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Superfluidity of an Ideal Bose-Einstein Gas

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An equilibrium superfluid is defined as a substance whose moment of inertia in thermal equilibrium is less than the classical value. The ideal Bose-Einstein gas below its transition temperature is shown to be an equilibrium superfluid. However, a more detailed analysis of the relation between angular momentum and angular velocity shows that experiments carried out with practical angular momenta and velocities would give an abnormally *large* apparent moment of inertia in thermal equilibrium, i.e., the gas would appear to be an "infrafluid." Nevertheless, a weakly interacting Bose-Einstein gas can be used as a model of a superfluid, if the superfluid state is assumed to be metastable rather than the thermodynamic equilibrium state.

1. INTRODUCTION

LONDON[1] first proposed that the peculiar phase transition (λ point) in liquid helium at 2.186°K is essentially the same as the condensation phenomenon exhibited by an ideal Bose-Einstein gas,[2,3] only slightly disturbed by the interactions between the particles. It is the purpose of this paper to study the properties of an ideal Bose-Einstein gas below the λ point when the vessel in which the gas is contained is subjected to a uniform rotation. This "rotating bucket" experiment was first discussed by Landau[4] on the basis of his theory of the superfluidity of liquid helium. Landau predicted that liquid helium would show an abnormal relation between angular momentum L and angular velocity ω in *statistical equilibrium*. The moment of inertia $I=\partial L/\partial\omega$ would be less than its classical value I_0. Subsequent experimental work by Andronikashvili[5] seemed to bear out this prediction.

We shall define the equilibrium moment of inertia of a substance as

$$I=\lim_{\omega\to 0}\left[\frac{\partial\bar{L}(\omega)}{\partial\omega}\right],\tag{1.1}$$

where $\bar{L}(\omega)$ is the *equilibrium* value of the angular momentum of the substance in a container rotating uniformly with angular velocity ω. We shall further define an *equilibrium superfluid* as a substance for which I is less than the classical moment of inertia I_0, the latter being computed in the usual way; in a cylindrical vessel of radius R, filled with N particles each of mass M, I_0 is given by

$$I_0=\tfrac{1}{2}NMR^2.\tag{1.2}$$

There are 3 different ways of defining equilibrium superfluidity, in increasing order of stringency of the requirement:

(1) The initial slope of the curve of $\bar{L}$ *vs* ω, evaluated at $\omega=0$, is less than the classical value I_0. This is our requirement.

(2) $\bar{L}$ maintains this nonclassical slope for a finite range of values of the circumferential velocity $v=R\omega$; however, this range may become smaller and smaller as the radius R of the vessel increases. This is the suggestion of London[6] who expects an upper limit to superfluid flow of order $v=R\omega\lesssim\hbar/MR$.

(3) $\bar{L}$ maintains the nonclassical slope for values of the circumferential velocity less than some "critical velocity," this latter being independent of the radius of the vessel. This (most stringent) requirement is assumed to be satisfied in the usual two-fluid model of liquid helium.

This paper proves that the ideal Bose-Einstein gas below its condensation point is an equilibrium superfluid in the sense of the first and second requirements above, but does not satisfy the third requirement. The nonclassical slope of $\bar{L}$ is maintained until the angular velocity ω reaches the value ω_1 given by

$$\omega_1=4.45\hbar/MR^2.\tag{1.3}$$

Thereafter, however, the picture changes suddenly. At $\omega=\omega_1$, the angular momentum $\bar{L}(\omega)$ has a discontinuity, due to the fact that the condensed particles shift suddenly from the state with angular momentum quantum number $m=0$ to the state with $m=1$. Thermodynamically, this is a phase transition of the first kind, with a latent heat. Similar phase transitions occur at higher values of ω. We shall show that a conventional measurement of the moment of inertia of an ideal Bose-Einstein gas in thermal equilibrium would yield an averaged value I_{eff} which is larger than the classical value I_0, i.e., the gas would appear to be an "infrafluid" rather than a superfluid. Thus, although the ideal Bose-Einstein gas is an equilibrium superfluid in the sense of London,[1,6] it fails to exhibit the kind of equilibrium superfluidity postulated by Landau[4] and Tisza.[7]

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[1] F. London, Phys. Rev. **54**, 947 (1938).

