu B_z)^2},$$

we can write Eq. (9) as

$$\alpha_\pm^2=\frac{\omega^2\mu_0}{2}\left\{(\epsilon_{st}'-(\Delta\epsilon)_\pm)\left[1+\left(\frac{\sigma_0/\omega}{\epsilon_{st}'-(\Delta\epsilon)_\pm}\right)^2\right]^{\frac{1}{2}}+\epsilon_{st}'-(\Delta\epsilon)_\pm\right\}. \quad (11)$$

If we neglect terms involving $(\Delta\epsilon)_\pm^2$, obvious reductions give

$$\alpha_-^2-\alpha_+^2=\frac{\omega^2\mu_0}{2}\chi[(\Delta\epsilon)_+-(\Delta\epsilon)_-], \quad (12)$$

where

$$\chi=\chi\left(\frac{\sigma_0}{\omega},\epsilon_{st}'\right)=\left[1+\left(\frac{\sigma_0/\omega}{\epsilon_{st}'}\right)^2\right]^{\frac{1}{2}}+1-\frac{1}{1+\left(\dfrac{\omega\epsilon_{st}'}{\sigma_0}\right)^2}.$$

Now

$$\alpha_--\alpha_+\cong(\alpha_-^2-\alpha_+^2)/2\alpha_{\text{Av}} \quad (13)$$

with

$$\alpha_{\text{Av}}=\omega(\tfrac{1}{2}\epsilon_0\mu_0)^{\frac{1}{2}}[K_{st}'\eta-(\Delta\epsilon)_{\text{Av}}\chi]^{\frac{1}{2}},$$

where

$$K_{st}'=\epsilon_{st}'/\epsilon_0,\quad (\Delta\epsilon)_{\text{Av}}=\sigma\tau/\epsilon_0,$$

and

$$\eta=\left[1+\left(\frac{\sigma_0/\omega}{\epsilon_{st}'}\right)^2\right]^{\frac{1}{2}}+1.$$

For weak magnetic fields and small values of $\omega\tau$ (μB_z and $\omega\tau\ll1$), we obtain for the angle of rotation, using Eqs. (5), (7), (12), and (13)

$$\theta=\frac{\chi}{2\sqrt{2}}\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{\sigma_0\mu B_z t}{[\eta K_{st}'-\chi(\Delta\epsilon)_{\text{Av}}]^{\frac{1}{2}}}. \quad (14)$$

For most semiconductors, even at microwave frequencies, $(\Delta\epsilon)_{\text{Av}}$ can be neglected and we then obtain

$$\theta=\frac{\chi}{2\sqrt{2}}\left(\frac{\mu_0}{\epsilon_0\eta}\right)^{\frac{1}{2}}\frac{\sigma_0\mu B_z t}{\sqrt{K_{st}'}}. \quad (15)$$

At microwave frequencies for most semiconductors $\sigma_0/\omega\epsilon_{st}'\simeq1$ so that, as a rough estimate

$$\theta\simeq0.44\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{\sigma_0\mu B_z t}{\sqrt{K_{st}'}}. \quad (16)$$

For the case of small losses ($\sigma_0/\omega\ll\epsilon'$),

$$\theta=\frac{1}{2}\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{\sigma_0\mu B_z t}{\sqrt{K'}}, \quad (17)$$

where K' is the dielectric constant of the material at the frequency employed. In general, values of sufficient accuracy for the angle of rotation can be obtained from Eq. (15). If greater accuracy is desired a rough estimate of the relaxation time τ (i.e., $(\Delta\epsilon)_{\text{Av}}$) is sufficient to obtain very accurate values for the mobility from measurements of the angle of rotation θ, using Eq. (14).

The degree of ellipticity is a second-order effect and cannot be readily approximated in the general case. For $\sigma_0/\omega\ll\epsilon'$, however, we can approximately set [see Eq. (10)]

$$\beta_+-\beta_-=\frac{1}{2}\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\left\{\frac{\sigma_+}{\sqrt{K_+'}}-\frac{\sigma_-}{\sqrt{K_-'}}\right\}. \quad 18)$$

Again letting $\sqrt{K_+'}=\sqrt{K_-'}=\sqrt{K'}$ in the denominator we have for the degree of ellipticity

$$\mathcal{E}=\frac{1}{4}\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{1}{\sqrt{K'}}(\sigma_+-\sigma_-)t, \quad (19)$$

which with the aid of Eq. (5) reduces to (if $\omega\tau$ and $\mu B_z\ll1$)

$$\mathcal{E}=\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{\sigma_0(\mu B_z)(\omega\tau)}{\sqrt{K'}}t. \quad (20)$$

Hence, for small losses the ellipticity depends linearly on the relaxation time.

