choice of matrix elements as above, we find: $\rho_1=0.18$, $\rho_2=0.44$, $\rho_3=0.39$, and $\rho_4=0.98$. Thus we conclude that $|\bar{H}|^2=1$ (isotropic and energy independent) appears to be ruled out by experiment, whereas the other choices of matrix elements are probably not inconsistent with the observations, considering the limited statistics of the experiment and the approximations made in the calculations. In this connection, we note that some of the K mesons emitted from a heavy nucleus may have been produced in a secondary reaction, i.e., pions were formed which subsequently collided with a nucleon within the same nucleus to produce a K particle. It is also possible that some of the K mesons emerging at 45° or 90° were produced near the forward direction but were scattered through large angles by collision with nucleons within the target nucleus.

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High-Energy Electromagnetic Phenomena in Cosmic Radiation*

M. Koshiba and M. F. Kaplon
Department of Physics, University of Rochester, Rochester, New York
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Isolated high-energy electron showers in photographic emulsion have been investigated and have yielded the following conclusions: (1) out of 16 cases of isolated electron showers observed to originate from single electron pairs of energy greater than 1 Bev, 2 cases have been found to be anomalous in the sense that they seem to have been initiated by more than 2 photons; one of the two has been analyzed in detail. (2) The discrepancy between the experimental observations and theoretical predictions on the trident process found in a previous work has been obtained again with the additional experimental data of this experiment.

1. INTRODUCTION

SINCE the work of Schein *et al.*[1] in 1954 on an anomalous electron shower pointed to some possible difficulties in high-energy cascade showers, it has been felt that this field requires a more detailed survey. The purpose of this work is twofold; to check the results obtained previously[2] pertaining to the trident process with additional experimental data and to investigate the high-energy cascade shower development in emulsion under the more favorable experimental conditions of single γ-ray initiated showers to see whether there actually exists some anomaly or not. It seems pertinent, however, to give some general comments on the difficulties inherent in this field before the presentation and discussion of this experiment.

First of all, it must be emphasized that in the development of individual electron-photon cascades, the fluctuations in the numbers of electrons and photons (referred to as number fluctuations) from the average value can be enormously large. In fact, the number fluctuations in the cascade process were, under some simplified assumptions, shown to be similar to that of the Polya distribution, instead of the familiar Poisson distribution of random events. This means that the number fluctuation from the average can be as large as the average itself. In order to illustrate the situation more clearly the results of a Monte Carlo calculation[3] on cascade showers have been given in the Appendix. The original results on 100 showers with a single photon primary were obtained by one of us (M.F.K.) in collaboration with D. M. Ritson, using the cross sections of Approximation A of Rossi and Griesen.[4] The other results with different initial conditions were derived from the original results by a change of shower origin or by a superposition of different initial conditions. As can be seen from these results the number fluctuations are quite large compared with those encountered in random processes.

There is another difficulty which arises when we attempt to measure electron energies. The available methods of energy measurement for electrons or photons, by determination of the multiple Coulomb scattering or of the opening angle of a converted electron pair, have, when applied to high-energy electrons or photons, some defects which usually lead to an underestimation of the energy. That is, in the conventional method of multiple Coulomb scattering, no account is taken of the bremsstrahlung energy loss of the electron. Also, in the energy estimation of a γ ray from the opening angle of its converted electron-positron pair, some care must be

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[1] Schein, Haskin, and Glasser, Phys. Rev. **95**, 855 (1954); A. Debenedetti *et al.*, Nuovo cimento **12**, 954 (1954); N. Dallaporta (private communication).

[2] M. Koshiba and M. F. Kaplon, Phys. Rev. **97**, 193 (1955), hereafter referred as I.

[3] This method was first proposed by S. Ulam and J. Von Neumann, Bull. Am. Math. Soc. **53**, 1120 (1947).

[4] B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 240 (1941).

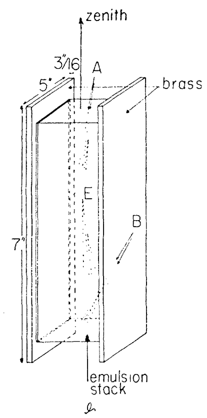


FIG. 1. Condition of exposure of emulsion stack; the events labeled A, B, and E characterize the observational origin of the electron showers.

given to the effect of the scattering of the electrons as well as to the energy loss due to bremsstrahlung; for high-energy electron pairs we have to go several millimeters away from the pair origin to measure the opening angle and in this distance the separation due to the scattering cannot, *a priori*, be neglected.[5]

In Sec. 2, the experimental results on isolated showers[6] in the cosmic radiation observed in photographic emulsion will be presented and discussed. In Sec. 3, an apparently anomalous case of an electron shower is analyzed in detail and in Sec. 4 some additional results on the trident process, direct pair production by an electron, will be given. Finally, in Sec. 5 the results of this work are summarized and the possible inferences of the results are discussed.

2. ISOLATED ELECTRON SHOWERS

A stack of 49 stripped emulsions (Ilford G–5) were flown in Sky Hook Balloons at 41° Geomagnetic latitude. These emulsions were processed and aligned by the method described by Crussard *et al.*[7] and the condition of exposure is shown in Fig. 1.

In order to see whether there are cases of electron showers with anomalously large numbers of electrons occurring more frequently than predicted by the Monte Carlo calculation, these plates were scanned for parallel tracks of minimum grain density (hereafter these tracks will be referred simply as minimum tracks). The parallel minimum tracks thus found were followed from plate to plate to find their origin. This tracing was done with a 40× immersion objective and 10× eyepiece. Target diagrams including neighboring tracks were drawn at each emulsion surface to insure correct tracing, which otherwise would be quite difficult, especially when the number of minimum tracks becomes small.

In the process of tracing showers back to an origin, some of them were found to have come from outside the emulsion stack either via the brass plates used to press the stack tightly together during the exposure or directly from the air, passing through a very small quantity of packing material (wood and foam rubber). These cases were classified as B (brass) and A (air), respectively. In addition, some were found to originate from a pair of minimum tracks and some from nuclear interactions in the emulsion. The latter cases were classified as N (nuclear) and are omitted from this analysis. When a pair of minimum tracks starting in the emulsion was found to be the origin of the shower under consideration, scanning for further minimum tracks parallel to this pair was done within 150 μ from the pair (this is an experimental criterion imposed by the limitation of the field of view). If such additional tracks were found they were traced further in the same manner until no further possibly associated tracks in the above specified vicinity of the pair were found. These cases were then classified as E (emulsion). Among these E cases, the origin of which are most probably electron-positron pairs converted in the emulsion, those cases in which the first pair energy was lower than 1 Bev have been discarded from the later analysis. The reason for doing this is that because of the poor scanning efficiency for low-energy cases we have to expect a considerable number of them to be missed in the initial scan. Only those E cases with the first pair energy greater than 1 Bev were used for the study of the cascade development. As for the B cases, we cannot be sure about their initial conditions at the entrance point to the sensitive volume. Some of the A cases might actually have originated in the emulsion, for in the A cases we can trace them only up to a few millimeters from the outer emulsion edge, the vicinity of which is usually quite distorted and blackened by the excess deposit of silver grains. B and A cases were therefore omitted from the detailed analysis due to the uncertainty in initial conditions. Finally, some cases of unsuccessful tracing are designated as U. The appearance in the stack of these various cases, A, B, and E, is illustrated in Fig. 1.

TABLE I. Classification of results of scanning and tracing of electron showers. The numbers represent the number of cases of showers of a given origin (B, A, E, or U defined in the test) having 1, 2, 3, or ≥ 4 minimum tracks at the observational shower origin.

Class	No. of initial tracks 1	2	3	4	≥ 1
B	8	15	2	5	30
A	4	4	0	1	9
E	0	16	0	0	16
U	7	1	0	1	9
Total					64

[5] These points will be discussed in a forthcoming paper in which an improvement of the scattering method for high-energy electrons will be presented with experimental data (if available).