[2] A. Einstein, Ber. Berl. Akad. **261** (1924); 3 (1925).

[3] B. Kahn and G. E. Uhlebeck, Physica **5**, 399 (1938).

[4] L. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941).

[5] E. L. Andronikashvili, J. Phys. (U.S.S.R.) **10**, 201 (1946).

[6] H. London, Phys. Soc. Camb. Conference Report, 48, 1946 (Physical Society, London, 1947).

[7] L. Tisza, Phys. Rev. **72**, 838 (1947).

Nevertheless, the Bose-Einstein gas can be used as a model of a superfluid if we drop the requirement of thermal equilibrium; the nonequilibrium superfluid behavior which results is highly suggestive, and we believe contains the essence of the superfluid properties observed in actual liquid helium.

2. MOMENT OF INERTIA OF AN IDEAL BOSE-EINSTEIN GAS

The Hamiltonian of an ideal Bose-Einstein gas in a stationary container is

$$H_0=\sum_{i=1}^{N} p_i^2/2M=\sum_{i=1}^{N} h_{0i}. \tag{2.1}$$

In order to discuss thermal equilibrium of the gas inside a rotating container, it is necessary[8] to use a system of coordinates rotating along with the container. The operator for the z component of the angular momentum of particle number i is

$$l_i=-i\hbar\left(x_i\frac{\partial}{\partial y_i}-y_i\frac{\partial}{\partial x_i}\right), \tag{2.2}$$

where x_i, y_i, z_i are the coordinates of particle i measured in the rotating frame of reference. It can be shown, however,[8] that the expectation value of l_i is the angular momentum of particle i with respect to the *stationary* frame of reference. The z-component of the total angular momentum of all the particles will be denoted by L:

$$L=\sum_{i=1}^{N} l_i. \tag{2.3}$$

The canonical transformation to the rotating system of coordinates leads to the new Hamiltonian

$$H'=\sum_{i=1}^{N}(h_{0i}-\omega l_i)=H_0-\omega L. \tag{2.4}$$

This Hamiltonian determines the properties of the gas in statistical equilibrium within the rotating container, in the usual way. We shall be particularly interested in the statistical average value of L, given by

$$\bar{L}(\omega)=\frac{\mathrm{Trace}[L\exp(-\beta H')]}{\mathrm{Trace}[\exp(-\beta H')]}, \tag{2.5}$$

where $\beta=(kT)^{-1}$, and the trace is to be taken over all permissible quantum states of the N-particle system (all states with symmetric wave functions for Bose-Einstein statistics).

Since there are no interactions between the particles, the calculation of $\bar{L}$ can be simplified. We define the symbol "trace" to mean a trace over all quantum states of a single particle in the container (rather than over quantum states of the full N-particle system). The chemical potential μ is then defined by the relation:

$$\mathrm{trace}\left\{\frac{1}{\exp[\beta(h_0-\omega l-\mu)]-1}\right\}=N, \tag{2.6}$$

where N is the total number of particles. By reflection symmetry $(\theta'=-\theta)$ μ is a symmetric function of ω. The equilibrium value of L, (2.5) is then given by

$$\bar{L}(\omega)=\mathrm{trace}\left\{l\frac{1}{\exp[\beta(h_0-\omega l-\mu]-1}\right\}. \tag{2.7}$$

In a noncylindrical container, h_0 and l fail to commute, and so do H_0 and L for the N-particle system. In the N-particle formulation (2.5) the error made by assuming that H_0 and L commute is of order N^{-1} and hence unimportant. The corresponding error in the one-particle formulation (2.7) is very significant for temperatures below the condensation point of the Bose-Einstein gas. Since these complications add nothing to the argument, we shall consider cylindrically symmetric containers.