If $\epsilon''>\epsilon'$ it is still possible to determine the relaxation time from a measurement of the ellipticity if the mobility has been previously determined from a measurement of the angle of rotation. It can also be shown that the angle of rotation reverses when the sign of the charge carriers or the direction of the magnetic field is reversed.

The whole derivation is based on the assumption that τ is energy independent. For more complicated cases where the relaxation time is a sensitive function of the velocity, the appropriate averages must be calculated in a manner similar to that proposed by Goldey and Brown[13] and Herring.[14]

Because of the small values of the quantities μB_z and $\omega\tau$ employed here, the cyclotron resonance condition $\omega\tau=\mu B_z$ cannot be expected to lead to any sharp maxima in the rotation *vs* B curve discussed by Suhl and Pearson.[5] The derivation also assumes the existence

[13] J. M. Goldey and S. C. Brown, Phys. Rev. **98**, 1761 (1955).
[14] C. Herring, Bell Syst. Tech. J. **34**, 237 (1955).

of only one effective carrier mass while in germanium the effective mass is anisotropic and more complex models of the energy surfaces apply in this case. For ellipsoidal energy surfaces, the Faraday rotation was discussed by Suhl and Pearson[7] and the equations developed by Lax and Roth[15] can also be applied to this problem.

In this case, the conductivity tensor in the coordinate system of each of the ellipsoids (in n-type germanium, 8 ellipsoids in the [111] direction) must be transformed to a coordinate system defined by the applied magnetic field and the transformed tensors for each of the ellipsoids added and divided by the number of ellipsoids, to obtain the conductivity tensor relevant to this problem. From this tensor a complex effective conductivity $[(\sigma_{\text{eff}})_\pm]$ can be calculated, which determines the propagation constants for right-handed and left-handed circularly polarized waves by the equation

$$k_\pm^2=\mu_0\omega^2\epsilon_{st}'-i\mu_0\omega(\sigma_{\text{eff}})_\pm.$$

The real part of $[(\sigma_{\text{eff}})_\pm]$ then corresponds to the conductivities $\sigma_\pm$ and the imaginary part to the products $\omega(\Delta\epsilon)_\pm$ which were derived for the case of spherical energy surfaces in Eq. (5). It is, therefore, simple to determine the angle of rotation of the plane of polarization and ellipticity of the transmitted radiation if the appropriate values of $(\sigma_{\text{eff}})_\pm$ have been calculated for the case under consideration.

In general, the equations for the Faraday rotation and the ellipticity for ellipsoidal energy surfaces are much more complicated than for spherical energy surfaces and are dependent on the direction of the applied magnetic field. It can be shown however, that in n-type germanium for small magnetic fields applied in the [100] or [110] directions, and small values of $\omega\tau$,[16] the relation for the Faraday rotation derived for spherical energy surfaces [see Eq. (14)] applies to a first order of approximation also to ellipsoidal energy surfaces, if the mobility μ is interpreted as the Hall mobility (assuming energy independent relaxation times τ).

In n-type germanium, where there are 8 ellipsoidal energy surfaces in the [111] directions, the effective complex conductivity for a magnetic field applied in the [100] direction parallel to the direction of propagation of the electromagnetic wave is given by the relation[15]

$$(\sigma_{\text{eff}})_\pm=\sigma_\pm+i\omega(\Delta\epsilon)_\pm =\sigma_0\left[1+i\left(\frac{m_1+2m_2}{2m_1+m_2}\right)\frac{qB}{m_2[(1/\tau)+i\omega]}\right]\Bigg/ 1+\left(\frac{m_1+2m_2}{3m_1}\right)\frac{q^2B^2}{m_2^2[(1/\tau)+i\omega]^2}, \quad (21)$$

where m_1 and m_2 are the longitudinal and transverse components of the mass tensor. It is easy to show that this equation leads to the correct values for $\sigma_\pm$ and $(\Delta\epsilon)_\pm$ given by Eq. (5), if $m_1=m_2$.