[6] By an isolated electron shower in this paper, we mean a shower of high-energy electrons in emulsion, which, under our experimental conditions, is not obviously associated with any other types of particles or events.

[7] Crussard, Kaplan, Klarmann, and Noon, Phys. Rev. **93**, 253 (1954).

TABLE II. Summary of results of E-type shower analysis. The first column gives the shower designation, the second the cosine of its zenith angle, the third the longitudinal distance of observation from the initiating pairs, the fourth the designation of individual pairs, tridents and compton electrons observed within the first 0.5 radiation length, the fifth the distance of the event in column 4 from the first observed pair, the sixth gives the energies of the corresponding events of column 4 (for the tridents the energy is the sum of the two lowest energy electrons) and the 7th column summarizes the information obtained after the first 0.5 radiation length.

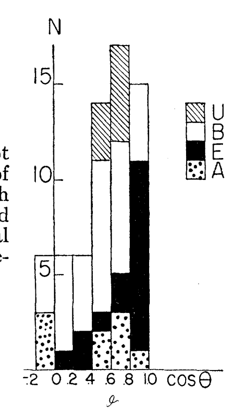
Shower No.	Cosine of the zenith angle	Total distance followed in mm	Pair No.	Distance (mm.) from first pair	Energy (Bev)	Secondaries after 0.5 rad. unit
P*–1	0.22	100.0	P_1	0	7.0	* See II–3
			T_1	2.5	1.4	
			T_2	7.6	0.79	
			P_2	9.0	1.7	
			T_3	11.8	0.23	
P*–7	0.80	36.7	P_1	0	1.9	Total opening angle 3×10^{-3} radian P_2 (4.0 Bev) P_3 (0.23 Bev) P_4 (5.3 Bev) T_1 (1.2 Bev) and P_5 (?) are observed after 0.5 rad. length * See the text
P–10	0.88	18.9	P_1	0	15.0	T_2 (1.4 Bev) and
			T_1	12.3	0.96	T_3 (0.14 Bev)
P–12	0.90	15.0	P_1	0	3.3	
			P_2	6.4	0.38	
P–15	0.51	16.1	P_1	0	48.0	P_3 (0.20 Bev) and
			P_2	5.4	0.07	P_4 (0.70 Bev)
P–25	0.995	23.5	P_1	0	1.7	P_3 (0.16 Bev)
			P_2	13.8	0.24	
L–5	0.27	34.1	P_1	0	13.0	2 more T's and
			T_1	5.0	6.0	2 more P's
			P_2	10.0	0.03	
			T_2	12.0	23.0	
L–12	0.82	16.9	P_1	0	6.3	P_3 (0.11 Bev)
			T_1	3.3	1.3	
			T_2	5.5	0.8	
SH–6	0.88	19.4	P_1	0	90.0	T_2 (0.07 Bev)
			T_1	12.6	10.0	
SH–7	0.91	23.0	P_1	0	30.0	P_4 (0.02 Bev),
			T_1	4.2	0.07	P_5 (0.4 Bev) and
			T_2	9.5	1.6	P_6 (0.05 Bev)
			P_3	12.5	0.05	
SH–15	0.93	26.0	P_1	0	4.7	T_1 (0.33 Bev) and K_1 (or C_1)
SH–17	0.82	21.8	P_1	0	10.0	
			P_2	4.0	0.092	
			P_3	11.1	0.018	
SH–23	0.97	21.6	P_1	0	3.0	P_4 (0.58 Bev)
			T_1	9.7	0.15	
			P_2	13.0	0.18	
			P_3	13.3	0.15	
SH–27	0.93	29.1	P_1	0	2.9	P_3 (0.25 Bev),
			P_2	10.4	0.90	P_4 (0.021 Bev), P_5 (0.074 Bev), P_6 (0.10 Bev) and C_1
SH*–29	0.75	19.0	P_1	0	24.0	P_5 (0.11 Bev) and
			P_2	4.6	24.0	P_6 (0.042 Bev)
			P_3	9.8	0.45	* See the text
			P_4	14.3	1.5	
SH–35	0.045	15.4	P_1	0	1.0	
			P_2	3.4	0.15	
			P_3	4.6	0.085	
			P_4	11.7	0.86	
			C_1	12.4	?	
			P_5	12.5	0.02	
SH–22	0.89	15.8	P_1	0	0.65	P_4 (0.25 Bev)
			C_1	5.9	?	* See the text
			P_2	8.7	0.34	
			P_3	9.2	8.0	
			T_1	10.2	0.4	

The results of scanning, tracing and classification are presented in Table I and Fig. 2. In Table I, cases in each classification are tabulated according to their initial number of minimum tracks and in the last column the total number of cases in each class are given.

The angular distribution of the showers (θ is the zenith angle) is given in Fig. 2 and is not corrected for the geometrical factor; the reference axis was taken to be the vertical axis of the stack. There is possibility of different scanning efficiencies for different angles which is rather hard to estimate. It does not seem that much information is obtainable from the angular distribution in this type of experiment except for one feature relevant to the origin of the showers. It may be noticed that there are cases of showers coming upwards which are not observed for the high-energy nuclear showers in the same stack. This presumably indicates the secondary nature of the isolated electron showers (say, γ rays from locally produced π^0's). In 1950 the Bristol group[8] concluded from their experiment on electron pairs in emulsions exposed to the high altitude cosmic radiation that the observed γ rays of energy up to around 800 Mev could be interpreted as due to the π^0 meson-2γ decay. Though our γ-ray energy is appreciably greater than theirs, it seems reasonable to assume that our high-energy γ rays arise also mainly from the π^0 decay process.

The cases classified as E were then traced from their origin to follow their development in detail up to at least a half-radiation length. (This minimum observational distance is a function of the shower geometry and energy.) The energies of the secondary pairs as well as of the initial pair were estimated from their opening angles using Borsellino's[9] formula. The results are summarized in Table II, where P's are pairs, T's are apparent tridents, and C's are Compton electrons. These secondaries were found within a cone having an opening angle of 10^{-2} radian in each case. In Table II, the first, second, and third columns give the shower number, cosine of the zenith angle and the total observed distance from the initiating pair origin. In the fourth column, pairs, tridents, and Compton electrons observed within 0.5 radiation unit are given with their distances from the first pair and their energies in the fifth and sixth columns, respectively. In the last column, the development after 0.5 radiation length is given. (1 radiation length in emulsion is 2.9 cm.)

FIG. 2. Angular distribution (not corrected for geometrical bias) of electron showers. θ is the zenith angle. The labelings U, B, E, and A characterize the observational origin of the showers and are defined in the text.



[8] A. G. Carlson *et al.*, Phil. Mag. **41**, 701 (1950).
[9] A. Borsellino, Phys. Rev. **89**, 1023 (1953).

TABLE III. Comparison of observed cascade development of E-type showers with results of a Monte Carlo calculation. The first row represents the number of additional electrons, N_e, within 0.5 radiation length of the initiating pair origin. The rows labeled A and B represent respectively the number of observed cases having a given N_e with the convention of counting only ordinary pair electrons (A) or ordinary pair electrons plus trident electrons (B). Row C represents the results of the Monte Carlo calculation. The numbers in parentheses represent relative frequency.

