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Energy Distribution of K Mesons Produced in Nuclei

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K mesons produced by internal target bombardment at the Brookhaven Cosmotron and Berkeley Bevatron are assumed to be made by the reaction: $p+N\rightarrow Y+N+K$, where N=nucleon, Y=hyperon. The expected energy distribution of the K particles is calculated for several laboratory angles at proton bombarding energies of 2.9 Bev and 4.8 Bev, assuming various angular and energy distributions of the K mesons in the center-of-mass system of the incoming proton and target nucleon. The internal motions of the target nucleons were taken into account by using both the Gaussian and the Fermi momentum distributions.

I. INTRODUCTION

In view of the recent Cosmotron[1–3] and Bevatron[4] experiments on the production of K mesons by internal proton bombardment of heavy nuclei targets, it seems of interest to calculate the expected laboratory energy distribution of the K particles as a function of laboratory angle and beam energy, making suitable assumptions about the center-of-mass system (c.m.) energy and angular distribution of the K mesons. A comparison of the calculated laboratory energy distributions with the experimental spectra from heavy nuclei should give information about the mode of production and the angular and energy distribution in the c.m. system for the individual nucleon-nucleon collisions.

Associated production[5] in nucleon-nucleon collisions[6,7] has been assumed, and computations have been made for the following typical reactions:

$$p+p\rightarrow\Sigma^{+}+K^{+}+n, \qquad (1)$$

$$p+p\rightarrow\Lambda^{0}+K^{+}+p, \qquad (2)$$

where the Σ^+ is the charged hyperon of mass 2340 m_e, and Λ^0 is the neutral hyperon of mass 2180 m_e, and the K mass is taken to be 975 m_e. Using either a Fermi or Gaussian internal momentum distribution, an average has been taken over the motion of the target nucleons; the results should be applicable to all but the lightest nuclei, since the internal momentum distribution is approximately independent of atomic number. The procedure of the calculation is essentially that employed by Block, Passman, and Havens[8] to deduce pion emission spectra from targets bombarded by 381-Mev protons.

II. METHOD

For a given beam kinetic energy T_p, the total energy $\bar{W}$ available in the c.m. system can be shown to be dependent primarily on u, the component of the target nucleon velocity parallel to the incident beam. For a particular u, the differential cross section in the c.m. system (of the beam proton and target nucleon) is given by[9]

$$\frac{d^2\bar{\sigma}}{d\bar{\omega}d\bar{T}}=\frac{2\pi}{\hbar}\frac{|\bar{H}|^2}{\bar{v}_r}\frac{d^2\bar{\rho}(u)}{d\bar{\omega}d\bar{T}}, \qquad (3)$$

where $\bar{H}$ is the matrix element for production, $\bar{\omega}$ and $\bar{T}$ are the K meson solid angle and kinetic energy, $d\bar{\rho}^2(u)/d\bar{\omega}d\bar{T}$ is the phase space factor for the 3-body final state, and $\bar{v}_r$ is the relative velocity of the colliding nucleons. The laboratory differential cross section is obtained by transforming (3) and averaging over u, and is

$$\frac{d^2\sigma}{d\omega dT}=\int_{u_{\min}}^{u_{\max}}\frac{d^2\bar{\sigma}(u)}{d\bar{\omega}d\bar{T}}JP(u)du, \qquad (4)$$

where J is the Jacobian of the transformation, given by $p/\bar{p}$, and p is the K meson momentum. $P(u)du$ is the probability that u lies in du and is obtained from the assumed internal momentum distribution. It can be shown that the exact relativistic 3-body phase

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9 Barred quantities, throughout, will be in the c.m. system, while unbarred symbols will refer to the laboratory system.

space factor is[10]

$$\frac{d^2\bar{\rho}(u)}{d\bar{\omega}d\bar{T}}=\frac{\pi}{6(2\pi\hbar)^6}\bar{p}\bar{E}$$
$$\times\left[1-2\frac{(m_1^2+m_2^2)}{(\bar{W}-\bar{E})^2-\bar{p}^2}+\left(\frac{m_1^2-m_2^2}{(\bar{W}-\bar{E})^2-\bar{p}^2}\right)^2\right]^{\frac{1}{2}}$$
$$\times\left\{3(\bar{W}-\bar{E})^2\left[1-\left(\frac{m_1^2-m_2^2}{(\bar{W}-\bar{E})^2-\bar{p}^2}\right)^2\right]\right.$$
$$\left.-\bar{p}^2\left[1-2\frac{(m_1^2+m_2^2)}{(\bar{W}-\bar{E})^2-\bar{p}^2}+\left(\frac{m_1^2-m_2^2}{(\bar{W}-\bar{E})^2-\bar{p}^2}\right)^2\right]\right\}, \quad (5)$$

where $\bar{E}$ and $\bar{p}$ are the K particle total energy and momentum, respectively, and m_1 and m_2 are the rest masses of the other two reaction products.

