Plasma Resonance in Crystals: Observations and Theory*

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Observations are reported of an electron plasma resonance absorption line in a magnetic field in an n-type crystal of InSb at 9000 and 24 000 Mc/sec and at 4° and 77°K. The magnitude of the plasma frequency and its dependence on rf frequency and on the orientation of the figure axis of the specimen relative to the static magnetic field are in approximate agreement with the elementary theory of the plasma resonance developed previously by the authors. The line shape is discussed for experiments carried out under conditions of carrier modulation and magnetic field modulation. A theoretical discussion is given of the effects of specimen shape and of the possibility of detection of minority carrier cyclotron resonance in the presence of a majority carrier plasma. Eddy current effects are discussed, with reference to the possibility of the detection of cyclotron resonance in metals and semimetals.

WE have observed an electron plasma resonance line in a magnetic field in an n-type indium antimonide crystal at microwave frequencies at 4°K and 77°K. Plasma magnetic resonance effects as first observed were discussed and reported briefly by the present authors[1] in a paper on cyclotron resonance in silicon and germanium. In the present paper, we extend the theory in several directions and report the experimental observations in detail. We employ a classical drift-velocity treatment; the results can be extended easily, if required, by using the Boltzmann equation. We believe that the most serious shortcoming of the present treatment is the absence of any treatment of conditions near the surface of the specimen; our discussion is not expected to be valid when either the smallest dimension of the specimen or the skin depth is comparable with the carrier mean free path or the radius of the motion.

I. THEORETICAL

We first review the general nature of a plasma resonance in a magnetic field. This is a simple extension of standard magneto-ionic theory. We consider a spheroidal particle of a crystalline material of dielectric constant ϵ with N conduction electrons per unit volume, the electrons having an isotropic effective mass m^* and an isotropic collision or relaxation frequency ρ $(=\tau^{-1})$. We treat in the present paper only an isotropic effective mass; the electron mass in InSb is known to be isotropic.[2] The particle is placed in a uniform external rf electric field $\mathbf{E}$ perpendicular to the figure or z-axis of the spheroid. The dimensions of the particle are assumed to be small in comparison with the skin depth and the wavelength of the radiation. A static magnetic field $\mathbf{H}$ is applied parallel to the z-axis. The equation of motion of the average or drift velocity of the conduction electrons is given by

$$m^*(d\mathbf{v}/dt+\rho\mathbf{v})=e(\mathbf{E}-L\mathbf{P})+(e/c)\mathbf{v}\times\mathbf{H}, \tag{1}$$

where L is the depolarization factor (analogous to the demagnetization factor) normal to the figure axis of the specimen, and $\mathbf{P}$ is the dielectric polarization. The term $-L\mathbf{P}$ gives the depolarizing electric field associated with the dielectric polarization of the system. We neglect the questionable contribution of a Lorentz or cavity contribution to the local field acting on a carrier; we also neglect the effect of charges induced in the cavity walls by the polarization of the specimen. If χ is the dielectric susceptibility per unit volume of the host crystal, exclusive of carriers, we have for the effective internal electric field acting on the carriers

$$\mathbf{E}_{\text{eff}}=\mathbf{E}-L\mathbf{P}=\mathbf{E}-L(\chi\mathbf{E}_{\text{eff}}+Ne\mathbf{r}), \tag{2}$$

where N is the conduction electron concentration, and $\mathbf{r}$ is the vector from the center of charge of the ionized donor atoms in the crystal to the center of charge of the conduction electrons. From Eq. (2), we have

$$\mathbf{E}_{\text{eff}}=(\mathbf{E}-LNe\mathbf{r})/(1+L\chi). \tag{3}$$

Introducing the notation

$$\mathbf{E}_i=\mathbf{E}/(1+L\chi), \tag{4}$$

$$L_i=L/(1+L\chi), \tag{5}$$

the effective internal electric field becomes

$$\mathbf{E}_{\text{eff}}=\mathbf{E}_i-L_iNe\mathbf{r}. \tag{6}$$

The equation of motion in the xy-plane (normal to the figure axis) may be written

$$m^*\left(\frac{d^2\mathbf{r}}{dt^2}+\rho\frac{d\mathbf{r}}{dt}+\frac{L_iNe^2}{m^*}\mathbf{r}\right)=e\mathbf{E}_i+\frac{e}{c}\frac{d\mathbf{r}}{dt}\times\mathbf{H}. \tag{7}$$

The depolarization effect is equivalent to a harmonic restoring force, of force constant L_iNe^2. If $\mathbf{E}_i\sim e^{i\omega t}$, the equation of motion becomes

$$(-\omega^2+\omega_p{}^2+i\omega\rho)\mathbf{r}=(e/m^*)\mathbf{E}_i+i\omega\mathbf{r}\times\boldsymbol{\omega}_c, \tag{8}$$

* This work has been supported in part by the Office of Naval Research and the U. S. Signal Corps.

[1] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

[2] Dresselhaus, Kip, Kittel, and Wagoner, Phys. Rev. **98**, 556 (1955).

where $\boldsymbol{\omega}_c = e\mathbf{H}/m^*c$, or

$$(i\omega'+\rho)\mathbf{v} = (e/m^*)\mathbf{E}_i + \mathbf{v}\times\boldsymbol{\omega}_c, \tag{9}$$

where

$$\omega_p = (L_i Ne^2/m^*)^{\frac{1}{2}} = [LNe^2/m^*(1+L\chi)]^{\frac{1}{2}} \tag{10}$$

is the plasma frequency. We see that the influence of the plasma depolarization may be taken into account by substituting $\omega' = \omega - \omega_p^2\omega^{-1}$ in place of ω in the integrated equations of motion.

