consistent wave functions as seen in Fig. 1 and Fig. 2. Rapid convergence resulted when

$$x+2y-2z=4 \tag{2}$$

and

$$4x-2y+2z=16. \tag{3}$$

These are the minimum Wigner conditions for saturation.

On the other hand, the Yukawa case,[2] with $f(r_{12}) = -35.6[\exp(-r_{12}/1.4)]/r_{12}$ Mev, converged slowly, but the most stable solution resulted for the same values of the exchange strengths. We think that more rapid convergence would have resulted had we used primitive orbitals with long tails.

The interesting result is that the rms radius of both the self-consistent collective potential and the particle density are within a few percent of each other: 6.44 and 6.23 (in units of 10^{-13} cm) respectively in the Gaussian case, and 5.90 and 6.16 respectively in the Yukawa case. The potential does not extend beyond the particles because the exchange forces cancel the direct forces in the outer regions of the nucleus.

It is important to point out that only integer values of the exchange strengths were used, as seen in Figs. 1 and 2. The right-hand sides of Eqs. (2) and (3) are probably slightly greater than 4 and 16. For this reason the difference in the radii quoted in the last paragraph is not significant, so that one may assume that matter and potential radius are the same.

If the rms radii of the particle densities are compared to the uniform density model, $R=r_0A^{\frac{1}{3}}$, we obtain $r_0=1.37\times10^{-13}$ and 1.31×10^{-13} cm for the Gaussian and Yukawa cases, respectively.

The 90 to 10 percent fall-off distance of the matter density, r_{90-10}, obtained for the Gaussian case is about 2.7×10^{-13} cm which is compatible with the fall-off obtained by Hofstadter *et al.*[3] for gold, about 2.4×10^{-13} cm. The value of r_{90-10} obtained with Yukawa interactions was 3.1×10^{-13} cm.

Drell[4] has offered an argument in favor of long-range attractive and short-range repulsive forces in order to account for the discrepancy between neutron and electron measurements of the nuclear radius, yet preserve a homogeneous mixture of neutrons and protons. In our calculations, we used repulsions (exchanges) of the same range as the attractions and obtained the same radius for matter and potential. This is in agreement with Drell's result.

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Strong Pion-Pion Interaction Model Applied to Pion-Nucleon Scattering in the 1-Bev Region*

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THE total cross section for scattering of pions by protons has been measured using high-energy pion beams at Brookhaven.[1] The $\pi^- - p$ cross section shows a pronounced maximum at about 900 Mev, while the $\pi^+ - p$ cross section shows a minimum at about 600 Mev. (See Fig. 1.) Here we discuss the qualitative aspects of the $\pi^\pm - p$ cross section in the 1-Bev region based on a strong pion-pion interaction model.

(1) A nucleon is a compound system of a nucleon core and a virtual pion cloud, whose probability of having more than one pion is neglected.

(2) There is a strong interaction between the incident pion and a pion in the cloud. In particular we assume the presence of a resonance state of the two pions, when they make the isotopic spin $T=1$ state.[2]

(3) The incident pion and the nucleon core do not interact strongly, because the wavelength of the incident pion is very small and the nucleon core is localized in a small region.

Using the impulse approximation for the pion-pion interaction and assuming the charge independence of our system, the total cross sections are given by

$$\sigma(p,\pi^-) = \mathcal{P}\{(2/9)\bar{\sigma}_0 + \tfrac{1}{2}\bar{\sigma}_1 + (5/18)\bar{\sigma}_2\} \tag{1}$$

and

$$\sigma(p,\pi^+) = \mathcal{P}\{\tfrac{1}{6}\bar{\sigma}_1 + \tfrac{5}{6}\bar{\sigma}_2\}, \tag{2}$$

where $\mathcal{P}$ is the probability of the one-pion state in the cloud and σ_T is the pion-pion collision cross section for

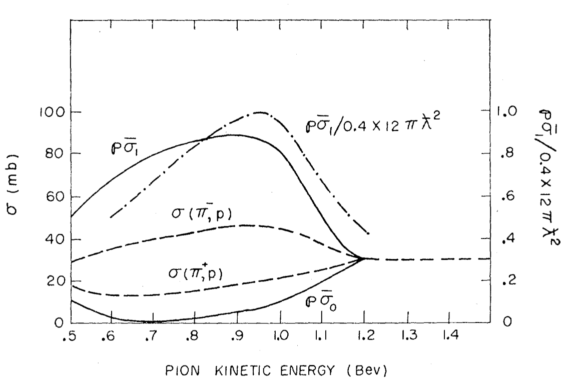

FIG. 1. The pion-pion cross section averaged over the momentum distribution of the bound pion. The dashed curves $\sigma(\pi^-,p)$ and $\sigma(\pi^+,p)$ are experimentally measured curves of $\pi^\mp$ cross sections by protons. (See reference 1.) The solid curves $\mathcal{P}\bar{\sigma}_1$ and $\mathcal{P}\bar{\sigma}_0$ are obtained from the $\sigma(\pi^\mp,p)$ curves by using Eqs. (1) and (2) and the assumption $\bar{\sigma}_0=\bar{\sigma}_2$. The dot-dashed curve is the ratio of $\mathcal{P}\bar{\sigma}_1$ to $\mathcal{P}\times12\pi\lambda^2$ ($\mathcal{P}=0.40$), the maximum *P*-wave collision cross section of the incident pion with the bound pion neglecting the internal motion of the bound pion.

an isotopic spin T ($=0$, 1, or 2) state of the two pions. $\bar{\sigma}_T$ is the average of σ_T over the momentum distribution of the pion cloud.

Since the wavelength of the incident pion in the center-of-mass system of the two pions is still large ($\sim 2\hbar/m_\pi c$) even at 900-Mev incident energy, only S and P wave interactions of the two pions are sufficiently strong to explain the large pion-nucleon cross sections in the 1-Bev region.