In (3), we will ignore the slow variation of $\bar{v}_r$ with u, and will make various assumptions about the dependence of $|\bar{H}|^2$ on the K meson momentum $\bar{p}$ and angle of emission $\bar{\theta}$. In particular, we assume the following cases:

(1) $|\bar{H}|^2=1$, i.e., isotropic and energy independent.

(2) $|\bar{H}|^2=\cos^2\bar{\theta}$.

(3) $|\bar{H}|^2=\bar{\gamma}^2-1$, where $\bar{\gamma}=\bar{E}/m_K$.

(4) $|\bar{H}|^2=(\bar{\gamma}^2-1)\cos^2\bar{\theta}$.

The normalization constants arising from the above cases were chosen so that the c.m. cross section $d\bar{\sigma}/d\bar{\omega}$ at 0° was equal to 1 mb/sterad for a bombarding energy $T_p=2.9$ Bev.

The momentum distributions employed were:

(1) The Fermi degenerate gas model, which yields

$$P_F(u)=\frac{3}{4u_F}\left(1-\frac{u^2}{u_F^2}\right)=3.4-67.9u^2, \quad (6)$$

where $u_F=0.22$ is the value of u corresponding to the cutoff kinetic energy (22 Mev);

(2) The Gaussian distribution,

$$P_G(u)=\left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}}\exp(-\alpha u^2)=3.41\exp(-36.4u^2), \quad (7)$$

which corresponds to an average kinetic energy of 19.3 Mev.[11]

III. RESULTS

$A\cdot T_p=2.9$ Bev

Using reaction (1), and Eqs. (3)–(7), we have numerically calculated curves for a proton beam energy of 2.9 Bev. Figure 1 shows the normalized energy distributions $d^2\sigma/d\omega dT$ at a laboratory angle of 90°. The notation in this and the succeeding figures is as follows. In the labeling triplet $\xi:\eta:\zeta$ the first symbol ξ refers to the momentum distribution employed (F for Fermi, G for Gaussian), while η is the energy dependence of $|\bar{H}|^2$, being 1 if independent of $\bar{T}$, or $\bar{\gamma}^2-1$, or $\bar{\beta}^2/\bar{\gamma}^2$, where $\bar{\beta}=(\bar{\gamma}^2-1)^{\frac{1}{2}}/\bar{\gamma}$. The last symbol, ζ, represents the angular dependence of $|\bar{H}|^2$ and is $\cos^2\bar{\theta}$, or 1 for isotropic. In Fig. 1, the spectral shapes are roughly independent of the choice of matrix elements, all curves giving a maximum in the neighborhood of 40 Mev. The principal difference between the Gaussian and Fermi distributions is reflected in the high-energy end of the spectrum, where the Gaussian has a long tail in contrast to the sharp cutoff of the Fermi distribution. Further, there is also little difference in *magnitude*, as well as shape, between for example, G:1:1 and G:1: $\cos^2\bar{\theta}$, which is due to the fact that the K mesons emitted at a laboratory angle of 90° are emitted at angles of $\bar{\theta}$ in the c.m. system near 180°, so that $\cos^2\bar{\theta}\approx 1$. The cases where $\eta=\bar{\gamma}^2-1$ and $\bar{\beta}^2/\bar{\gamma}^2$ correspond to matrix elements behaving like $\bar{p}$ for low velocities, which might be expected of gradient coupling. By contrast, the energy independent matrix element may be applicable if the K meson is scalar. Also, this case corresponds to the Fermi theory[12] of meson production. The case of $\eta=\bar{\beta}^2/\bar{\gamma}^2$ was included because this matrix element decreases for large energies, and this compensates in part for the rapid increase of the phase space factor with increasing beam energy.

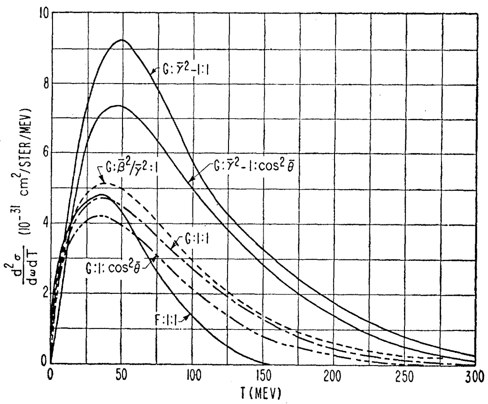

FIG. 1. Laboratory differential cross section $d^2\sigma/d\omega dT$ as a function of the K meson laboratory kinetic energy T, at a 90° laboratory angle and proton beam energy $T_p=2.9$ Bev, for various matrix elements and internal momentum distributions.