We label the common eigenstates of h_0 and l by the two indices m and γ, where $\hbar m$ is the eigenvalue of l and γ stands for the two additional quantum numbers necessary to specify the quantum state uniquely. The eigenvalue of h_0 in the quantum state (m,γ) will be called $\epsilon_{m\gamma}$; the lowest $\epsilon_{m\gamma}$ for a given value of m will be called ϵ_{m0}.

We first evaluate the initial slope of $\bar{L}$ as a function of ω; that is, we differentiate (2.7) with respect to ω and then set $\omega=0$. In the range of ω for which the differentiation is permitted, the chemical potential μ is also a differentiable function of ω; furthermore, since μ is an even function of ω, the derivative $\partial\mu/\partial\omega$ vanishes at $\omega=0$. We therefore obtain the following expression for the moment of inertia I:

$$\begin{aligned} I=\left(\frac{d\bar{L}}{d\omega}\right)_{\omega=0}&=\beta\,\mathrm{trace}\left\{l^2\frac{\exp[\beta(h_0-\mu_0)]}{(\exp[\beta(h_0-\mu_0)]-1)^2}\right\}\\ &=\hbar^2\beta\sum_{\gamma}\sum_{m=-\infty}^{\infty} m^2\frac{\exp[\beta(\epsilon_{m\gamma}-\mu_0)]}{\{\exp[\beta(\epsilon_{m\gamma}-\mu_0)]-1\}^2}. \end{aligned} \tag{2.8}$$

The sum in (2.8) is rather awkward to handle since $\epsilon_{m\gamma}$ is a complicated expression for a cylindrical container. However, we can simplify the evaluation of (2.8) tremendously provided that the main contribution to the trace comes from quantum states with de Broglie wavelengths much smaller than the dimensions of the bucket. This is obviously so above the Bose-Einstein condensation point; it is also true below the condensation point, because the ground state of the single particle spectrum is the state ϵ_{00}, which has $m=0$ and therefore contributes nothing to (2.8). If the de Broglie

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[8] For detailed arguments see the following paper, Blatt, Butler, and Schafroth, Phys. Rev. **100**, 481 (1955).

wavelength is small compared to the linear dimensions of the bucket, the boundary condition $\psi=0$ at the wall of the container is unimportant. We can work with plane wave functions $\exp(i\mathbf{p}\cdot\mathbf{r}/\hbar)$ which form a complete set, but fail to satisfy the boundary condition. The error is of order $\lambda/\mathfrak{L}$ where λ is the de Broglie wavelength and $\mathfrak{L}$ is the linear dimension of the container. We then get

$$I=\beta\sum_{p}{}'\langle\mathbf{p}|l^2|\mathbf{p}\rangle\frac{\exp[\beta(p^2/2M-\mu_0)]}{\{\exp[\beta(p^2/2M-\mu_0)]-1\}^2},\qquad(2.9)$$

where the prime on the sum indicates that very small values of p ($\hbar/p\sim\mathfrak{L}$) are to be excluded. The diagonal matrix element of the operator l^2 in a plane wave state is given by (V = volume of the container)

$$\begin{aligned}\langle\mathbf{p}|l^2|\mathbf{p}\rangle&=V^{-1}\int(xp_y-yp_x)^2d^3r\\&=\langle x^2\rangle p_y^2+\langle y^2\rangle p_x^2-2\langle xy\rangle p_xp_y.\end{aligned}\qquad(2.10)$$

The volume integral in (2.10) is over the interior of the container. For a cylindrical container of radius R we get

$$\langle x^2\rangle=\langle y^2\rangle=R^2/4\quad\langle xy\rangle=0.\qquad(2.11)$$

However, this part of our argument applies to a container of arbitrary shape. The shape of the container becomes important only when we consider quantum states with de Broglie wavelengths comparable to the linear dimensions of the container.