N_e	0	1 and 2	3 and 4	5 and 6	7 and 8	≥ 0
A	7 (0.50)	5 (0.36)	1 (0.07)	0 (0.00)	1 (0.07)	14 (1.00)
B	3 (0.21)	5 (0.36)	3 (0.21)	1 (0.07)	2 (0.15)	14 (1.00)
C	25 (0.71)	7 (0.20)	3 (0.09)	0 (0.00)	0 (0.00)	35 (1.00)

We now compare these cascade developments with the results of the Monte Carlo calculation given in the Appendix. Table III was prepared from Table II, using secondary electrons of energy greater than 10^{-2} of the initiating pair energy. In this way, we can be fairly sure of a uniform detection efficiency for various degrees of electron number multiplication, because those electrons with energy lower than 10^{-2} of the initiating energy serve only for the purpose of efficient detection. In Table III the first row, N_e, is the number of additional electrons observed within 0.5 radiation length from the initiating pair origin and of energy specified above. The second and third rows, (A) and (B), give the number of cases having a given N_e with the respective conventions of counting only (A) ordinary pair electrons and (B) trident secondary electrons as well as the ordinary pair electrons. The fourth row, (C), gives the results of the Monte Carlo calculation. The Monte Carlo calculation to be compared with our data was taken from Table A-2-C of the Appendix for the following reasons. The minimum opening angle of the two γ rays from a π^0 meson is equal to $2/\gamma\beta$ (γ and β refer to the primary π^0 meson); this means, for example, that the minimum lateral separation of the two γ rays from a 14 Bev π^0 meson at a distance of 1 cm from the decay (the conversion length in the emulsion is 3.75 cm) is 200 μ and is even larger for lower energy π^0's. By our tracing conventions, it seems appropriate to assume that the other γ ray is outside the investigated area. In fact, the observed energy distance relations in each shower are quite consistent with this assumption except for two cases which were not included in Table III. They are SH–22 and SH–29 of Table II. In SH–22, both pairs $P1$ and $P3$ must be of a primary nature and in SH–29 pairs $P1$ and $P2$ must also be of such a nature because the latter pair is greater in energy than the corresponding first pair ($P1$) in each case. We also note that the further development of shower P–7 beyond a half-radiation length seems to indicate the existence of at least three photons initially present for similar reasons. SH–22 and SH–29 can be understood as originating from the 2γ decay of a single π^0 meson and in these two cases both γ rays contribute to the development. The energies of the two high-energy γ rays in SH–29 and their angular separation are quite consistent with this assumption; in SH–22, no accurate measurement of the angular separation can be made. As to the shower P–7, the existence of more than two photons will be discussed later in connection with the similar nature of shower P–1.

The results given in Table III seem to indicate agreement between the observed development and the Monte-Carlo calculation if we take only pairs (A) and not apparent tridents. If we include the apparent tridents (B) the distribution seems to extend too far in favor of large electron numbers. The assumption of a low shower detection efficiency, as low as $\frac{2}{3}$ which is rather improbable in the procedure described above, does not seem to alter this conclusion. Even though the statistics are poor in both the experimental data and the Monte Carlo results, they seem to indicate that the majority of the apparent tridents may actually be genuine tridents which were not included in the Monte-Carlo calculation. This result was suggested by a similar analysis in a previous experiment[2] (referred to as I).

It is instructive at this point to comment about the cascade development observed in I. The situation there is rather different because of the much higher shower energies and the fact that it occurs in a penetrating shower. When we see a high-energy electron pair in the narrow cone of a penetrating shower, the most reasonable assumption is to assume that this arises from the conversion of a γ ray from the decay of a π^0 meson produced in the primary act. The very high energy of these π^0 mesons as estimated from the opening angle of the cone and the rather short observational length at our disposal requires us to assume that another high-energy γ ray is most probably spatially associated with this high-energy electron pair. (The opening angle of 2 γ rays from a 200-Bev π^0 meson is $\sim 1.4\times 10^{-3}$ radian.) The energy-distance relations of the actually observed cascade development are quite consistent with this idea. Therefore, the initial conditions for these electron cascade showers should be taken as one converted electron pair plus one photon incident at the first high-energy pair origin. Further decomposition into single-photon-initiated showers was not possible in I due to the high degree of the two γ-ray collimation.

The cascade development with the above initial conditions was studied using 8 groups of electron pairs in I which were so located that they could be followed at least 0.5 radiation length from the first pair in each group. The results are given in Table IV and are compared with the results of the Monte Carlo calculation given in the Appendix for the same initial conditions. The numbers in parentheses in the second, third, and fourth rows represent the percentage occurrences. In the second row, Case A, only the electrons of the ordinary pairs were taken whereas in Case B all electrons (ordinary pair electrons and secondary electrons from apparent tridents) were counted. The cut-off energy for acceptance ($10^{-2}E_0$) was chosen to insure good detection efficiency and to agree with the conventions taken in the Monte Carlo calculation.

Because of the small sampling, we cannot obtain any firm conclusions except that the inclusion of the apparent trident electrons, Case B, seems to make the distribution extend to larger electron numbers, slightly more than expected from the Monte Carlo results in which the trident processes were not taken into account. The poor statistics in the experimental results come from the superposition of the two photon showers while for the Monte Carlo results they arise from the fact that the given initial conditions could only be met by superimposing the results from 100 single-photon-initiated showers, since the original calculation had been done only to this extent.

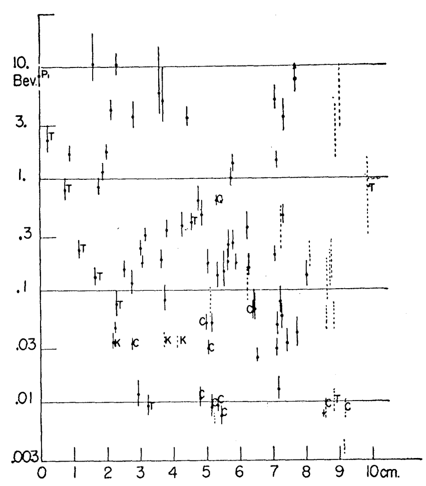


FIG. 3. Energy-distance plot of electron secondaries from shower $P-1$. Unlabeled points are pairs. T means trident, K, knock-on and C, Compton electron. Q is a pair conversion in the field of an atomic electron.

The problem of the trident process and its relationship to the general question of the validity of the Bethe-Heitler[10] high-energy quantum electrodynamics will be discussed later.

3. ANALYSIS OF SHOWER P-1

Because of the anomaly in the energy-distance relation exhibited in Table IV for showers P-1 and P-7, one of them, P-1, due to its very favorable geometry, was investigated in more detail. This shower is inclined at 78° with respect to the vertical axis of the stack and lies almost parallel to the emulsion surface. The beginning of this shower consists of two electrons entering plate $S104$ from the glass side at a point 8.25 mm from its edge. After 48.1 mm traversal in the same plate it entered plate $S103$, which was the neighboring quarter of the same emulsion before cutting, and then entered plate $S93$ which is part of the next emulsion. Every effort to trace these two electrons back into $S114$ (their continuation from $S104$) failed. A detailed investigation of the emulsion showed that the portion of tracks in the upper layer of the emulsion was removed in the process of wiping off the surface silver deposit after development. This layer is about 5μ in depth and because of the very good parallelism between this shower and the emulsion plane, this depth gives a projected length of about 1.8 mm. Therefore, it is almost certain that the pair was created in the upper layer of $S114$ and entered into $S104$ as two separated tracks. The portion of the track thus wiped out should be less than 1.8 mm in length. There is another fact which supports this assumption; that is, there was no appreciable amount of matter before the entrance to the emulsion except for a negligible quantity of wood and foam rubber.

In order to insure good efficiency for detecting secondary events in following the development of this shower, scanning was done at every 1 mm for tracks of minimum grain density using 40× immersion objective and diagrams were made at these points. Every minimum track thus found and recorded in the diagrams was now traced back and forth to get a one to one correspondence between the tracks in each diagram. In this way, all minimum tracks were identified with respect to their origin and their genetic relation to the others. The energies of these identified tracks were measured by their multiple Coulomb scattering. When there was any possibility of noise or distortion, the relative scattering method described in I[11] was used. The results of these measurements are given in Fig. 3. The abscissa is the distance from the entrance point of the first two electrons to $S104$ and the ordinate is the γ-ray energy of each pair or the electron energy in the case of a

TABLE IV. Comparison of observed cascade development of showers from I with results of a Monte Carlo calculation. The first row represents the number of additional electrons, N_e, within 0.5 radiation length of first pair. The rows labeled A and B represent respectively the number of observed cases having a given N_e with the convention of counting only ordinary pair electrons (A) or ordinary pair electrons plus trident electrons (B). Row C represents the results of the Monte Carlo calculation with appropriate boundary conditions (1 pair plus an incident γ ray). The numbers in parentheses represent relative frequency.