A plot of ω_p as a function of the depolarization factor L and of the axial ratio of a spheroid is given in Fig. 1, taking for convenience $\chi = 1$ which corresponds to $\epsilon = 13.6$, a value not unrepresentative of several classes of semiconductors. The shape dependence of the plasma frequency for this value of the dielectric susceptibility is not very marked, except for quite flat geometries.

The rate of energy dissipation per unit volume is given by

$$\mathcal{P} = \tfrac{1}{2}\,\mathrm{Re}[\mathbf{E}\cdot(d\mathbf{P}^*/dt)]. \tag{11}$$

Now

$$\mathbf{P} = \chi\mathbf{E}_{\mathrm{eff}} + Ne\mathbf{r}, \tag{12}$$

or, using Eq. (3),

$$\frac{d\mathbf{P}}{dt} = \frac{i\omega\chi}{1+L\chi}\mathbf{E} + \frac{1}{1+L\chi}Ne\mathbf{v}.$$

If a tensor conductivity component, σ_{xx} is defined in terms of the solution of Eq. (9) by

$$\sigma_{xx} = Nev_x/E_i^x,$$

we have, for the applied electric field in the x-direction,

$$\mathcal{P} = \frac{\frac{1}{2}|E^x|^2}{(1+L\chi)^2}\,\mathrm{Re}(\sigma_{xx}) = \tfrac{1}{2}|E_i^x|^2\,\mathrm{Re}(\sigma_{xx}). \tag{13}$$

The same expression is obtained on calculating $\mathcal{P}$ as $\frac{1}{2}\,\mathrm{Re}(\mathbf{E}_i\cdot\mathbf{j}^*)$, where $\mathbf{j}$ is the current density. We may note that $\omega_p^2 = L_i\sigma\rho$, where ρ is the relaxation frequency.

The real part σ_R of the conductivity σ_{xx} is given by

$$\frac{\sigma_R}{\sigma_0} = \frac{1+\nu'^2+\nu_c^2}{(1+\nu_c^2-\nu'^2)^2+4\nu'^2}, \tag{14}$$

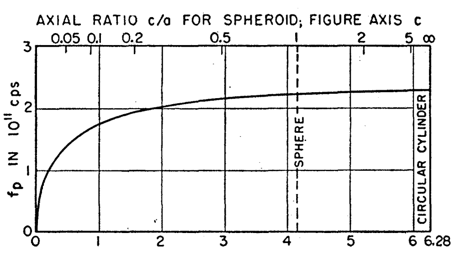

FIG. 1. Plasma frequency $f_p = \omega_p/2\pi$ for $m^* = 0.01m$, $\chi = 1$, and $N = 10^{14}$ carriers/cm³, calculated as a function of depolarization factor L_a.

where

$$\sigma_0 = Ne^2\tau/m^*;$$
$$\nu = \omega\tau;\quad \nu_p = \omega_p\tau;$$
$$\nu_c = \omega_c\tau = (eH/m^*c)\tau;$$
$$\nu' = (\omega - \omega_p^2\omega^{-1})\tau = \nu - \nu_p^2\nu^{-1}.$$

The variation of σ_R with the ratio of plasma frequency to experimental frequency is plotted in Fig. 2 for several values of the line sharpness parameter $\omega\tau = \nu$. When $\omega_p = \omega$ the conductivity is a maximum and equal to the value at zero frequency. In the absence of plasma effects ($\omega_p = 0$) the rf conductivity is lower than the dc conductivity, as is well known. When the plasma frequency is much greater than the experimental frequency ($\omega_p \gg \omega$), the electron motion is stiffened by the plasma effects, and the conductivity is reduced; to the limit $\sigma_R \cong \omega^2/L_i^2\sigma_0$. In this limit the resonance frequency is independent of the effective mass and is $\omega \cong L_i Nec/H$. The reduction in rf losses at high conductivities was pointed out in connection with some earlier experimental observations.[3]

We should note that plasma effects may be expected to have an important influence on microwave conductivity measurements on semiconductors. In such experiments the geometry is usually planned with the hope that the wave guide or cavity walls will act effectively to short-circuit the depolarization fields, so plasma effects may be neglected ($L = 0$). It has not been demonstrated theoretically or experimentally to our knowledge that the assumption $L = 0$ is sufficiently valid for all of the diverse experimental arrangements which have been employed. We suggest it is possible that plasma effects may be responsible in part for the discrepant results reported by various observers for the microwave conductivity of germanium.

The development of a magnetoplasma resonance from a cyclotron resonance as the carrier concentration (plasma frequency) is increased is illustrated in Fig. 3, where the conductivity is plotted against the magnetic field intensity, both in normalized units. The relaxation

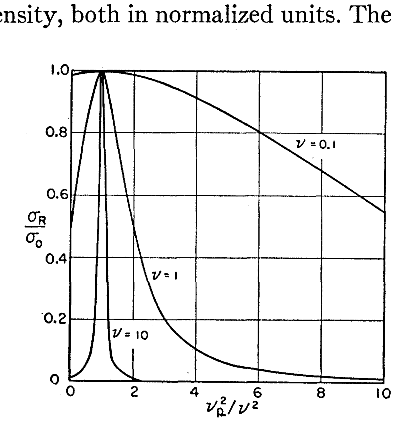

FIG. 2. Calculated variation of σ_R in zero magnetic field, as a function of $(\nu_p^2/\nu^2) = (\omega_p^2/\omega^2)$, for several values of $\nu = \omega\tau$.

[3] Portis, Kip, Kittel, and Brattain, Phys. Rev. 90, 988 (1953).