We substitute (2.10) into (2.9) and replace the sum over quantum states p by an integral in the conventional way. Consider the term proportional to $\langle x^2\rangle$; this term can be transformed by an integration by parts:

$$\begin{aligned}-\beta V(2\pi)^{-3}\langle x^2\rangle\int d^3p\,p_y(M/\beta)\frac{\partial}{\partial p_y}&\\\times\{\exp[\beta(p^2/2M-\mu_0)]-1\}^{-1}=M\langle x^2\rangle V(2\pi)^{-3}\int d^3p&\\\times\{\exp[\beta(p^2/2M-\mu_0)]-1\}^{-1}.&\end{aligned}\qquad(2.12)$$

As a result of the factor p^2 in the volume element d^3p, states with very small values of p make no significant contribution to the last integral, even if μ_0 is very close to zero. Thus the integral correctly represents the primed sum (2.9).

Above the condensation point, the second integral in (2.12) represents the trace (2.6), i.e., we get the expected result for the moment of inertia:

$$I=I_0=NM\langle x^2+y^2\rangle\quad T>T_\lambda.\qquad(2.13)$$

Below the condensation point, however, we get a different result: the integral in (2.12) still represents the primed sum (2.9), and hence the trace (2.8); but it is no longer simply related to the trace (2.6), because the ground state ϵ_{00} contributes appreciably to(2.6) but is excluded from (2.12). The integral on the right side of (2.12) represents the number of noncondensed particles, N'. The well-known expression for N' is

$$N'=N(T/T_\lambda)^{\frac{3}{2}},\qquad(2.14)$$

and hence the moment of inertia of the ideal Bose-Einstein gas below the condensation point is

$$I=N'M\langle x^2+y^2\rangle=(T/T_\lambda)^{\frac{3}{2}}I_0.\qquad(2.15)$$

This concludes the proof that the ideal Bose-Einstein gas *in thermal equilibrium* has a moment of inertia less than the classically expected value, i.e., the ideal Bose-Einstein gas is an equilibrium superfluid. As far as we know, this is the only system known which exhibits equilibrium superfluidity.

There have been many attempts in the literature to derive equilibrium superfluidity from the assumption of an energy gap between the ground state of the N-particle system and some or all of the excited states. For example, Landau[4] assumes that the only states contiguous to the ground state in energy are phonon states in which the circulation vector $\nabla\times\mathbf{v}$ vanishes. However, no one has been able to demonstrate the existence of such an energy gap from first principles. Thus the only system known at present to be an equilibrium superfluid does *not* have an energy gap.

3. ANGULAR MOMENTUM OF AN IDEAL BOSE-EINSTEIN GAS

The definition (1.1) of the moment of inertia I in thermal equilibrium is reasonable from a theoretical point of view. Experimentally, however, the angular velocity ω cannot be made smaller than some minimum value, determined by the sensitivity of the measuring equipment. Thus the experimentalist determines not the derivative $\partial\bar{L}/\partial\omega$ but rather a difference quotient

$$I_{\text{eff}}=\Delta\bar{L}/\Delta\omega.\qquad(3.1)$$

We now propose to study the angular momentum $\bar{L}(\omega)$ of the ideal Bose-Einstein gas in thermal equilibrium as a function of the angular velocity ω, rather than merely the derivative $\partial\bar{L}/\partial\omega$ at $\omega=0$. This will enable us to find the quantity I_{eff}, of (3.1).

According to (2.7), $\bar{L}(\omega)$ is given by

$$\bar{L}(\omega)=\sum_{\gamma}\sum_{m=-\infty}^{\infty}\frac{\hbar m}{\exp[\beta(\epsilon_{m\gamma}-\hbar\omega m-\mu)]-1}.\qquad(3.2)$$

Let us now see what happens as ω is increased slowly from zero. Initially, the macroscopic number of particles in state ϵ_{00} does not contribute to (3.2), since $m=0$ for these particles. Thus initially $\bar{L}$ is a linear function of ω with the nonclassical slope (2.15).

However, this behavior does not persist. As ω increases smoothly, a point is reached at which the lowest energy level with $m=1$, i.e., ϵ_{10}, fulfills the equation

$$\epsilon_{10}-\hbar\omega=\epsilon_{00}.\qquad(3.3)$$

At this value of ω the thermal equilibrium distribution changes abruptly. A macroscopic number of particles shifts from state ϵ_{00} to state ϵ_{10}. Correspondingly, there occurs a finite jump in the value of the equilibrium angular momentum $\bar{L}$; the discontinuity in $\bar{L}$ is equal to $N_0\hbar$ where $N_0=N-N'$ is the number of condensed particles. There is also a discontinuous shift in the internal energy of the system.