	No. of additional electrons $E \geq 10^{-2}E_0(\gamma)$ within 0.5 rad. unit						
	0	1	2	3	4	>4	≥0
Case A	3 (0.38)	0 (0)	3 (0.38)	1 (0.12)	1 (0.12)	0 (0)	8 (1.00)
Case B	3 (0.38)	0 (0)	0 (0)	2 (0.25)	2 (0.25)	1 (0.12)	8 (1.00)
Table A–3–C	1625 (0.46)	195 (0.06)	1010 (0.29)	295 (0.08)	244 (0.07)	131 (0.04)	35×100 (1.00)

[10] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).

[11] This method was first used by Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950).

Compton or knock-on electron. The trident energy has been taken to be the sum of the energies of the two lower energy electrons of the emerging electrons. The statistical probable errors in the scattering measurements are indicated by vertical lines. Some of the low-energy electrons were not measured and in these cases rough estimates were made on the basis of the opening angle and are indicated by vertical dotted lines with the estimates obtained by using Borsellino's[9] and Stearns'[12] formula as end points.

In addition, a survey was made to find any possible event associated with this shower. For this purpose, scannings were made across the shower axis for tracks parallel to the shower at distances 1 cm apart longitudinally and up to 1.2 cm in width on both sides of the shower axis using a 40× objective. In this scanning a high-energy pair was found which originated in the same emulsion ($S104$) 5332 μ away from the shower axis. The opening angle and the projected inclination of this pair with respect to the shower axis of P–1 were found to be 0.82×10^{-3} and $(1.30\pm0.04)\times10^{-2}$ radian, respectively. This pair is also almost parallel to the emulsion plane and its opening angle gives an energy estimate of 1.8 or 5.9 Bev using Bosellino's or Stearns' formula, respectively. If we admit that this pair comes from the same primary event which gave rise to shower P–1, we can estimate the position of this primary event as 41.0 ± 1.0 cm away from the entrance point of shower P–1 to plate $S104$.

The total opening angle of shower P–1 as viewed from the entrance point to $S104$ is 2.8×10^{-3} radian for all pairs and 0.82×10^{-3} radian for pairs of energy greater than 1 Bev observed within four cm from the entrance point. The total visible energy of this shower up to 10 cm from the origin is 80 Bev. This value is obtained by adding the energies of individual pairs as measured by scattering or by opening angle and hence does not include the energy loss of electrons after the terminal points of the scattering measurements. This procedure overestimates the instantaneous visible energy but may be compensated for by the invisible energy carried in the form of γ rays and the actual total energy will not be much different from $\sim$100 Bev.

The anomaly in the energy-distance relation exhibited in Fig. 3 seems to suggest that at least 4 γ rays were present initially. This conclusion should not and does not rest upon the large number of electrons observed, because, as was seen in the previous section, as far as the number of electrons is concerned, this shower is well inside the fluctuation tail if the initial energy is large enough (at least around 100 Bev). The energy of the initial pair give in Fig. 3 has been estimated in the following way. One of its electrons ($P1^2$) was measured directly by the scattering method using 500 and 1000 μ cells, both giving quite consistent energies, their mean being $1.30_{-0.28}^{+0.48}$ Bev. The other track of the initial pair ($P1^1$) made a trident after 2.46 mm. The scattering measurements on the portion of $P1^1$ before producing $T1$ gave a lower limit of 5 Bev. The energies of $T1^1$ and $T1^3$ were measured to be $0.93_{-0.20}^{+0.34}$ and $0.45_{-0.11}^{+0.12}$ Bev, respectively, by the relative scattering method,[11] using 250 and 500 μ cells. The energy of $T1^2$ was still too high to be estimated and it gave rise to a further trident, $T2$, after 5.117 mm traversal. $T2^1$ and $T2^3$ were measured in a similar way; $T2^2$ was found to be about 5 Bev and it gave still a third trident, $T3$, after 4.13 mm. The highest-energy track of $T3$, $T3^3$, was consistently measured to be 4.9, 4.0, and 4.9 Bev using 500, 1000, and 2000 μ cells, respectively. The energy of $P1^1$ was considered to be the sum of all these secondaries (tridents) and is thus $7.1_{-1.1}^{+1.4}$ Bev. Of course, there is a possibility that electron $P1^1$ had initially an energy of about 100 Bev and became an electron of about 5 Bev by emitting quite a number of γ rays including those higher energy ones of from 5 to 10 Bev whose conversion to pairs accounts for the energy distance anomaly. However, this is a very drastic behavior when we recall that the available distance is about 0.3 radiation length for the electron energy to go from 100 Bev to 5 Bev and that this is not accomplished by the emission of single γ ray of about 95 Bev but by the emission of a number of γ rays of rather small energy as compared with the initial energy. Therefore, it seems more likely to assume that there were at least 4 γ rays of energy around 10 Bev present initially. We also note here that the direction and the energy of the pair found about 5 mm away from shower P–1 would give about 22 Bev or less for the initial single γ-ray energy of shower P–1 if we assume these two pairs are due to the two γ rays from the same π^0-meson decay. This possibility is thus excluded and we must consider that they (pair P–1 and the separated correlated pair) are due to different γ-ray sources, possibly different π^0 mesons.

Let us now ask about the meaning of this event. For the sake of illustration, imagine that we had observed the shower discussed in I without any associated charged penetrating particles. Then we would have concluded

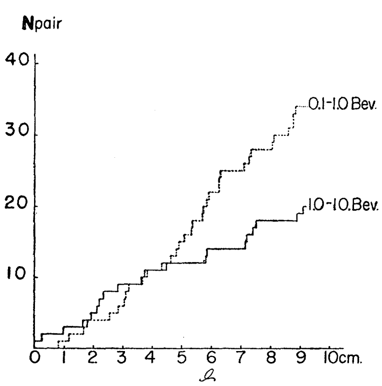


FIG. 4. Integral electron numbers for shower $P-1$ as a function of distance from the origin for electrons in the energy intervals 0.1–1 Bev and 1–10 Bev.

[12] M. Stearns, Phys. Rev. **76**, 836 (1949).

from Fig. 2 of I, by arguments similar to those presented above, that there must have been at least several γ rays present initially on the basis of the energy distance relation and the total energy of this apparently anomalous shower would have been estimated to be around 1000 Bev. However, when we look at the total opening angle of this shower it is found to be 1.1×10^{-2} radian as compared with 2.8×10^{-3} radian for shower P–1 whose total energy is one order of magnitude less than that of this fictitious anomalous shower. Therefore, even if we could explain this fictitious anomalous shower in terms of a fluctuation from the average prediction of charge independence as applied to multiple meson production, the fluctuation giving rise to multiple production of neutral mesons without charged ones, it is much more difficult to apply this kind of argument to shower P–1. The angle of the cone including electron pairs of energy higher than 1 Bev in shower P–1 was 0.82×10^{-3} radian. Since the minimum opening angle of two γ rays from a 20-Bev π^0 meson is 1.4×10^{-2} radian, such a source for these γ rays does not seem quite compatible with the observed data. This argument, however, does not hold if the actual energy of the initiating pair was much higher than that estimated and a fluctuation of the type mentioned previously in the number of emitted γ rays within the first $\frac{1}{3}$ of a radiation length had occurred; this is not entirely impossible. However, when we recall the existence of another possibly anomalous case, P–7, this latter explanation seems less probable. (Shower P–7 exhibits the same anomaly in the energy distance relationship and in the total opening angle as does shower P–1.)