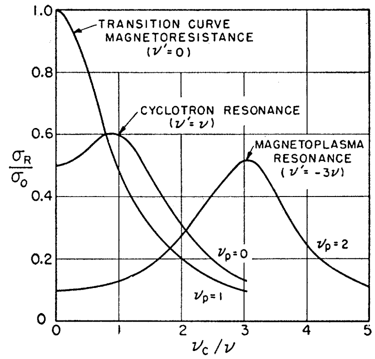

FIG. 3. Development of magnetoplasma resonance; theoretical plot of effective conductivity σ_R *vs* $\omega_c/\omega=\nu_c/\nu$ for $\nu=1$ and several values of ν_p.

time is kept constant with $\nu=1$. At zero plasma frequency we have the curve $\nu'=\nu$ for which a cyclotron resonance is just resolved For $\nu_p=\nu$ we have the transition curve $\nu'=0$ which is of the form of an ordinary dc magnetoresistance effect. For $\nu_p=2\nu$ we have the curve $\nu'=-3\nu$ which exhibits a distinct magnetoplasma resonance.

The striking consequences for the magnetoplasma resonance of a decrease in the experimental frequency are shown by Fig. 4. The two curves are drawn for a constant relaxation time and constant plasma frequency ($\nu_p=2$), but for two experimental frequencies $\nu=1.0$ and $\nu=0.5$. It is seen that a *reduction* in frequency by a factor of two almost *doubles* the actual value of the magnetic field for which the maximum of the magnetoplasma resonance occurs.

It is sometimes convenient to work with the components of the conductivity tensor. We write

$$j_x=\sigma_{xx}E_x+\sigma_{xy}E_y;\quad j_y=\sigma_{yx}E_x+\sigma_{yy}E_y. \tag{15}$$

For the problem considered above,

$$\sigma=\sigma_{xx}=\sigma_{yy}=\left(\frac{Ne^2}{m^*}\right)\frac{i\omega'+\rho}{(i\omega'+\rho)^2+\omega_c^2}; \tag{16}$$

$$\sigma_{xy}=-\sigma_{yx}=\left(\frac{Ne^2}{m^*}\right)\frac{\omega_c}{(i\omega'+\rho)^2+\omega_c^2}. \tag{17}$$

The off-diagonal components satisfy the reciprocity relation

$$\sigma_{xy}(H)=\sigma_{yx}(-H).$$

The tensor formulation is useful particularly in treatments of the nonreciprocal circuit properties of semiconductor devices. It is possible, for example, to utilize the cyclotron or magnetoplasma resonance effects in a semiconductor gyrator.

Elliptical Cross Section

If the depolarization factors L^x, L^y in the xy plane—the plane of the cyclotron motion—are not equal, we are to replace L_i in Eq. (10) for the plasma frequency by an appropriate function. We have

$$\begin{aligned}E_{\text{eff}}^x&=(E^x-L^xNex)/(1+L^x\chi);\\ E_{\text{eff}}^y&=(E^y-L^yNey)/(1+L^y\chi).\end{aligned} \tag{18}$$

The determinantal equation for free undamped oscillations is

$$\begin{vmatrix} i(\omega-\omega_{px}^2\omega^{-1}) & -\omega_c \\ \omega_c & i(\omega-\omega_{py}^2\omega^{-1})\end{vmatrix}=0, \tag{19}$$

so resonance occurs when

$$\omega_c^2=(\omega-\omega_{px}^2\omega^{-1})(\omega-\omega_{py}^2\omega^{-1})=\omega_x'\omega_y', \tag{20}$$

where $\omega_{px}^2=L_i^xNe^2/m^*$; $\omega_{py}^2=L_i^yNe^2/m^*$; $L_i^x=L^x/(1+L^x\chi)$; $L_i^y=L^y/(1+L^y\chi)$. When ω_{px}^2, $\omega_{py}^2\gg\omega^2$, we have

$$\omega_c\cong(\omega_{px}\omega_{py}/\omega)=(L_i^xL_i^y)^{\frac{1}{2}}Ne^2/m^*\omega, \tag{21}$$

so when plasma effects are dominant the magnetic field for magnetoplasma resonance is determined by the square root of the product of the effective depolarization factors L_i^x, L_i^y.

If one depolarization factor, say L_i^y, is zero, the apparent cyclotron frequency is given by $\omega_c^2=\omega^2-\omega_{px}^2$. This shows that when one of the plasma frequencies (ω_{py}) is small, a *resonance* may only be obtained when the other (ω_{px}) is also small, in order that ω^2 may be larger than ω_{px}^2.

Minority Carriers

We wish now to study the behavior of a low concentration N_s of minority carriers (which, for example, may be injected optically) in the presence of a high concentration N_h ($\omega_p\gg\omega$) of majority carriers h. In particular, we wish to determine the effect of the majority carriers on the cyclotron resonance of the minority carriers. In our work on cyclotron resonance in indium antimonide we observed the cyclotron resonance of a low concentration (possibly 10^4-10^8 carriers/cm^3) of high-mobility electrons and holes produced by optical excitation against the background of a relatively high concentration ($\sim4\times10^{14}$ per cm^3) of low-mobility holes, presumably in an impurity band.[4] The present calculation examines the possible interaction effects in such a system.

The effective electric field is

$$\mathbf{E}_{\text{eff}}=(\mathbf{E}-LN_he\mathbf{r}_h)/(1+L\chi), \tag{22}$$

if we neglect the contribution of the minority carriers. The equations of motion are

$$(i\omega+\rho_s)\mathbf{v}_s=(e/m_s^*)\mathbf{E}_i+i(m_h^*/m_s^*)(\omega_p^2/\omega)\mathbf{v}_h+\mathbf{v}_s\times\boldsymbol{\omega}_{cs}; \tag{23}$$

$$(i\omega+\rho_h)\mathbf{v}_h=(e/m_h^*)\mathbf{E}_i+i(\omega_p^2/\omega)\mathbf{v}_h+\mathbf{v}_h\times\boldsymbol{\omega}_{ch}, \tag{24}$$

[4] H. J. Hrostowski (private communication); H. Fritzsche and K. Lark-Horovitz (private communication). We are indebted to all three workers for helpful communication of their unpublished results.