Thus, thermodynamically speaking, we have a *phase transition of the first kind*.[9] The chemical potential μ is continuous with discontinuous derivative $\partial\mu/\partial\omega$; the same is true of the free energy. The internal energy and the angular momentum have discontinuities, and there is therefore a latent heat.

Let us determine the value of ω at which the transition occurs. The energy ϵ_{m0} of the lowest state with angular momentum quantum number m is given by

$$\epsilon_{m0}=\hbar^2x_m^2/2MR^2 \qquad (3.4)$$

where x_m is the first zero of the Bessel function $J_m(x)$. We therefore get

$$\omega_1=(\epsilon_{10}-\epsilon_{00})/\hbar=4.45\hbar/MR^2. \qquad (3.5)$$

An upper limit to the angular velocities at which equilibrium superfluidity can occur, of precisely this order of magnitude, was predicted on qualitative grounds by London.[6] Thus the ideal Bose-Einstein gas is a model of an equilibrium superfluid in London's sense, as well as in the sense of our definition (which refers only to the initial slope of $\bar{L}$ as a function of ω). However, not too much stress should be laid on this point, since the ideal Bose-Einstein gas is a very artificial system, and we shall see (in subsequent papers) that real physical systems do not show equilibrium superfluidity, no matter how small the angular velocity.

Consider a vessel of radius 1 cm and height 1 cm filled with an ideal gas of particles, each of mass equal to the mass of a helium atom. Then ω_1 is of the order of 10^{-3} radian/sec, and the discontinuity in $\bar{L}$ is of the order of 10^{-4} erg-sec. It is clear that these values are much too small for most experimental work.

The phase transition at $\omega=\omega_1$ is not the only one, but is the first of an infinite sequence. The next phase transition occurs when the lowest level with $m=2$ becomes the center of condensation, i.e., when

$$\epsilon_{20}-2\hbar\omega=\epsilon_{10}-\hbar\omega. \qquad (3.6)$$

In order to estimate the value of ω at which the mth phase transition occurs, we need an estimate for the first zero x_m of $J_m(x)$. We use the asymptotic formula[10]

$$x_m=m+1.856m^{\frac{1}{3}}+O(m^{-\frac{1}{3}}), \qquad (3.7)$$

and the definition

$$\hbar\omega_m=\epsilon_{m0}-\epsilon_{m-1,\,0} \qquad (3.8)$$

to get the following value for the angular velocity at which the mth phase transition occurs:

$$\omega_m=(m+2.474m^{\frac{1}{3}}-\tfrac{1}{2}+\text{order } m^{-\frac{1}{3}})(\hbar/MR^2). \qquad (3.9)$$

At each critical point ω_m the angular momentum $\bar{L}$ increases suddenly by $N_0\hbar$, where N_0 is the number of condensed particles.[11] The over-all curve of $\bar{L}$ against ω is shown in Fig. 1.