Next, we consider the other features of the shower development of P–1. In Fig. 4, the numbers of electrons of energies from 1 to 10 Bev and from 0.1 to 1 Bev are plotted against their distances from the initial observation point of the first two electrons. What we can say from this figure is that the effect of pairs of the third generation seems to become important at about 5 cm (1.7 radiation lengths) and this is a quite reasonable behavior in view of the ordinary cascade shower theory. We might also notice that there is a bump in the curve of the 1.0–10 Bev electrons over that of 0.1–1.0 Bev group at small distances, where we should expect the same behavior for the two curves if the electrons of both categories are cascade secondaries of the same original electron of much higher energy. The electron pairs, tridents, and Compton electrons of energy smaller than 0.1 Bev were not considered here because the detection efficiency for these low-energy electrons is thought to be poorer than for the higher energy ones. However, from Fig. 3 we can safely say that we cannot neglect the Compton electrons when we are dealing with γ rays of energy smaller than 100 Mev. In addition, the photonuclear reaction will not be negligible if we have to take the Compton effect into account because of the so-called giant dipole maximum of the photonuclear reaction around 20 Mev. To see the effect of this, a strip of 500 μ width with shower P–1 as its central axis was scanned for stars, including low-energy one-prong proton stars and this scan was compared with the background taken from randomly chosen strips of the same width. The results obtained were 4.4 ± 1.7 and 10.8 ± 4.1 stars per 1 cm length of the strip for the regions 0 to 4 and 4 to 8 cm from the origin compared with 5.3 ± 1.0 for background stars in an equal random area. Even though the statistics are poor the low-energy photonuclear effect seems to be observable and this is consistent with the conclusions drawn from Fig. 4. The projected lateral distributions of electrons of energy greater than 100 Mev at 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 radiation lengths from the observational origin of the shower ($\sim$0.06 radiation length from the first pair origin: see the beginning of this chapter) are presented in the form of histograms in Fig. 5 (parts 1, 2, 3, 4, 5, and 6, respectively). The two-dimensional projection of shower P–1 on a plane parallel to the emulsion surface (which is approximately parallel to the shower axis) was divided into strips of equal width (24.3μ) parallel to the projected shower axis, and the number of electrons of energy greater than 100 Mev in each strip is indicated by the number of blocks (the width 24.3μ was taken for observational convenience). The horizontal lines at the bottom of these histograms indicate the scanned area at each distance. Numbers in parentheses in each of the figures are the number of electrons of energy greater than 100 Mev which were produced within the observed area but were scattered out of it before reaching the distance under consideration. These numbers give a crude

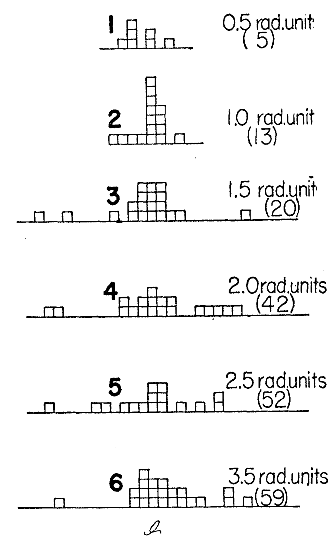


FIG. 5. Lateral distribution of electrons from shower $P-1$ with energy >0.1 Bev. Each histogram is labeled with the longitudinal distance in radiation units from the origin. The numbers in parentheses refer to the number of electrons of energy >100 Mev produced in observed area but scattered out before reaching distance given above. Each block in the histogram is 24.3 μ wide perpendicular to the shower axis.

idea of the extent that the lateral spread of the cascade development is affected by the electron scattering and the emission angle of γ rays of a tertiary nature.

4. TRIDENT PROCESS

The importance of the trident process in the high-energy region was suggested in I. We will consider the problem again in this chapter with the additional data obtained from the analysis of isolated showers.

From the isolated nonanomalous showers investigated in Sec. 2, we get

$$\sum t_i = 20.6 \text{ radiation lengths.}$$
$$\langle t \rangle = 0.395 \text{ radiation length.}$$
$$\langle E_i \rangle = \sum t_i E_i / \sum t_i = 8.0 \text{ Bev } (E_i \geqq 1.0 \text{ Bev}),$$

where t_i and E_i are the track length and the energy of individual electrons of energy greater than 1 Bev. The results on trident formation from these showers are presented in Table V. Here E in Column 1 is the energy of the pair or of the trident (the sum of the two lower energy electrons of the emerging three is taken for the trident energy). Column 2 gives the number of observed apparent tridents, 3 the number of observed bremsstrahlung pairs, 4 the corrected number of tridents by the method described in Appendix B of 1, and 5 the resulting mean free path in radiation lengths (2.92 cm in emulsion) for trident production.

It must be emphasized that in the type of correction we have applied to the data, the low detection efficiency for low-energy pairs, the inadequacy of the assumption of complete screening and asymptotic conversion length for low-energy photons and the intervention of other effects, e.g., Compton scattering or photonuclear effects, make it necessary to restrict ourselves to those electron pairs of energy above a certain lower limit. One might think that the mean free path for trident production for a sufficiently lower energy limit would represent a lower limit for the trident mean free path on the basis of a poorer detection efficiency for low-energy bremsstrahlung pairs in contrast with the almost 100% detection efficiency for apparent tridents irrespective of their energy. However, this is not necessarily true, because the longer conversion length of the low-energy photons makes the calculated correction smaller, which would compensate for the increased correction due to the inclusion of missed low-energy pairs. The lower limit in this table was taken to be 10^{-2} and 10^{-3} of the initiating pair energy in each shower and it is felt that the former lower energy limit is a more suitable one. (In energy units this is on the average of 80 Mev.)

TABLE V. Results on trident mean free path from E-type showers.

1 Energy lower limit	2 No. of apparent tridents	3 No. of B. S. pairs	4 Corrected No. of tridents	5 m.f.p. for trident production
$E \geq 10^{-2} E_0$	11	10	6.7	$3.1_{-0.8}^{+1.4}$
$E \geq 10^{-3} E_0$	12	15	5.6	$3.7_{-1.0}^{+2.0}$

Additional data can be obtained from shower P–1. We use the first 4.8 cm of this shower development (the reason for the restriction to this portion is twofold; this part is the most thoroughly measured and if we take too long a path we cannot use the energy values determined by scattering measurements performed on the first 1 or 2 cm of each electron track and would have to take into account the electron energy degradation due to bremsstrahlung), and find:

$$a \begin{cases} \sum t_i = 5.85 \text{ radiation lengths,} \\ \langle t \rangle = 0.835 \text{ radiation length,} \\ \langle E_i \rangle = 4.75 \text{ Bev } (E_i \geqq 2.5 \text{ Bev}), \end{cases}$$

or

$$b \begin{cases} \sum t_i = 14.2 \text{ radiation lengths,} \\ \langle t \rangle = 0.65 \text{ radiation length,} \\ \langle E_i \rangle = 3.3 \text{ Bev } (E_i \geqq 1.0 \text{ Bev}). \end{cases}$$

The results are presented in Table VI which is similar in nature to Table V. The difference between a and b in Table VI is due to the difference in the lower limit of electron energies accepted as a primary track for trident production, consequently the difference in the average electron energy of the primary electrons, $\langle E_i \rangle$.

All the results on tridents are presented in Fig. 6, in which the data from I is also given as well as the results of Bhabha's[13] calculation. The results obtained in Appendix B of I (the correction for B.S. pairs) were revised in the following way. The electron deflection at the instant of bremsstrahlung has been neglected and this amounts to dividing B–9 of I by $\sqrt{2}$. The numerical factor to be used in B–9 was taken to be the one corresponding to the average γ-ray energy for $E\gamma \geq 10^{-2} E_0$. This leads to a change of the factor given in I–B from 0.67 to 0.72. After these revisions we obtain $5.4_{-1.9}^{+6.2}$ and $2.2_{-0.7}^{+2.0}$ radiation lengths for average energies of 4.2 and 50 Bev, respectively of I. The previous results in I were 4.5 and 1.1 radiation lengths, respectively. The revised curves are given in Fig. 7.