where $\omega_p^2 = L_i N_h e^2/m_h^*$. We note that in our approximation the motion of the majority carriers is independent of the motion of the minority carriers. Thus, if $\mathbf{E}$ is in the x-direction,

$$v_{hx} = \frac{(e/m_h^*)E_i^x(i\omega'+\rho_h)}{(i\omega'+\rho_h)^2+\omega_{ch}^2}; \tag{25}$$

$$v_{hy} = -\frac{(e/m_h^*)E_i^x\omega_{ch}}{(i\omega'+\rho_h)^2+\omega_{ch}^2}. \tag{26}$$

In this degree of generality, the equations of motion for the minority carriers are rather tedious and unrevealing. We examine first the equations in the approximation $\omega_{ch} \ll \rho_h, \omega'$; that is, magnetic effects on the majority carriers are neglected. We have

$$(i\omega+\rho_s)v_{sx} - \omega_{cs}v_{sy} = (eE_i^x/m_s^*)\left[1+\frac{i\omega_p^2}{(i\omega'+\rho_h)\omega}\right]; \tag{27}$$

$$(i\omega+\rho_s)v_{sy} + \omega_{cs}v_{sx} = 0; \tag{28}$$

so

$$\frac{v_{sx}}{E_i^x} = \left(\frac{e}{m_s^*}\right)\frac{(i\omega+\rho_h)(i\omega+\rho_s)}{[(i\omega+\rho_s)^2+\omega_{cs}^2](i\omega'+\rho_h)}. \tag{29}$$

The total current density is

$$j_x = N_s e v_{sx} + N_h e v_{hx}. \tag{30}$$

If we set

$$\sigma_{Rh} + i\sigma_{Ih} = \frac{N_h e^2/m_h^*}{i\omega'+\rho_h}, \tag{31}$$

$$\sigma_{Rs} + i\sigma_{Is} = \frac{(N_s e^2/m_s^*)(i\omega+\rho_s)}{(i\omega+\rho_s)^2+\omega_{cs}^2}, \tag{32}$$

we have for the total conductivity $\sigma(=j_x/E_i^x)$:

$$\sigma = \sigma_{Rh} + i\sigma_{Ih} + (\sigma_{Rs}+i\sigma_{Is})\frac{i\omega+\rho_h}{i\omega'+\rho_h}, \tag{33}$$

so the power dissipation associated with the minority carriers is determined by

$$\mathrm{Re}\Delta\sigma = \mathrm{Re}(\sigma-\sigma_{Rh}) = \frac{\sigma_{Rs}(\omega\omega'+\rho_h^2)+\sigma_{Is}(\omega'-\omega)\rho_h}{\omega'^2+\rho_h^2}. \tag{34}$$

If plasma effects are dominant

$$\mathrm{Re}\Delta\sigma \approx -\sigma_{Rs}(\omega/\omega_p)^2. \tag{35}$$

If the relaxation frequency of the majority carriers is dominant,

$$\mathrm{Re}\Delta\sigma = \sigma_{Rs}. \tag{36}$$

In intermediate situations the power dissipation by the minority carriers may contain a mixture of absorptive (σ_{Rs}) and dispersive (σ_{Is}) components. In the p-type indium antimonide crystal produced by the Bell Telephone Laboratories and used for our cyclotron resonance work, we have approximately, at 4°K, $N_h \approx 4 \times 10^{14}/\mathrm{cm}^3$ and $\rho_h \approx 2\times 10^{14}$ sec^{-1} as calculated from the conductivity $\sigma_h = 4\times 10^{-4}$ (ohm-cm)$^{-1}$ and an effective mass[5] of m; taking $L_i \approx 1$, we have $\omega_{ph} \approx 3\times 10^{11}$ sec^{-1}. We see therefore that at the experimental frequency $\omega = 1.5\times 10^{11}$ the relaxation frequency may be dominant in this specimen, and the energy absorption by minority carriers probably is not affected by the presence of the majority carriers.

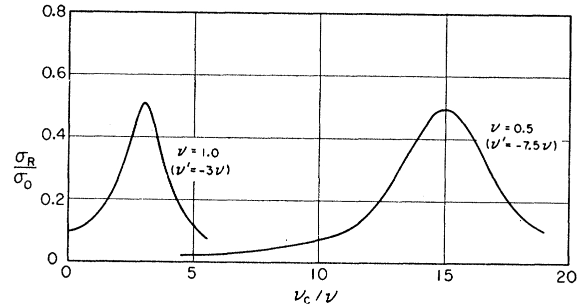

FIG. 4. Calculated effect of reducing experimental frequency by a factor of two. Plot of conductivity *vs* $\omega_c/\omega = \nu_c/\nu$ for $\nu = 1.0$ and $\nu = 0.5$, keeping $\nu_p = 2$.

Cyclotron and Plasma Resonance Under Eddy Current Conditions

We calculate now the magnetic field dependence of the rate of energy absorption in an infinite slab of material thick in comparison with the rf skin depth.[6] Our primary interest is in the shape of the absorption line under eddy current conditions. We assume explicitly that the carrier mean free path is small in comparison with the skin depth. This assumption limits the region of validity of the treatment to moderately low carrier concentrations. Two geometries are of interest, one with the static magnetic field parallel to the plane of the slab and the second with the magnetic field normal to the plane. We take the plane of the slab to be the xy plane. The results suggest that under eddy current conditions and the above assumptions it is not possible to obtain a distinctive cyclotron resonance line. With carriers produced by optical absorption it should be possible to get around the difficulty for the perpendicular geometry if the injected carriers can be localized in the surface region of the specimen, as by recombination.

Parallel Geometry

The static magnetic field is in the y-direction. The equations to be solved are the cyclotron equations (15) through (17), and the wave equation in a conducting

[5] The effective mass should be that of holes in the impurity band; we have taken this as m in complete ignorance of what to do. It may be that a much higher mass is applicable; in the limit of high mass the majority carriers will have little effect on the cyclotron resonance of the minority carriers.