For small values of the magnetic field, so that the second term in the denominator of Eq. (21) may be neglected, and small values of $\omega\tau$

$$(\Delta\epsilon)_+-(\Delta\epsilon)_-=2\sigma_0\left[\left(\frac{m_1+2m_2}{2m_1+m_2}\right)\frac{q\tau}{m_2}\right]B \quad (22)$$

leading to an expression for the angle of rotation equivalent to that given by Eq. (14), if

$$\mu=\left(\frac{m_1+2m_2}{2m_1+m_2}\right)\frac{q\tau}{m_2}. \quad (23)$$

But, for energy independent relaxation times, the ratio of the Hall mobility μ_H to the drift mobility μ is given by the relation[14]

$$\frac{\mu_H}{\mu}=\frac{3m_1(m_1+2m_2)}{(2m_1+m_2)^2}.$$

Hence,

$$\mu_H=\frac{3m_1(m_1+2m_2)}{(2m_1+m_2)^2}\cdot\frac{q\tau(2m_1+m_2)}{3m_1m_2}=\frac{m_1+2m_2}{2m_1+m_2}\frac{q\tau}{m_2}.$$

As far as the anisotropy of the effective masses is concerned, the electron mobility in germanium obtained from measurements of the Faraday rotation with small magnetic fields in the [100] direction and low frequencies or relaxation times (small values of $\omega\tau$) is, therefore, the Hall mobility.

In the case of n-type germanium, the complex effective conductivity $(\sigma_{\text{eff}})_\pm$ was also calculated for magnetic fields applied in the [110] direction (the direction in which the field was applied in our experiments) and, to the order of approximation discussed above, Eq. (14) was found to be valid in this case also.

For the case of holes in germanium the energy surfaces were, as a first approximation, assumed to be spherical, although they should be more accurately represented by warped spheres.[9]

III. CONNECTION WITH THE HALL EFFECT AND MAXWELL'S EQUATIONS

Making the assumption that the definition of the Hall field for the dc case

$$\mathbf{E}_H=R\mathbf{B}\times\mathbf{J}=\mu\mathbf{B}\times\mathbf{E},$$

where R is the Hall constant and $\mu=R\sigma_0$ (μ being the Hall mobility) is also valid for sinusoidally varying currents at low frequencies, the results of the Drude-Zener analysis, described in the previous section can, for the case of low frequencies, also be obtained by adding Hall-type fields to Maxwell's equations as

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[15] Benjamin Lax and Laura M. Roth, Phys. Rev. **98**, 5, 549 (1955).

[16] The above approximations are valid in our experiments, where $\mu B\cong 0.06$ and $\omega\tau\cong 0.07$ (see Sec. V).

follows:

$$\nabla\times(\mathbf{E}+\mathbf{E}_H)=-\mu_0\frac{\partial\mathbf{H}}{\partial t},$$

$$\nabla\times\mathbf{H}=\sigma_0\mathbf{E}+\epsilon'\left[\frac{\partial\mathbf{E}}{\partial t}+\frac{\partial\mathbf{E}_H}{\partial t}\right].$$

For plane wave solutions of angular frequency ω, with the static magnetic field applied along the direction of propagation, the above equations lead to the following propagation constants for right- and left-handed circularly polarized light:

$$k_\pm^2=\mu_0\left\{\frac{\omega^2\epsilon'[1+(\mu B)^2]\mp\omega\sigma_0\mu B-i\omega\sigma_0}{1+(\mu B)^2}\right\}, \tag{24}$$

which are identical with the results derived from the Drude-Zener model at low frequencies ($\omega\tau\ll\mu B$).

The fact that the Hall field must not be added to the first term on the right-hand side of the second equation is because, in analogy with the dc case, the total current contains a displacement current, but no conduction current contribution from the Hall field.