Figure 6 seems to indicate a systematic deviation of the experimental mean free path for trident production below that of Bhabha's result which gives the shortest mean free path among all the theoretical calculations of this process. Let us now investigate the situation more closely. First of all, we must recognize that the calculated correction of B.S. pairs in Appendix B of I after the above revision is an overestimate in that the deflection of the primary electron at the instant of γ-ray

TABLE VI. Results on trident mean free path obtained from electrons of shower P–1.

1 Energy lower limit	2 No. of apparent tridents	3 No. of B. S. pairs	4 Corrected No. of tridents	5 m.f.p. for trident production
$E \geq 10^{-2} E_0$	4	13	2.4	$2.4_{-1.0}^{+4.3}$
$E \geq 10^{-3} E_0$	5	14	3.2	$4.4_{-1.5}^{+4.5}$

[13] H. Bhabha, Proc. Roy. Soc. (London) A152, 559 (1935).

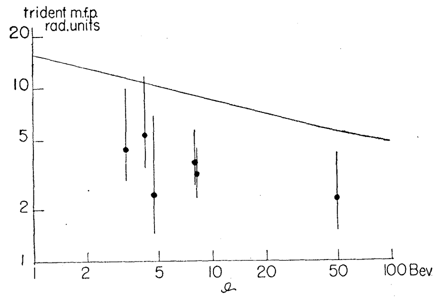


FIG. 6. Experimental results on trident mean free path as a function of energy. The solid line is that calculated by Bhabha.

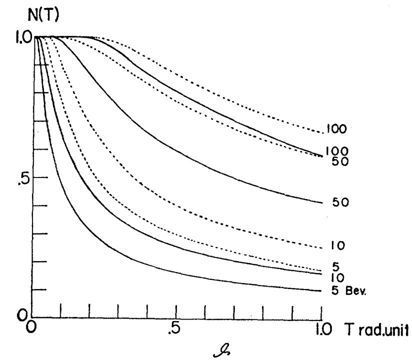


FIG. 7. Calculated correction curves for the trident process (solid curves obtained using the rms emission angle of Stearns: dotted curves represent a zero emission angle). Details of the calculation appear in the Appendix of I.

emission and the energy loss of this electron have not been included. The latter effect, the energy loss of the primary electron, can, within the portion of the electron track on which the energy measurements have been made, partially be taken into account. That is, we use the energy value as estimated by the scattering measurement in choosing a particular correction curve from Fig. 7, since the magnitude of the correction is determined by both the average γ-ray emission angle and the average electron scattering over the observed distance and not directly by the initial instantaneous energy of the primary electron. This argument also applies roughly to the energy estimation by the opening angle determination together with the assumption of energy equipartition, because the separation of the two electrons at a certain distance includes the effect of the electron scattering and the electron energy loss up to this distance. Now, there may be an argument against the use of a Gaussian distribution with the root-mean-square angle of emission as calculated by Stearns for the angular distribution of the emitted γ ray. It is true that there is no direct experimental verification of Stearns' results, while the most probable opening angle of a converted pair as given by Borsellino has been verified experimentally for energies up to 800 Mev.[14] However, in our problem of the bremsstrahlung pair correction to the trident process, we are dealing with the behavior of an ensemble of electrons and photons as a whole and the root mean square angle is better suited for the purpose of representing the whole distribution. As for the approximate use of a Gaussian distribution for the true angular distribution of emitted γ rays we have the following considerations. Although there is no direct calculation of this distribution, we can attempt an estimate of how well its salient features are accounted for by a Gaussian by considering the similarity of bremsstrahlung to pair production together with the results given by Borsellino and Stearns. The true angular distribution would probably be similar to that given by Borsellino for the opening angle distribution of converted pairs. The errors introduced by the use of a Gaussian distribution can be estimated in the following way. We can calculate the mean distance which the emitted γ ray has traveled before it is converted into an electron pair within the observational distance of t radiation lengths and from equation B–4 of I we find it to be about $0.3t$ in the approximation of small t. This means that the important contribution comes from angles smaller than 0.43×10^{-4} radian for an observational distance of 0.5 radiation length and an observational accuracy of 0.2 μ. The percentages of the distributions below this angle are 37% for a Gaussian-type and 40% for a Borsellino-type distribution for an electron energy of 50 Bev. For electron energies smaller than 50 Bev, the Gaussian-type distribution has more percentage within the above angle than the Borsellino-type distribution does; this is because the Guassian type distribution has a nonzero valve at zero angle while the Borsellino type distribution goes to zero with zero angle. Therefore, in our energy region, it seems almost certain that the use of a Gaussian function for the angular distribution of emitted γ rays does not cause an underestimation of the bremsstrahlung correction to the trident process. We also note that the root-mean-square angle for our energy region is much smaller, by at least a factor of 10, than the angle in which our observations have been restricted, so that the number of γ rays converted outside the observed area would not be sufficient to affect the magnitude of the correction appreciably. (See Appendix B of I.)

The remaining problem is to what extent we can trust the energy values as determined by scattering measurements, or by the opening angle determination of converted pairs, as representing the true initial energy. Figure 6 shows that if we want the experimental data to be in harmony with Bhabha's results, an over-all underestimation of electron energy by at least a factor of 30 must be assumed. Although this is not absolutely impossible, it is rather hard to admit that all of these electron energies are systematically underestimated by a factor of $\sim$30 when we consider that our lower energy limit is 1 Bev. We cannot attribute the energy underestimation to a few electron tracks, for we have seen in Table II that the tridents are rather uniformly distributed among the isolated showers considered there and the short observational

[14] K. Hintermann, Phys. Rev. 93, 898 (1954).

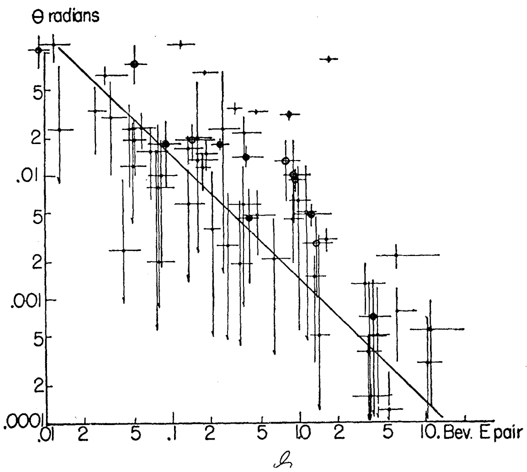


FIG. 8. Opening angles of pairs (solid circles) and tridents (open circles) plotted against their energy as determined by multiple scattering.

length makes it difficult for this discrepancy to be due to a few cases of very high-energy showers. Therefore, it seems rather more probable to conclude that there actually exists a discrepancy of a factor of about 2.5 between the experimental data and the theoretical mean free path for trident production (as calculated by Bhabha).

Additional information is obtained from the opening angle distribution of electron pairs and tridents. This is presented in Figs. 8 and 9. In Fig. 8, the opening angles of individual pairs are plotted against their energies as determined by the scattering measurement. The horizontal lines indicate the statistical probable errors in the scattering measurements while the vertical lines give the uncertainties in the opening angle determination which have been taken to be due to the root-mean-square deviation of the electrons from their original directions as determined by scattering measurements. In Fig. 9, the results are presented following Hintermann[14] in a form directly comparable to Borsellino's calculation: that is, individual cases have been expressed in terms of Borsellino's characteristic angle $W_0=4kmc^2/(E_+E_-)$ where k, E_+, E_-, and mc^2 are the energies of primary photon, positive electron, negative electron and the rest mass of the electron respectively. The good agreement with Borsellino's results, which has been verified previously by K. Hintermann for energies below some 800 Mev, is thus apparently extended to the higher energy region of around 10 Bev. This, however, does not necessarily imply that both types of energy measurements give an accurate initial energy, for the effect of bremsstrhalung would be expected to cause a systematic error in favor of an energy underestimation in the high-energy region for both types of measurements. In Figs. 8 and 9, the apparent trident cases, of which a considerable fraction are expected to be genuine ones (from the preceding analysis), are also plotted considering the lower energy two of the emerging electrons as the produced pair. (The apparent tridents from I are also included.) There seems to be a rather marked difference between the two categories, ordinary pairs and apparent tridents; i.e., the trident cases seem to favor larger opening angles than the ordinary γ-ray pairs. Whether this is actually so or not will be determined only upon obtaining increased data in the future.