[6] See also B. Donovan and E. H. Sondheimer, Proc. Phys. Soc. (London) **A66**, 849 (1953).

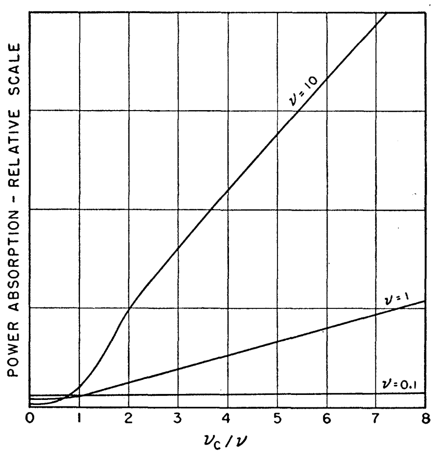

FIG. 5. Calculated power absorption *vs* magnetic field intensity under eddy current conditions with H parallel to the surface.

medium

$$\nabla^2\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t}+\frac{\epsilon}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}. \tag{37}$$

The right-hand side of this equation is zero for the z component. We assume E_x has a solution of the form $e^{i(\omega t-kz)}$; the wave equation gives us

$$k^2=-i\frac{4\pi\omega\sigma}{c^2}+\frac{\epsilon\omega^2}{c^2}. \tag{38}$$

With the appropriate restrictions this expression gives the usual low frequency eddy current result.

The electromagnetic behavior of the material is described conveniently by the intrinsic surface impedance

$$Z=\left.\frac{E_x}{H_y}\right|_{z=0}=-\frac{i\omega}{c}\frac{E_x}{\partial E_x/\partial z}=\frac{\omega}{ck}. \tag{39}$$

The rate of energy loss per unit area is

$$S=(c/8\pi)|H_y|^2\,\mathrm{Re}(Z), \tag{40}$$

from the Poynting vector. If we may neglect $\epsilon\omega^2/c^2$ in (38), as is often justified, we have that the shape of the absorption line is determined by

$$\mathrm{Re}(Z)\propto\mathrm{Re}(i/\sigma)^{\frac{1}{2}}=\left[\frac{(\sigma_R{}^2+\sigma_I{}^2)^{\frac{1}{2}}+\sigma_I}{2(\sigma_R{}^2+\sigma_I{}^2)}\right]^{\frac{1}{2}}, \tag{41}$$

where

$$\sigma=\sigma_R+i\sigma_I. \tag{42}$$

The unusual line shape is shown in Fig. 5 for a pure cyclotron resonance, with no specific plasma effects—that is, the ratio of the skin depth to the breadth of the specimen is taken to be vanishingly small. There is no recognizable cyclotron resonance line.

Perpendicular Geometry

The static magnetic field is in the z-direction. From the electromagnetic wave equation

$$\begin{aligned}-k^2E_x&=i(4\pi\omega/c^2)(\sigma E_x+\sigma_{xy}E_y)-(\epsilon\omega^2/c^2)E_x;\\ -k^2E_y&=i(4\pi\omega/c^2)(\sigma_{yx}E_x+\sigma E_y)-(\epsilon\omega^2/c^2)E_y.\end{aligned} \tag{43}$$

We obtain k from the following determinantal equation, where $p=4\pi\omega/c^2$:

$$\begin{vmatrix} k^2+ip\sigma-\dfrac{\epsilon\omega^2}{c^2} & ip\sigma_{xy} \\ -ip\sigma_{xy} & k^2+ip\sigma-\dfrac{\epsilon\omega^2}{c^2}\end{vmatrix}=0, \tag{44}$$

so

$$k^2=(\epsilon\omega^2/c^2)-ip\sigma\pm p\sigma_{xy}, \tag{45}$$

and

$$E_x=\pm iE_y. \tag{46}$$

If we enforce the condition that the radiation field at the surface shall be linearly polarized, the field components are given by

$$\begin{aligned}E_x&=\tfrac{1}{2}E_0(e^{-ik_+z}+e^{-ik_-z});\\ iE_y&=\tfrac{1}{2}E_0(e^{-ik_+z}-e^{-ik_-z}).\end{aligned} \tag{47}$$

Thus, $H_y=(ic/\omega)(\partial E_x/\partial y)=(c/2\omega)E_0(k_++k_-)$ at the surface, and, neglecting $\epsilon\omega^2/c^2$,

$$\mathrm{Re}(Z)\propto\mathrm{Re}\left\{\left(\frac{i\omega}{4\pi\sigma_0}\right)^{\frac{1}{2}}\times\frac{(1+i\omega'\tau-i\omega_c\tau)^{\frac{1}{2}}(1+i\omega'\tau+i\omega_c\tau)^{\frac{1}{2}}}{(1+i\omega'\tau-i\omega_c\tau)^{\frac{1}{2}}+(1+i\omega'\tau+i\omega_c\tau)^{\frac{1}{2}}}\right\}. \tag{48}$$

A plot of the power absorption as a function of the magnetic field intensity is given in Fig. 6. There is no recognizable cyclotron resonance line.

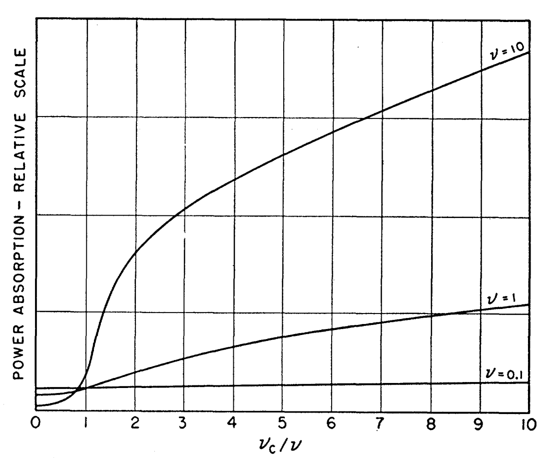

FIG. 6. Calculated power absorption *vs* magnetic field intensity under eddy current conditions with H normal to the surface.