By adding Hall-effect terms to Maxwell's equations Wicher[4] has predicted a Faraday rotation for artificial dielectrics. It can, however, be shown that if the dielectric constant is independent of $\mathbf{B}$ any current entirely in phase with $\mathbf{E}$ produces no rotation. If

$$\mathbf{J}=\alpha\frac{\partial\mathbf{E}}{\partial t}(\alpha\text{ real}),$$

and

$$\mathbf{E}_H=R\mathbf{B}\times\mathbf{J},$$

then

$$\nabla\times(\mathbf{E}+\mathbf{E}_H)=-\mu_0\frac{\partial\mathbf{H}}{\partial t}$$

$$\nabla\times\mathbf{H}=(\alpha+\epsilon_0)\left(\frac{\partial\mathbf{E}}{\partial t}+\frac{\partial\mathbf{E}_H}{\partial t}\right).$$

Assuming solutions identical to those giving Eq. (24)

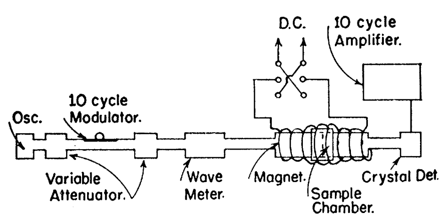
$$k_+^2=k_-^2=\omega^2\mu_0\epsilon'$$

and no Faraday rotation can be expected. The rotation predicted by Wicher[4] for artificial dielectrics would not occur if a Hall field term had been added to the left-hand side of the first of Maxwell's equations.

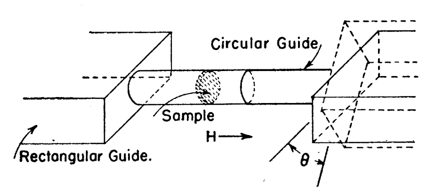
IV. EQUIPMENT AND MEASURING TECHNIQUES

A block diagram of the equipment appears in Fig. 1. The signal generator was a Klystron 723 *AB* tube and it was isolated from the rest of the circuit by two variable attenuators and one rotating attenuator, the latter modulating the power at 10 cycles per second so that a high-impedance 10-cycle amplifier (Electro-Mechanical Research Company) could be used in conjunction with the crystal detector. The wave guide was of the standard rectangular X band variety and transmitted. the standard TE_{10} mode except for the section containing the sample. This consisted of two lengths of cylindrical hollow brass pipe which fitted together inside a brass sleeve thus permitting the rotation of one of the circular guides relative to the other. With no sample in the guide the power in the rotating guide varied as $\cos^2\theta$, where θ is the angle between the principal planes of the two guides. With the sample inserted and no magnetic field applied this same $\cos^2\theta$ variation was observed except that there was a slightly larger minimum. In all cases the power ratio of maximum to minimum was 10^4 or more. The reflections from the junctions of the rectangular and circular guides should have no effect on the measured angle of rotation if they are isotropic. The $\cos^2\theta$ variation mentioned above indicates this to be the case. A solenoid slipped over the circular guide supplied the longitudinal magnetic field. The angle of rotation of the second guide relative to the first was measured by reflecting light from an attached mirror.

The sample was inserted in the nonrotating guide. It was found that the result did not depend on whether the sample made electrical contact (as determined by an ohm-meter) with the guide or not, although the spacing was not made more than about 0.5 mm. This is probably explained by the fact that the size of spacing and thickness of the sample (about 4.5 mm) makes propagation around the center impossible. The result was found to be unvaried when the sample was rotated about an axis along the direction of propagation illustrating the cubic nature of the single crystals used. An arbitrary zero ellipticity and rotation both independent of the magnetic field were introduced by inserting into the rotating guide a rectangular post of lucite inclined at an arbitrary angle relative to the original direction of polarization. This changed the original angle of the minimum by about 2.0° but the



(a)—Experimental Set-up.



(b)—Detail Drawing of Sample Chamber.

FIG. 1. Measuring equipment.

measured rotation caused by the field about this new minimum remained the same. These angles are of the order of 3.5°. When the attenuators isolating the oscillator were varied it was found that the rotation caused by the magnetic field was independent of the power delivered by the oscillator over a range of 10^2 in power. The angle was determined by taking about 6 points around the minimum and drawing a symmetric curve. The angle was found to reverse when the magnetic field was reversed. The measured angle was reproducible to within about 4 percent. The germanium utilized had a skin depth of about 3 mm so that with samples about 4.5 mm thick all problems of multiple transmission due to internal reflections and standing waves should be eliminated.