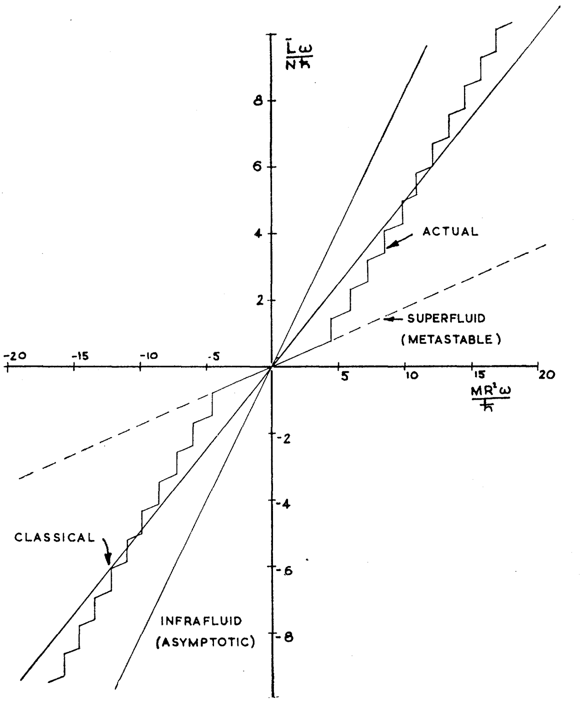

FIG. 1. Thermal equilibrium value of the angular momentum of an ideal Bose gas in a rotating container, *vs* the angular velocity of the container. The portion of the curve near $\omega=0$ has a slope less than the classically expected value, corresponding to superfluid behavior. Asymptotically, for large ω, the mean slope becomes larger than the classically expected value, corresponding to "infra-fluid" behavior, i.e., an abnormally high moment of inertia. The sharp breaks in the curve are thermodynamic transitions of the first kind, due to a macroscopic number of particles shifting from one quantum state to another. If this change-over takes a time much longer than the time in which the experiment is carried out, the angular momentum is given by the dashed curve, and exhibits superfluid behavior throughout.

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[9] An objection may be raised against this terminology on the grounds that the critical value of ω, (3.3) and (3.4), depends on the radius R of the vessel, and approaches 0 as the radius of the vessel is increased. Phase transitions have a meaning only in the limit as the number of particles in the system approaches infinity, and this means a vessel of infinite volume, other things being equal. However, the volume of the vessel can be increased indefinitely without changing the radius R, merely by increasing the height. More physically, we can estimate the range $\Delta\omega$ of angular velocities over which the transition occurs for a vessel of finite height. For a typical vessel (radius=height=1 cm) $\Delta\omega$ is of order 10^{-9} radian/sec compared to a critical value ω_1 of order 10^{-4} radian/sec. Thus the transition is already almost perfectly sharp.

[10] G. N. Watson, *Bessel Functions* (Cambridge University Press, London, 1948), p. 516.

[11] Strictly speaking, N_0 is itself a function of ω, since the number of particles which can be accommodated in the normal fluid part of the Bose-Einstein distribution changes as a result of the centrifugal force. This effect, however, can be shown to be quite negligible mathematically; the physical reason is that for typical containers (radius=1 cm) filled with particles of the mass of a helium atom, and rotating with angular velocities of the order of 1 rad/sec, the kinetic energy due to the rotating motion, of order $MR^2\omega^2$, is many orders of magnitude less than kT (T being of order 1°K).

For a vessel of reasonable dimensions these phase transitions are very closely spaced. Hence, even if we had an ideal Bose-Einstein gas at our disposal, and even if thermal equilibrium could be established quickly enough to allow normal experiments to proceed, these phase transitions would be missed. The experimentally observed moment of inertia would be the difference quotient $\Delta\bar{L}/\Delta\omega$, (3.1) evaluated over a range $\Delta\omega$ large compared to ω_1, (3.5). Since $\bar{L}$ increases suddenly by $N_0\hbar$ each time a new transition point ω_m is reached, the contribution of these jumps to the mean slope $\Delta\bar{L}/\Delta\omega$ is

$$I_s=\frac{N_0\hbar}{\omega_m-\omega_{m-1}}=N_0MR^2(1-0.825m^{-\frac{2}{3}}+\cdots). \quad (3.10)$$

For a vessel of radius 1 cm rotating with $\omega=1$ rad/sec, and filled with particles of mass equal to that of a helium atom, the value of m is of order 10^4, and hence the correction term in (3.10) is of order 2×10^{-3} of the main term. The observed moment of inertia is obtained by adding to (3.10) the contribution of the normal fluid part of the Bose-Einstein distribution, i.e., (2.15). The total is

$$I_{\text{eff}}=(2N_0+N')\tfrac{1}{2}MR^2=\left(1+\frac{N_0}{N}\right)I_0. \quad (3.11)$$

Thus, the ideal Bose-Einstein gas is not a normal fluid even if one averages over the phase transitions; rather, it now appears to be an "infrafluid," with a moment of inertia *larger* than the classical one, in the ratio $1+(N_0/N)$.[12]