Line Shape

It is often convenient in microwave resonance experiments to present as the output signal the modulated component of the rf power absorption in the specimen. The modulation in our experiments may be produced by modulating the carrier concentration,[7] which may be accomplished conveniently by modulating a light source which produces internal photoelectrons in the specimen, or by modulating the magnetic field intensity, a standard method in electron and nuclear spin resonance experiments.

In carrier modulation, the signal S is proportional to $\partial\sigma_R/\partial N$, where σ_R is given by Eq. (14) and N is the carrier concentration. We find that

$$S \propto \mathrm{Re}\left[\sigma - \frac{\nu_p^2}{\nu}\frac{\partial\sigma}{\partial\nu'}\right], \tag{49}$$

or

$$S \propto \mathrm{Re}\left\{\frac{1+i\nu'}{(1+i\nu')^2+\nu_c^2} - \frac{i\nu_p^2[\nu_c^2-(1+i\nu')^2]}{\nu[\nu_c^2+(1+i\nu')^2]^2}\right\}. \tag{50}$$

If $\nu' \gg \nu$

$$S \propto \left\{\frac{1+\nu_c^2-\nu'^2}{(1+\nu'^2-\nu_c^2)^2+4\nu'^2} + 8\nu'^2\nu_c^2\frac{(1+\nu_c^2-\nu'^2)}{[(1+\nu'^2-\nu_c^2)^2+4\nu'^2]^2}\right\} \tag{51}$$

and the phase of the signal changes when

$$\nu_c^2 = \nu'^2 - 1. \tag{52}$$

A plot of calculated line shapes is given in Fig. 7, in the approximation $\nu' \gg \nu$. The line shapes are quite distinctive and are somewhat reminiscent of the shapes of ordinary spin resonance absorption lines under eddy

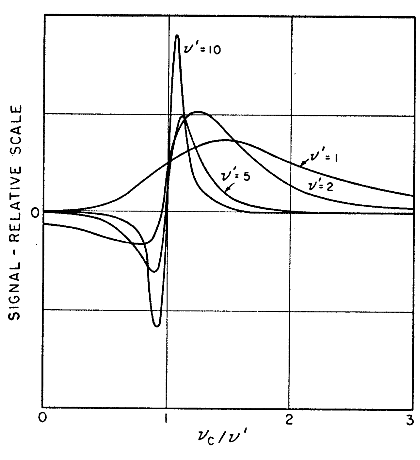

FIG. 7. Plot of theoretical line shape *vs* magnetic field when the carrier concentration is modulated and the detected signal is the modulated component of the power absorption. The plot is for the limiting case $\nu \ll \nu_p$, so $\nu' = -\nu_p^2/\nu$.

[7] A. F. Kip, Physica **20**, 813 (1954).

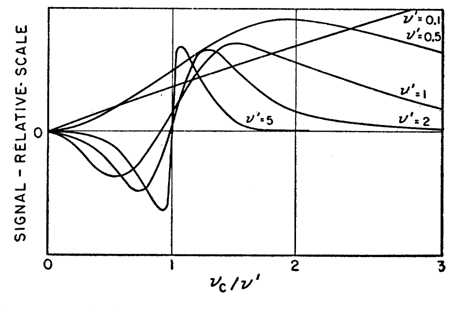

FIG. 8. Plot of theoretical line shape *vs* magnetic field when the magnetic field is modulated and the detected signal is the modulated component of the power absorption.

current conditions. We note that if plasma effects may be neglected the signal is simply of the form of σ_R, an absorption shape. It should therefore be possible to distinguish in most instances plasma resonances from cyclotron resonances by the line shape.

With magnetic field modulation the signal S is of the form

$$S \propto \partial\sigma_R/\partial H, \tag{53}$$

or

$$S \propto \nu_c\left\{\frac{(1+\nu_c^2-\nu'^2)^2+4\nu'^2(\nu_c^2-\nu'^2)}{[(1-\nu'^2+\nu_c^2)^2+4\nu'^2]^2}\right\}. \tag{54}$$

The calculated line shapes are plotted in Fig. 8; it is seen that the lines tend to be more antisymmetric about the resonance frequency than was true with carrier modulation.

II. EXPERIMENTAL RESULTS

Although the plasma resonance absorption line is in general relatively strong, it is not entirely a trivial matter to arrange to observe the resonance experimentally. It is not convenient to sweep in frequency over a wide range, so we have preferred to use a magnetic field to sweep the plasma frequency through the experimental frequency. It is desirable to use a crystal in which the carrier concentration is independent of temperature,[8] so the heating or ionization effects of the microwave field will not change the carrier concentration. We were fortunate enough to obtain from the Battelle Memorial Institute, through the kindness of Dr. A. C. Beer, an n-type indium antimonide crystal which is very well suited to our purpose. The crystal is described by Dr. Beer as having an average extrinsic carrier density of 1.4×10^{14} cm^{-3}; we are inclined to believe on the basis of our plasma measurements that the fragment used in our work actually had a carrier density of $(0.5\pm0.2)\times10^{14}$ cm^{-3}. The electron density at somewhat higher concentrations in n-InSb is known

[8] There is some interest in experiments on materials in which the carrier concentration can be varied by varying the temperature, and our original observation of a plasma resonance in germanium was made in this way, but we have not used the method in a systematic way.