The analysis above has been made for a plane wave while the waves used in the experiment were propagating in the TE_{11} mode in the cylindrical guide. This problem is covered by Suhl and Walker,[17] but to apply their solution to the present case of lossy materials would involve complex trial and error solutions of Bessel functions of complex arguments. We here present a semiquantitative analysis. While the guide mode differs from a plane wave by having (a) a longitudinal component of the magnetic field, (b) H not constant in a transverse plane, (c) E not constant in a transverse plane, it was felt that only the absence of a uniform electric field in the transverse plane would cause any important difference between the two solutions.

A diagram of the electric field is given in Fig. 2.[18] From this figure, it appears that the actual field configuration in the guide differs from that of a plane polarized wave by the decrease of the field vector in the plane of polarization. The former does not lead to a change of the angle of rotation as the wave passes through the sample, but only to a change in the amplitude or intensity of the field component parallel to the major axis of the ellipse. Although a field component at right angles to the plane of polarization will lead to a change of the angle of rotation compared with the plane polarized incident wave, it is easily seen from symmetry considerations that the contributions of this field component are equal and opposite at equal and opposite distances from the center of the guide and will, therefore, on the average, cancel out.

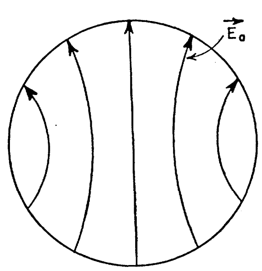


FIG. 2. Field configuration in circular guide (TE_{11} mode).

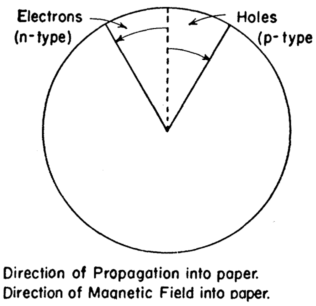


FIG. 3. Rotation of the plane of polarization for n and p type semiconductors.

We may, therefore, conclude that the plane wave solutions which we have used in our theoretical considerations are good approximations in determining the angle of rotation of the plane of polarization. The degree of ellipticity of the transmitted radiation, however, will be influenced by deviations from the assumption of strictly plane polarized incident waves.

V. RESULTS

Measurements were made on two single crystals of germanium, which were made available to us through the courtesy of the Philco Radio Corporation. The crystals were large enough to fill the entire cross section of the guide. One sample was n-type with a resistivity of 16 ohm-cm and the other p-type with a resistivity of 19.8 ohm-cm. The magnetic field was applied in the [110] direction. The observed sense of rotation was in agreement with the theoretical arguments given above. (Fig. 3.)

The experimental results for these two samples at room temperature are shown in Table I.

The effective Hall mobility was calculated from Eq. (14) using a value of 16 for the static dielectric constant K_{st}' and a value of 1.3×10^{-12} sec for the relaxation time τ. An approximate value for the effective Hall mobility can be obtained from Eq. (15) which does not contain the relaxation time. From Eq. (15) we obtain values of 3720 cm²/volt-sec and 3080 cm²/volt-sec for the effective Hall mobilities of the n- and p-type samples respectively. These values differ by only *ca* 2 percent from the more accurate values obtained from Eq. (14), showing that the dependence of the Faraday rotation on the relaxation time is not very large.

From the values for the effective Hall mobility of these two samples, it is easy to derive the Hall mobility of electrons and holes in germanium at room temperature.

An elementary analysis gives the following relations between the effective Hall mobilities $(\mu_{\text{eff}})_n$ and $(\mu_{\text{eff}})_p$ of the n- and p-type samples, respectively, obtained from our experiments, and the Hall mobilities μ_e and μ_h of

[17] H. Suhl and L. R. Walker, Phys. Rev. **86**, 122 (1952).
[18] N. Marcuvitz, *Wave Guide Handbook* (McGraw-Hill Book Company, Inc., 1951), Vol. 10, Radiation Laboratory Series, p. 71. This reference also contains an analytic expression for the field (p. 69).

TABLE I. Faraday rotation and effective Hall mobilities of n- and p-type samples of germanium at room temperature. Thickness of samples $=4.6\times10^{-3}$ meters. Wavelength $=3.434$ cm. Magnetic field $B_z=1430$ gauss.