5. CONCLUSIONS

The investigation of isolated electron showers of energy greater than 1 Bev in photographic emulsions exposed to the high-altitude cosmic rays has revealed the following results.

(1) Out of 16 isolated electron showers with an initial pair energy $\geqq 1$ Bev, 12 cases were found to be consistent with the assumption that they originated from a single γ ray (most probably from a locally produced π^0 meson), and 2 cases were consistent with the assumption that both γ rays from the π^0 decay contributed to the observed development.

In 2 cases, however, it seems quite difficult to reduce the required number of primary photons down to 2. One of them in fact seems to require at least 4 incoming γ rays of comparable energy ($\sim$10 Bev). The very narrow angular spread of these γ rays for their energy makes it difficult to explain this event as due to a nuclear production mechanism for π^0 mesons as the γ-ray source. There might be raised the question of whether this apparently anomalous electron shower could be interpreted in terms of the decay of a long lived neutral particle into several π^0 mesons: e.g., $\tau^0\rightarrow 3\pi^0$ or $\theta^0\rightarrow 2\pi^0$. In this way, we could explain the isolation of this event from other charged particles as well as the number of initial γ rays responsible for the event. However, the lower limit of the angular spread of the electron shower is fixed by the spread of the 2 γ rays from the π^0 decay rather than that of the π^0's from the decaying neutral particle (no known particle has been observed decaying directly into γ rays other than the π^0 meson). The minimum angular separation of 2 γ rays from a 20-Bev π^0 meson is 1.4×10^{-2} radian compared with the observed 0.82×10^{-3} radian. Therefore, the production of γ rays through the intermediary of π^0 mesons seems to be excluded.[15]

(2) The experimental mean free path in emulsion for the trident process, direct pair production by an electron, seem to be smaller by a factor of at least 2.5 than that predicted theoretically by Bhabha. In order to bring the experiment into harmony with his calculation, it would be necessary to admit a systematic underestimation, by a factor of about 30, of the initial electron energies by the scattering method, or by opening angle determination in the energy region observed (1 Bev to 15 Bev). There are two kinds of possible

[15] A mechanism proposed by S. Hayakawa in 1949, radiation accompanying the deceleration of primary charged particles [Phys. Rev. 75, 1760 (1949)], may be recalled in this connection. A charge exchange scattering of a very high-energy proton ($\sim$1000 Bev) with a neutron, or a π^- with proton, would probably give rise to this kind of event by his mechanism.

sources of error leading to a systematic under-estimation of energies. One is the inadequacy in the formula which relates the experimentally observed quantities to the energy value. This will be considered in a forthcoming paper in detail. The other possible source is a purely experimental one; that is, calibration of eyepiece scales, emulsion distortion and so on. The experimental procedures employed in this work were carefully checked and no possible source of this nature has been found to affect the results. Furthermore, if we admit that the majority of the energies were underestimated by a large factor, say 15, we should have observed the two photon primary nature in most of the isolated electron showers in Sec. 2 whereas it was observed in only 2 out of 14 cases. Therefore, it seems rather difficult to accept an explanation in terms of an overall energy underestimation by a large factor.

If this discrepancy is confirmed by increased experimental data with improved techniques of energy measurement, its inference would be quite embarrassing. Namely, we have assumed the validity of the Bethe-Heitler conversion length[10] and the angular distribution of emitted γ rays in deriving the correction to be applied to the observed number of apparent tridents; the result obtained is incompatible with Bhabha's calculation which had been carried out in the same theoretical framework as the Bethe-Heitler theory.[10] Thus in order to obtain agreement between the experimental data and the perturbational calculation, it would be necessary to admit a shorter conversion length or a smaller root mean square emission angle than those calculated on the above basis. This would probably mean that the assumption that the lowest order processes pertaining to the phenomena under consideration (bremsstrahlung and pair production in the cascade development) are larger than the higher order processes (such as trident process, simultaneous two γ-ray emission in bremsstrahlung or pair production with the emission of additional γ rays) by the appropriate factors in the perturbation expansion coefficient is no longer well justified in the high-energy region. In fact, if the trident cross section is larger than estimated by the perturbation calculation, there is no *a priori* reason for neglecting, for example, radiative electron-pair production (production of an electron-positron pair with an accompanying photon by an incident photon) as compared with ordinary pair production since this process is of the same order in the perturbation expansion as that of the trident process. In addition, if radiative pair production is not entirely negligible in the high-energy region, it will make the observable γ-ray conversion length shorter and tend to bring the experimental trident cross section in conformity with Bhabha's result.

There is another possible source which might lead to a shorter γ-ray conversion length; that is, if by any reason the screening by atomic electrons is not complete, the lowest order cross sections would increase with energy instead of approaching a constant value.

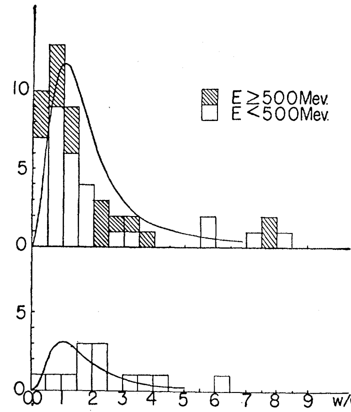


FIG. 9. Comparison of theoretical and experimental opening angle distributions for pairs (upper) and tridents (lower). W/W_0 is the pair energy in units of $W_0=4km_ec^2/(E_+E_-)$, Borsellino's characteristic angle.

We find approximately that in order to have an agreement between the experimental data on the trident process and the results given by Bhabha, it would be necessary to have a smaller conversion length than that given by the Bethe-Heitler theory by a factor of about 1.5 if the angular distribution is unchanged.

All these considerations, however, are tentative in that they require further experimental data with increased statistics combined with more refined techniques of energy measurement for high-energy electrons.

(3) Some additional information was obtained about the opening angle of electron paris and the verification of Borsellino's results was apparently extended up to around 10 Bev. The trident process seems to favor opening angles larger than those characteristic of ordinary electron pairs of equal energy.

In conclusion, almost all the difficulties will be avoided if we can obtain a more reliable method of energy estimation than available at the present time.

We wish to thank Professor K. Greisen for some discussions concerning the trident process and our method of correction; we also would like to express our appreciation to Miss B. Hull and to Mrs. J. Rutherford, Jr. for their scanning assistance, and to thank the Aero Medical Field Laboratory, Holloman Air Force Base for arranging the balloon flight.

Note added in proof.—Recently Peters and his co-workers (private communication) have investigated the limitations of the multiple Coulomb scattering method as applied to nuclear emulsions. They report the existence of an inherent emulsion noise, independent of distortion, presumably arising from small scale dislocations, that gives rise to a spurious scattering which sets an effective upper limit of approximately 1 Bev for the meaningful measurement of momenta of singly charged particles. They would thus ascribe the values of energy determined by the application of the conventional method (their criticism does not apply to the relative scattering method) as spurious and having no meaningful relation to the true energy, the true energy of a given track in general being higher by some indeterminate factor. Thus, they would say that in fact the energy has been underestimated (due to the spurious scattering) and that no discrepancy exists such as reported in this paper. We feel however that we cannot at this time accept this criticism as a resolution of the discrepancy reported in this paper for the following reasons. In this paper we have set forth some general arguments concerning reasons for our belief that we have not underestimated our energy by a factor as high as 30 and these arguments are independent of those advanced by Peters. In addition this view is supported by the agreement between energy determination of electron pairs by opening angle and scattering method as portrayed in Fig. 8—though this does not constitute the setting of an absolute scale, we believe it is a strong indication of the fact that the scattering measurements are not subject to the random uncertainties implied by Peters' work. In addition we have done some preliminary work on this (spurious scattering) problem in our laboratory and we do not find the effects described by Peters and his co-workers.