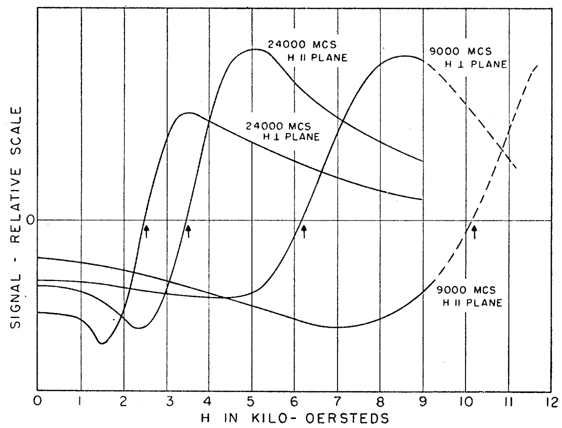

FIG. 9. Experimental plasma resonance absorption signals obtained with carrier modulation in a thin disk of n-type InSb at 77°K, at 9000 Mc/sec and 24 000 Mc/sec as indicated. The static magnetic field is directed normal to or parallel to the plane of the disk in separate runs. The broken lines connect the curves below 9000 oersteds with single terminal points determined at higher fields. The resonance condition is determined approximately by the crossover points.

to be independent of temperature[9] between the temperatures of liquid helium and liquid nitrogen. In our sample the plasma frequency did not appear to shift between these two temperatures, so we believe with some confidence that the electron concentration is indeed approximately constant. At high rf power levels at X-band at 77°K it appeared to be possible to alter the carrier concentration, however.

It is desired that the carrier concentration and effective mass correspond approximately to a convenient experimental frequency. As shown roughly in Fig. 1, the properties of our specimen put the plasma frequency close enough to the microwave range, so a reasonable laboratory magnetic field acting on the light effective mass of the carrier makes possible measurements at both X-band and K-band. It is also necessary that the relaxation time of the carriers be long enough to permit the resonance to be resolved distinctly. The mobility measured at Battelle at 80°K is reported to be about 500 000 cm^2/volt-sec. Taking the effective mass[10] to be $m^*=0.013m$, we have $\tau=\mu m^*/e\cong 4\times 10^{-12}$ sec. As at K-band we obtained plasma resonance at a cyclotron angular frequency of about 4×10^{12} radians/sec, we should expect a resolution $\omega_c\tau$ of about 16, more than adequate. The actual resolution was only about 2; we believe that the actual resolution is probably limited by unavoidable irregularities in the shape of the test specimen. The fact that the resolution did not improve on moving to X-band[11] supports our opinion.

It is required further that the thickness of the specimen be less than the rf skin depth. Based on the reported mobility $\mu=5\times 10^5$ cm^2/volt-sec and carrier concentration $N=1.4\times 10^{14}$ cm^{-3}, we calculate that the skin depth at K-band is about 0.1 mm. The value may be raised to 0.2 mm if the correct N should be 0.5×10^{14} cm^{-3}. The skin depth is too small, taken in combination with the poor mechanical properties of the material, for us to fabricate easily a suitable well-formed spheroidal test specimen. Through persistent efforts, Dr. Lawrence Hadley was able to isolate an irregular disk roughly 0.4 mm×0.4 mm×0.1 mm. This specimen was good enough to enable us to confirm approximately the principal predictions of the magnetoplasma theory, but we can claim none of the high accuracy and reproducibility often associated with other types of resonance experiments. We also checked qualitatively the general features of the results using a second specimen in the form of a short thin needle, making runs at 77°K and 4°K, with no significant differences at the two temperatures.

The experimental absorption curves obtained with the disk-like specimen are plotted in Fig. 9, for runs taken at 77°K under carrier (light) modulation.[12] The observed curves show the general features of the calculated curves, Fig. 7, but seem to be smudged out as might be expected if the depolarization factors vary somewhat from point to point within the irregular specimen. The resonance field at 24 000 Mc/sec for the normal field orientation is apparently near 2500 oersteds, as compared with 3500 oersteds in the parallel field orientation.

We note that in our specimen the effect of the plasma may be described as shifting the position of the cyclotron resonance to higher magnetic fields; the shift is by a factor of 20 or more at K-band and 200 or more at X-band.

The ratio of 0.7 between the two orientations is in approximate agreement with the ratio calculated using Eq. (21) and the approximate axial dimensions a, b, c of the specimen. We have $(L_a/4\pi)=0.70$; $(L_b/4\pi)=(L_c/4\pi)=0.15$; further, taking the value of the dielectric constant[13] of InSb as 14,

$$L_i^a=0.87;\quad L_i^b=L_i^c=0.64, \tag{55}$$

so the calculated ratio of resonance fields is

$$H(\perp)/H(\|)=0.64/[(0.64)(0.87)]^{\frac{1}{2}}=0.86, \tag{56}$$

with an appreciable range of error because of the uncertain depolarization factors of the specimen. The thickness actually varies between 0.07 and 0.12 mm;

[9] H. J. Hrostowski (private communication).

[10] See reference 2. We take advantage of this occasion to correct an error in the reference cited: the electron spin resonance reported for InSb is now believed to be associated instead with both cavities used in the work. It has been necessary to silver-plate the cavities to eliminate the spurious signal. The specimen reported as n-type in the reference is p-type extrinsic at low temperatures.

[11] We recall that decreasing ω increases ω_c, according to Eq. (14).

[12] The experimental methods employed were similar in general to those described in reference 1; in the plasma work some difficulty is caused by the high loss of the specimen.

[13] Avery, Goodwin, Lawson, and Moss, Proc. Phys. Soc. (London) **B67**, 761 (1954). Dr. Beer has called our attention to several unpublished results suggesting that the dielectric constant may be close to 17. If we use this value, Eq. (58) would give N close to 0.5×10^{14} cm^{-3}, which might be increased to 0.6×10^{14} cm^{-3} by the consideration which follows Eq. (58).

further, the injected carriers may not cover the specimen uniformly. The observed resonance field ratio 0.7 is somewhat low.