Sample	Dc conductivity σ_0 (ohm-meter)$^{-1}$	Measured angle θ	Effective Hall mobility μ_{eff} cm²/volt-sec
n-type	6.25	$-3°28'\pm4'$	3640 ± 70
p-type	5.05	$+2°22'\pm4'$	3040 ± 85

the electrons and holes:

$$(\mu_{\rm eff})_n=\frac{\mu_e}{b}\left\{\frac{bB(n_i/n_h)^2-1}{B(n_i/n_h)^2+1}\right\},$$

$$(\mu_{\rm eff})_p=\mu_h\left\{\frac{1-(n_i/n_h)^2Bb}{1+(n_i/n_h)^2B}\right\}.$$

Here n_h is the hole concentration in the sample, b and B are the Hall and drift mobility ratios respectively, while n_i is defined by the relation $n_en_h=n_i^2$. (n_e being the electron concentration.)

The ratio n_i/n_h can be obtained from the appropriate roots of the equation

$$\frac{\rho_{n\text{ or }p}}{\rho_i}=\frac{n_i/n_h(B+1)}{1+(n_i/n_h)^2B},$$

where $\rho_{n\text{ or }p}$ are the dc resistivities of our samples and ρ_i is the intrinsic resistivity of germanium, which was taken to be 47 ohm-cm.[19]

For the drift mobility ratio a value of 2.1 was chosen[19] while the Hall mobility ratio was taken to be 1.18 in agreement with the measurements of Morin[20] as well as our own results.

The Hall mobility values of the electrons and holes in germanium at room temperature thus obtained are given in Table II and are compared with those of Morin.[20]

The errors in the mobility values as stated in Table II apply only to the precision with which the angles of rotation could be determined. The relatively good agreement between our values and those of Morin is surprising considering the rather rough nature of our experiments and the many approximation inherent in our theoretical considerations and must be considered accidental.

Measurements of the ellipticity $\mathcal{E}$ of the transmitted radiation in these samples gave values of the correct order of magnitude. The latter were, however, too small to permit an accurate determination of the relaxation time, since the small elliptical component always present in the incident radiation in the guide was of the same order of magnitude as that produced by the transmission through the sample. This background ellipticity can be caused by imperfections in the circular guide or nonuniformity of the sample. Furthermore, the difference in the power received by the detector between radiation along the major and minor axes of the ellipse was too large to be measured accurately with our equipment. To obtain a value of the relaxation time, the tests will have to be carried out at lower temperatures or with somewhat larger magnetic fields. In any case, the anisotropy of the effective masses of the charge carriers will have to be taken into account even for weak magnetic fields, since the ellipticity is a second order effect (see Sec. II). In addition, the fact that the incident radiation in the guide is not strictly plane parallel will have a far larger influence on the ellipticity than on the angle of rotation and the applicability of the equations developed in Sec. II to this problem requires careful consideration (see Sec. IV).

VI. APPLICATION TO POWDERS

The measurements described in the previous sections can also be applied to powders. This method has the attractive feature that no electrodes need to be applied to the specimen.

In order to apply the technique, it is necessary to find the field existing within the powder particle in terms of the average field in the wave guide. A procedure which can be applied to specimens in which the displacement current is large in comparison with the conduction current has been worked out by Smith.[21] He gives a formula for the average field within randomly oriented particles of different shapes in terms of the microscopic field which is averaged over both particles and the space between them.

In the case of spheres, his formulas are the same as those obtained by the Clausius-Mosotti procedure but differ markedly when particles of other shapes such as prolate spheroids are considered.

Some preliminary measurements have been made on a germanium powder obtained by crushing in a mortar and pestle some single crystal germanium of high quality. The rotation observed was much less than that obtained from the same amount of crystalline material. The reason for this is that the maximum density obtainable is only about 50 percent of the single crystal value. Germanium has a high dielectric constant and hence most of the field appears in the interstices between the particles rather than in the particles them-

TABLE II. Hall mobilities of electrons and holes in germanium at room temperature.