The origin of this infrafluid behavior is easy to understand: the wave function of the condensed particles is proportional to $J_m(x_m r/R)$, and since x_m is the *first* zero of $J_m(z)$, this function is concentrated near the outside of the container. Unlike the normal fluid particles whose energy kT is much larger than the kinetic energy of the rotating motion, the condensed particles are pushed out immediately by the centrifugal force. Thus they contribute a moment of inertia MR^2 per particle, whereas the normal fluid component, with its uniform distribution in space, contributes $\frac{1}{2}MR^2$ per particle to the total moment of inertia. Of course, at extremely high angular velocities even the normal fluid component is pushed out toward the rim of the bucket, and the limiting value of the over-all moment of inertia per particle is MR^2. However, the ideal Bose-Einstein gas below the condensation point contains a non-negligible number of particles which are pushed out towards the rim practically immediately by the centrifugal force, long before the normal centrifugal effects are felt. Thus "infrafluidity" is not merely a particularly strong centrifugal effect understandable on a classical basis, but is a distinct quantum-mechanical effect of the Bose-Einstein statistics.

4. IDEAL BOSE-EINSTEIN GAS AS A MODEL OF A NONEQUILIBRIUM SUPERFLUID

The preceding section has shown that the ideal Bose-Einstein gas, in spite of its nonclassical moment of inertia I, is not an adequate model of an equilibrium superfluid. Unless abnormally small angular velocities are used (their size depending upon the radius of the bucket) the ideal Bose-Einstein gas would show equilibrium infrafluidity, not superfluidity.

However, the calculation is instructive in that it gives us a clue to the nature of a possible *nonequilibrium* superfluid state. Let us suppose that equilibrium is reached quickly for the highly excited states (excitation energy of order kT) but that the phase transitions discussed in Sec. 3, in which macroscopic numbers of particles shift from one very low-lying state to another, take a very long time to complete, a time much longer than the duration of usual experiments. Under these conditions, we get superfluid behavior of the ideal Bose-Einstein gas below its transition point. This nonequilibrium value of $\bar{L}$ as a function of ω is shown in Fig. 1 by the dashed line.

In this sense, therefore, the ideal Bose-Einstein gas can be used as a model of a superfluid: there is normal fluid behavior above the transition point, then a thermodynamic transition to a state in which superfluidity can show itself as a transport (nonequilibrium) phenomenon. Of course, the assumption of no interaction between the particles is not adequate to encompass transport phenomena; the concept of a mean free path has to be introduced, and the relaxation times for various kinds of momentum transfer must be studied in detail. Such a theory has been developed, following the suggestions made here, by Klemens.[13]

The results of this analysis of the ideal Bose-Einstein gas suggest strongly that the superfluid phenomena observed in actual liquid helium are also nonequilibrium phenomena; that the superfluid state is only metastable, not thermodynamically stable. The proof that this is indeed the case will be given in subsequent papers.

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We would like to thank Dr. M. R. Schafroth for innumerable, extremely helpful discussions, without which this work might not have been done at all. We are also indebted to Dr. P. G. Klemens for valuable discussions on estimates of relaxation times.

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[12] The contrary statement in our earlier Letter to the Editor [J. M. Blatt and S. T. Butler, Phys. Rev. 96, 1141 (1954)] was due to our having overlooked a factor 2.

[13] P. G. Klemens (to be published). We are grateful to Dr. Klemens for many valuable discussions on estimates of relaxation times, in particular for calling to our attention the great importance of the rapid variation of relaxation times with the relative velocity v_n-v_s between normal fluid and superfluid. While the weakly interacting Bose-Einstein gas provides a model of superfluidity with many of the qualitative features correctly reproduced, we do not believe that this model is sufficient for actual liquid helium. The thermodynamic properties of liquid helium are so different from those of an ideal (or almost ideal) Bose-Einstein gas that we believe it is unlikely that transport properties, which generally depend upon details of the model much more than thermodynamic properties, are given correctly by the weakly interacting Bose-Einstein gas model.