Assuming $\bar{\sigma}_0=\bar{\sigma}_2$, we obtained $\mathcal{P}\bar{\sigma}_1$ and $\mathcal{P}\bar{\sigma}_0$ from the experimental values of $\sigma(p,\pi^\pm)$. (See Fig. 1.) $\mathcal{P}\bar{\sigma}_1$ is much larger than $\mathcal{P}\bar{\sigma}_0$ and shows a pronounced peak at about 900 Mev.[3] Applying the one-level formula to the p-wave pion-pion scattering ($T=1$),

$$\sigma_1=12\pi\lambda\!\!\!^{-2}(\Gamma/2)^2/[(\epsilon-\epsilon_0)^2+(\Gamma/2)^2], \tag{3}$$

where $\lambda\!\!\!^{-}$ and ϵ are the wavelength and the energy of the incident pion in the center-of-mass system of the two pions, and neglecting the internal motion of the pion cloud, we can reproduce the $\mathcal{P}\bar{\sigma}_1$ curve between 500 Mev and 1100 Mev, if $\mathcal{P}\approx 40\%$, $\epsilon_0\approx 155$ Mev, and $\Gamma=50$–90 Mev.

The Doppler effect due to the internal motion of the pion cloud brings an additional width into the $\mathcal{P}\bar{\sigma}_1$ curve. For simplicity, we consider the case where the width due to this Doppler effect is larger than Γ, replacing Eq. (3) by

$$\sigma_1=12\pi\lambda\!\!\!^{-}{}_0^2(\pi\Gamma/2)\delta(\epsilon-\epsilon_0), \tag{3'}$$

where $\lambda\!\!\!^{-}{}_0$ is the wavelength corresponding to the resonance energy ϵ_0. $\mathcal{P}\bar{\sigma}_1$ is given by averaging Eq. (3′) over the momentum distribution of the pion cloud. If we take a momentum distribution of the form[4] $q^2\exp(-\alpha q^2/m_\pi^2)$, the experimental $\mathcal{P}\bar{\sigma}_1$ can be obtained by taking $\alpha=6$–10 and $\Gamma=55$–40 Mev with $\epsilon_0\approx 155$ Mev and $\mathcal{P}\approx 40\%$. Because of the large effect of the internal motion of the pion cloud, only a very narrow distribution of the pion cloud can be consistent with the experimental $\mathcal{P}\bar{\sigma}_1$.[5]

Our model predicts also the following features of pion-nucleon scattering. Since the probability of the pion cloud having more than one meson is small, the inelastic pion-nucleon scattering is predominantly one-pion production until the incident pion energy becomes high enough to produce a pion by a pion-pion collision. If the latter becomes appreciable when the pion energy in the center-of-mass system of the two pions is larger than 340 Mev, then the two-pion production in the pion-nucleon scattering becomes appreciable for a larger energy than 1.7 Bev.

If the pion-nucleon scattering occurs only through the $T=1$ state of the two pions, the ratios of the various types of inelastic scattering are

$$\sigma(p+\pi^-\to p+\pi^-+\pi^0):\sigma(p+\pi^-\to n+\pi^-+\pi^+):\sigma(p+\pi^-\to n+2\pi^0)=1:1:0$$

and

$$\sigma(n+\pi^-\to n+\pi^-+\pi^0):\sigma(n+\pi^-\to p+2\pi^-)=1:0$$

when there is no final interaction between the pions and the nucleon, while

$$\sigma(p+\pi^-\to p+\pi^-+\pi^0):\sigma(p+\pi^-\to n+\pi^-+\pi^+):\sigma(p+\pi^-\to n+2\pi^0)=17:26:2$$

and

$$\sigma(n+\pi^-\to n+\pi^-+\pi^0):\sigma(n+\pi^-\to p+2\pi^-)=13:2$$

when either one of the two pions composes a $T=\frac{3}{2}$ isobar state with the nucleon after the two-pion interaction. These ratios are consistent with the experimental data.[6] The ratio between the elastic scattering and the inelastic scattering depends on the strength of the interaction between the pions and the nucleon after the two-pion interaction.

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[2] F. J. Dyson [Phys. Rev. **99**, 1037 (1955)] has proposed independently a similar model which assumed the presence of a resonance state of the two pions in $T=0$ state. We are obliged to him for sending us his manuscript before publication and for comments he made on this paper. Also we are very grateful to Dr. M. Ross who informed us about Dyson's work.
[3] The arbitrariness of the assumption $\bar{\sigma}_0=\bar{\sigma}_2$ does not change these qualitative conclusions about $\mathcal{P}\bar{\sigma}_1$ as long as only S and P wave interactions are considered.
[4] This is proportional to the probability of a pion in the cloud for having a momentum between q and $q+dq$. The momentum distribution can be taken to be spherically symmetric, since the direction of spin of the nucleon is averaged out.
[5] This difficulty was pointed out to the author by Dr. O. Piccioni, who had suggested a similar model several years ago, and by Dr. M. Ross. If the strong pion-pion interaction works only when the bound pion has a very small momentum, this difficulty might be solved. This possibility was suggested to the author by Professor F. J. Dyson.
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Spatial Extension of the Proton Magnetic Moment from the Hyperfine Structure of Hydrogen

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UNTIL recently the measurement of the hyperfine splitting of the ground state of hydrogen, used in conjunction with the theoretical formula,[1] has provided the most accurate means of ascertaining the value of the fine structure constant, $\alpha=e^2/\hbar c$. In this note, we propose to employ the formula, together with an independent measurement[2] of α, to estimate a "magnetic radius" for the proton.

The doublet separation, $\Delta\nu_H$, can be represented to