We shall now estimate the carrier concentration N from the K-band data in the normal field orientation. The approximate resonance condition is, from Eq. (14),

$$-\omega\omega_c \cong \omega_p^2 = L_i N e^2/m^*. \qquad (57)$$

Here $\omega = 1.5\times 10^{11}$ radians/sec; $\omega_c = eH/m^*c \cong 3\times 10^{12}$ radians/sec for $H=2500$ oersteds and $m^*=0.013m$; and $L_i^c = 0.64$ from Eq. (55). Thus

$$N \cong m^*\omega\omega_c/L_i e^2 \cong 0.4\times 10^{14}\ \mathrm{cm}^{-3}, \qquad (58)$$

as compared with the electron concentration of 1.4×10^{14} cm^{-3} reported by Dr. Beer for a part of the crystal as grown. We do not believe the discrepancy necessarily is significant in view of the distribution in impurity concentration known to occur in other semiconductor crystals. The anisotropy discrepancy suggests that we have overestimated L_i^c, and a correction for this would make the observed $N \cong 0.5\times 10^{14}$ cm^{-3}.

Experimental results are shown in Fig. 10 for a second specimen which had a better geometry than that used in Fig. 9; the improvement in resolution is evident.

It is a remarkable feature of the plasma resonance condition (57) that the resonance moves to higher magnetic fields as the frequency is lowered. The frequency ratio 9000 Mc/sec/24 000 Mc/sec is 1/2.7; the corresponding resonance field ratio is expected to be 2.7. The observed increase on going from K-band to X-band is from 2500 oersteds to 6200 oersteds in the perpendicular orientation, a factor of 2.5. In the parallel orientation the results are less reliable because of limitations on the available magnetic field, but the corresponding factor appears to be about 2.9. The effects of frequency are thus in general agreement with expectation.

III. SUMMARY

We have given an elementary and simplified theory of plasma resonance in a magnetic field in semiconductor crystals for carriers having a single isotropic effective mass. The dependence of the magnetic field at resonance on microwave frequency, specimen shape and orientation, and carrier concentration has been confirmed approximately by observations on conduction electrons in indium antimonide. The plasma effect, apart from any intrinsic interest, is important in the interpretation of cyclotron resonance experiments at high carrier concentrations; that is, at concentrations over perhaps 10^{12} cm^{-3}. It remains to be seen whether the depolarizing electric fields which give rise to the plasma frequency can be short-circuited by suitable contact between the specimen and the walls of the resonant cavity. It may not be possible to carry out cyclotron resonance experiments at high carrier densities unless the plasma frequency can be suppressed. In normal metals the plasma frequency for $L=4\pi$ is in the near ultraviolet spectral region.

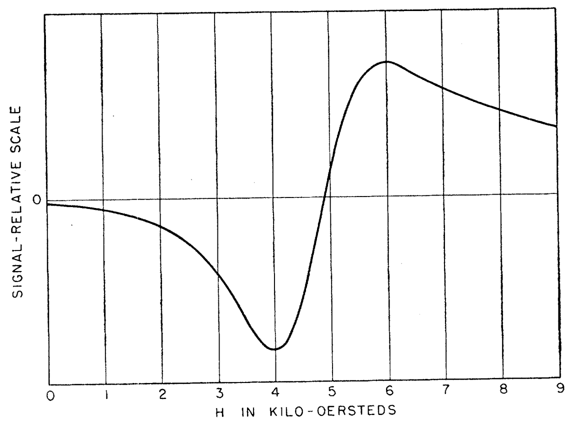

FIG. 10. Experimental plasma resonance absorption signal obtained with carrier modulation in a thin rectangular specimen ($a=0.04$ cm, $b=0.026$ cm, and $c=0.008$ cm) of n-type InSb at 77°K and 24 000 mc. The magnetic field is along the c-axis. The position of the resonance corresponds to an electron charge concentration of $\sim 0.8\times 10^{14}/\mathrm{cm}^3$. The width indicates an apparent relaxation time of about $\omega_c\tau \gtrsim 4$. The improvement in resolution with respect to the curves shown in Fig. 9 is attributed to the smoother geometry of the present specimen.

It may be possible that the properties of the electron plasma in indium antimonide may find application in oscillators in the millimeter and submillimeter region. It is relatively easy to obtain a pure plasma resonance in this region, and the Q of the resonance may be of the order of 50 to 100 at liquid nitrogen temperature. The oscillator might be tuned by means of a magnetic field.

Apart from the overriding importance of depolarizing fields in metals, we note from the calculations presented above that the specimen must, regardless of carrier concentration, be thinner than a skin depth if the plot of power absorption *vs* magnetic field is to exhibit a resonance maximum. This point is not considered correctly in Dingle's discussion[14] of diamagnetic resonance in metals, and we have at present no reason to believe that resonance absorption can occur under the conditions envisaged in his paper. This remark also applies to the reference by Dorfman[15] to cyclotron resonance in metals.

We are indebted to Mr. G. Wagoner for assistance in the measurements. The crystal of indium antimonide used in the work was supplied by the Battelle Memorial Institute through the kindness of Dr. A. C. Beer. It was developed as part of contractual work undertaken for Wright Air Development Center, U. S. Air Force. Dr. Lawrence Hadley, on leave from Dartmouth College, kindly undertook the difficult preparation of the specimen. We wish also to thank Mr. R. Behringer for checking the manuscript. We wish to thank Professor L. B. Loeb and Dr. D. Judd for helpful conversations.

[14] R. B. Dingle, Proc. Roy. Soc. (London) **A212**, 38 (1952); Phys. Rev. **98**, 550 (1955).

[15] J. G. Dorfman, Doklady Akad. Nauk (S.S.S.R.) **81**, 765 (1951); Phys. Rev. **96**, 1704 (1954).