	Hall mobility (cm²/volt-sec) Faraday rotation measurements	Hall effect measurements (Morin)
Electrons	3780 ± 70	3900
Holes	3330 ± 85	3300

[19] L. P. Hunter, Phys. Rev. 91, 579 (1953).
[20] F. Morin, Phys. Rev. 93, 62 (1954).

[21] R. S. Smith, Thesis, University of Pennsylvania (1955). See also Tech. Rept. 12, Contract N6-onr-24914, January 1, 1955.

selves. The effect did appear to be of the right order of magnitude, but no quantitative measurements were attempted because of the unknown particle shape distribution of our crushed specimen.

Further work on powders is planned for the immediate future.

VII. DISCUSSION

A measurement of the rotation of the plane of polarization when a plane polarized electromagnetic wave passes through a nonferromagnetic semiconductor and a small magnetic field is applied along the direction of propagation yields to a first order of approximation values of the product $\sigma_0\mu/\sqrt{K_{st}'}$ where σ_0 is the dc conductivity, μ the Hall mobility of the free charge carriers and K_{st}' the static dielectric constant of the material [see Eq. (15)]. Hence, if the dc conductivity and the static dielectric constant are known, the mobility can be determined by this technique. Conversely, K_{st}' can be obtained if the dc conductivity and the Hall mobility of the sample are known. The method has the advantage over dielectric measurements in that measurements of the angle of rotation of the plane of polarization are considerably easier than measurements of standing wave ratios. Furthermore, the method should be applicable to measurements on powdered samples without electrodes. Since the angle of rotation depends only in the second order of approximation on the relaxation time through the factor $\sigma\tau/\epsilon_0$ and since this is small for almost all semiconductors, even at microwave frequencies,[22] an accurate knowledge of the relaxation time of the charge carriers is not essential in order to obtain accurate values of the Hall mobility for most semiconductors.

For weak magnetic fields (at least when these are applied in the [100] or [110] directions) and small values of $\omega\tau$, the equations governing the Faraday rotation are, to a first approximation, the same for ellipsoidal or spherical energy surfaces, and, by the use of Eq. (14), measurements of the above quantity will lead to a determination of the Hall mobility of the charge carriers. In general, the angle of rotation may depend on the direction of the applied magnetic field because of the nonspherical nature of the energy surfaces.

Although a measurement of the angle of rotation of the plane of polarization cannot be expected to give accurate information on the relaxation time, the relaxation time and also the effective masses of the charge carriers can be obtained from a measurement of the phase shift or the degree of ellipticity of the transmitted radiation, if the mobility of the charge carriers has already been determined from a measurement of the angle of rotation of the plane of polarization [see Eq. (20)].[23] The phase shift should be appreciable, and could be measured accurately, if larger magnetic fields are employed or if the experiments are carried out at lower temperatures. Furthermore, if the phase shift is investigated as a function of the magnetic field strength B, the relaxation time could be obtained from measurements of small differences in the phase shift which should be detectable even at higher temperatures and low frequencies (small values of $\omega\tau$). Since the phase shift is a second order effect the anisotropy of the effective masses must be taken into account and the simple Eq. (20) is no longer valid (see Sec. II). Hence, the ellipticity is a function of the direction of the applied magnetic field. In principle, then, the effective masses of the charge carriers may be determined from measurements of the Faraday rotation and the phase shift as a function of the orientation of the applied magnetic field, as was attempted in the experiments of Suhl and Pearson.[5] While this method would be more inaccurate than the cyclotron resonance technique, it may be applicable to measurements at somewhat higher temperatures than the latter and to substances in which the cyclotron resonance method cannot be used.

For general applications, the theory has to be worked out in detail in cases where the relaxation time is a function of the electron energy and direction of motion, but this difficulty would also be experienced in the interpretation of dielectric measurements although the mathematical formalism may be simpler in the latter case.[24]

ACKNOWLEDGMENTS

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[22] R. Rau, Tech. Rept. 2, Contract AF 33(616)-78, January 15, 1955.

[23] For materials of small conductivity and spherical energy surfaces the phase shift depends linearly on the relaxation time.

[24] The mobility obtained from the Faraday rotation can probably not be interpreted simply as the effective Hall mobility of the charge carriers, even for weak magnetic fields, if the energy dependence of the relaxation time is taken into account, although such a interpretation seems correct as far as the anisotropy of the effective masses is concerned. (See reference 14.)