APPENDIX. MONTE CARLO CALCULATION OF CASCADE SHOWERS

We present here a set of tables representing the results of a Monte Carlo calculation on the one-dimensional number development of the electron-photon cascade. The original results, the cascade development of 100 single γ-ray initiated showers was done by one of us (MFK) in collaboration with Dr. D. M. Ritson under approximation A of Rossi and Greisen.[4] 100 showers were constructed and tabulations made of the number of showers having a given number of electrons, N_e, of energy greater than αE_0, E_0 = initial photon energy, in intervals of 0.1 radiation length from the origin to 1 radiation length. The results for the single photon initiated showers are presented in Tables A–1–a, A–1–b, and A–1–c, where a, b, and c mean respectively $\alpha = 10^{-4}$, 10^{-3}, and 10^{-2}.

Tables A–2–a, A–2–b, and A–2–c represent the development for two incident electrons and were obtained from Tables A–1 by using those cases in which the primary photon was converted within 0.5

TABLE A–1–a. ($E \geq 10^{-4} E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1.0	47	0	20	1	8	1	3	2	5	3	2	2	0	1	2	0	1	1	0	0	0	1	0
0.9	52	0	17	2	6	3	5	3	1	4	2	0	2	1	1	1	0	0	0	0	0	0	0
0.8	57	0	18	1	6	3	5	3	2	1	2	1	0	0	1	0	0	0	0	0	0	0	0
0.7	61	0	16	3	7	1	3	3	3	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0.6	62	0	19	3	7	2	1	3	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0
0.5	65	0	23	3	2	2	2	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0.4	72	0	19	1	5	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	77	0	19	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.2	79	0	20	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	87	0	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A–1–b. ($E \geq 10^{-3} E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1.0	47	0	23	2	8	3	5	4	4	1	1	1	1	0
0.9	52	0	20	3	8	7	6	0	1	1	2	0	0	0
0.8	57	0	21	2	11	6	1	1	1	1	0	0	0	0
0.7	60	0	21	3	6	6	3	0	0	1	0	0	0	0
0.6	62	0	23	2	7	3	2	0	1	0	0	0	0	0
0.5	65	0	27	0	3	3	2	0	0	0	0	0	0	0
0.4	72	0	21	1	6	0	0	0	0	0	0	0	0	0
0.3	77	0	20	1	2	0	0	0	0	0	0	0	0	0
0.2	79	0	20	0	1	0	0	0	0	0	0	0	0	0
0.1	87	0	13	0	0	0	0	0	0	0	0	0	0	0

TABLE A–1–c. ($E \geq 10^{-2} E_0$).

t \ N_e	0	1	2	3	4	5	6	7
1.0	47	1	28	6	11	4	3	0
0.9	52	0	29	6	11	1	2	0
0.8	57	0	30	4	7	2	0	0
0.7	60	0	28	4	6	2	0	0
0.6	62	0	30	2	4	2	0	0
0.5	65	0	30	3	2	0	0	0
0.4	72	0	25	1	2	0	0	0
0.3	77	0	22	0	1	0	0	0
0.2	79	0	20	0	1	0	0	0
0.1	87	0	13	0	0	0	0	0

TABLE A–2–a. ($E \geq 10^{-4} E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9	10	11
0.5	10	2	8	2	4	3	2	0	1	2	1	0
0.4	16	4	3	2	5	2	1	0	2	0	0	0
0.3	19	4	6	3	1	0	2	0	0	0	0	0
0.2	24	3	7	0	1	0	0	0	0	0	0	0
0.1	33	0	2	0	0	0	0	0	0	0	0	0

TABLE A–2–b. ($E \geq 10^{-3} E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8
0.5	14	4	6	5	4	0	1	1	0
0.4	20	1	6	5	2	0	1	0	0
0.3	24	0	3	2	1	0	0	0	0
0.2	28	2	5	0	0	0	0	0	0
0.1	33	0	2	0	0	0	0	0	0

TABLE A–2–c. ($E \geq 10^{-2} E_0$).

t \ N_e	0	1	2	3	4	5
0.5	25	3	4	2	1	0
0.4	26	3	5	1	0	0
0.3	30	1	4	0	0	0
0.2	32	2	1	0	0	0
0.1	33	1	1	0	0	0

TABLE A-3-a. ($E \geq 10^{-4}E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0.5	650	130	750	206	470	289	268	105	168	158	132	65	51	17	16	9	6	4	3	2	1	0
0.4	1152	288	520	236	501	221	204	60	211	22	53	7	16	4	3	0	2	0	0	0	0	0
0.3	1463	308	823	326	252	75	194	10	41	2	6	0	0	0	0	0	0	0	0	0	0	0
0.2	1896	237	1033	60	243	3	27	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	2871	0	603	0	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A-3-b. ($E \geq 10^{-3}E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0.5	910	260	768	433	464	189	231	106	66	49	11	6	5	2	0
0.4	1440	72	852	401	391	117	155	32	33	1	6	0	0	0	0
0.3	1848	0	1096	178	285	48	38	5	2	0	0	0	0	0	0
0.2	2212	158	955	40	128	2	5	0	0	0	0	0	0	0	0
0.1	2871	0	603	0	26	0	0	0	0	0	0	0	0	0	0

TABLE A-3-c. ($E \geq 10^{-2}E_0$).

t \ N_e	0	1	2	3	4	5	6	7	8	9
0.5	1625	195	1010	295	244	78	44	7	2	0
0.4	1872	216	1010	173	180	36	11	2	0	0
0.3	2310	77	968	22	118	1	4	0	0	0
0.2	2528	158	719	40	52	2	1	0	0	0
0.1	2871	87	516	13	13	0	0	0	0	0

radiation length and taking the point of conversion as the origin. The letters a, b, c here again refer to $\alpha = 10^{-4}$, 10^{-3}, and 10^{-2} respectively where now E_0 = initial pair energy. Here N_e is the number of additional electrons formed in the distance t radiation lengths from the first pair origin and the numbers in the rows opposite t represent the number of cases.

Tables A-3-a, A-3-b, and A-3-c represent the development for the initial condition of one electron pair and one photon of equal energy, E_0, incident at the origin and were made by a superposition of Tables A-1 and A-2. Here again a, b, c, refer respectively to $\alpha = 10^{-4}$, 10^{-3}, and 10^{-2} and N_e is the number of additional electrons made in the distance t radiation lengths from the origin and the numbers in the rows opposite t represent the number of cases.

With these results we clearly see the large fluctuations in electron numbers in the cascade development. As a matter of fact, these fluctuations can cover almost all anomalies in electron numbers in cascade showers thus far observed if the initial energy is sufficiently high.

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Causality in the Pion-Proton Scattering*

H. L. ANDERSON, W. C. DAVIDON, AND U. E. KRUSE
Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois
(Received May 11, 1955)

Dispersion relations applicable to particles with mass and charge have been used for analyzing pion-proton scattering data. In these relations, experimental values of the total cross sections for π^+ and π^- from 0 to 1.9 Bev were used to calculate the real part of the forward scattering amplitudes, and these were compared with the results of phase-shift analyses. With suitable choice of the pion-nucleon coupling constant, good agreement can be obtained for the phase-shift solutions with a resonant behavior for α_{33}.

INTRODUCTION

A RELATION between the real and imaginary parts of the forward scattering amplitude for the scattering of light on atoms has been known for some time. The relation, known as the Kramers-Kronig dispersion relation[1,2] is derived from the condition that the scattered wave should have zero amplitude until the incident wave reaches the scatterer. Following a suggestion by Kronig[3] that such a causality condition might be extended to apply to the scattering of particles

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