Figures 2 and 3 indicate the results for laboratory angles of 0° and 45°, respectively. The most striking feature of the 0° spectra is the presence of a minimum near 300 Mev for the energy dependent cases, in contrast to the smooth rise of the energy independent spectra. Thus, one may be able to distinguish experimentally between scalar and gradient coupling by observing the spectrum in the forward direction in the

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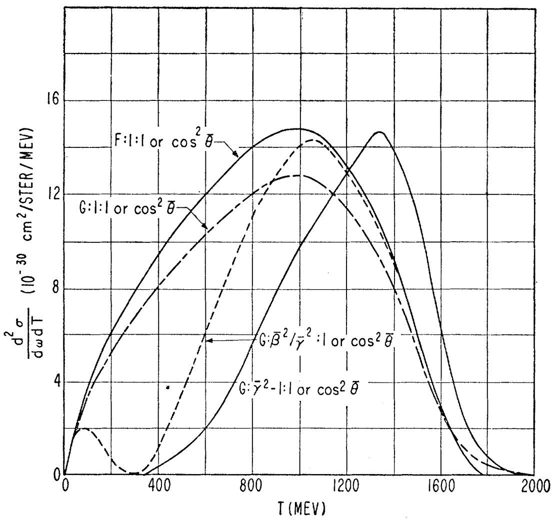

FIG. 2. Laboratory differential cross section $d^2\sigma/d\omega dT$ at $\theta=0°$ for beam energy $T_p=2.9$ Bev for various matrix elements and internal momentum distributions.

low-energy region ($T<800$ Mev). The presence of a minimum in $d^2\sigma/d\omega dT$ would indicate that the matrix element is energy dependent, going to zero as $\bar{p}\rightarrow 0$. The shapes and magnitudes of the 45° spectra are most sensitive to the choice of angular distribution, since the K particles emitted at 45° in the laboratory typically arise from c.m. angles $\bar\theta\sim 110$–140°, where $\cos^2\bar\theta$ is markedly different from unity.

$B\cdot T_p=4.8$ Bev.

Using reaction (1), and Eqs. (3)–(7), the energy spectra were numerically evaluated for a beam energy of 4.8 Bev, and are shown in Fig. 4 for a 90° laboratory

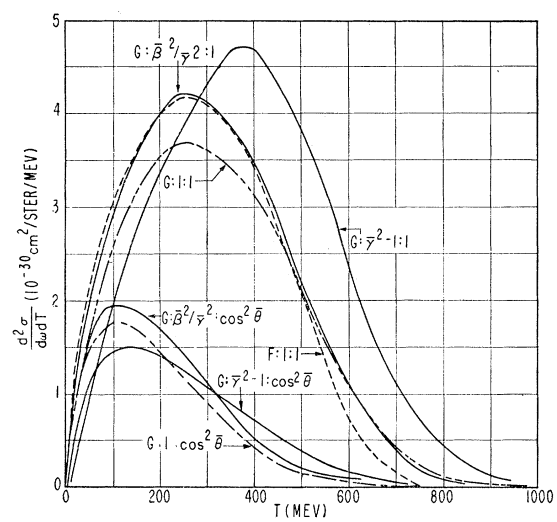

FIG. 3. Laboratory differential cross section $d^2\sigma/d\omega dT$ at $\theta=45°$ for beam energy $T_p=2.9$ Bev for various matrix elements and internal momentum distributions.

angle. The general features are quite similar to the 2.9-Bev results. The curves are normalized by the same procedure as the 2.9-Bev results; thus, a by-product of the calculation is the excitation function of the total cross section. Numerical integration of (4) gives the ratio of the *total* cross section at 4.8 Bev to that at 2.9 Bev as: (a) 9.1 for G:1:1, (b) 23.4 for G:$\bar\gamma^2-1$:1, (c) 8.3 for G:$\bar\beta^2/\bar\gamma^2$:1.

IV. COMPARISON WITH EXPERIMENT

Using nuclear emulsions, Hill, Salant, and Widgoff[1] have observed K mesons produced by internal proton bombardment of a copper target, at laboratory angles of 45° and 90° from the 3-Bev Cosmotron beam. The ratio of K particles at 90° to 45°, was given as $\rho=N(90°)/N(45°)=1.8_{-1.2}^{+0.4}$, where the energy intervals observed were 80 to 145 Mev at 90° and 260 to

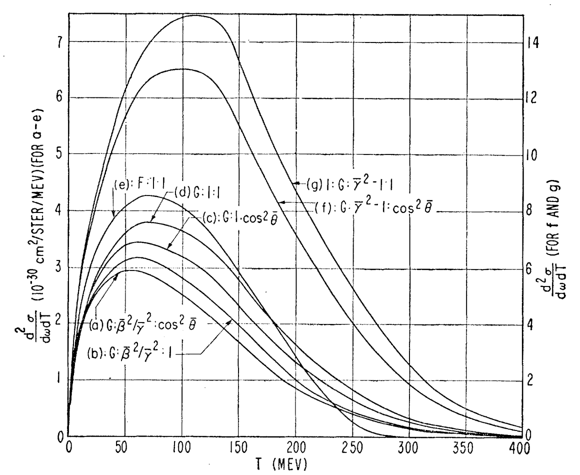

FIG. 4. Laboratory differential cross section $d^2\sigma/d\omega dT$ at $\theta=90°$ for beam energy $T_p=4.8$ Bev for various matrix elements and internal momentum distributions.

300 Mev at 45°. The corresponding calculated results, obtained from numerically integrating the appropriate energy regions in Figs. 1 and 3, are:

(i) $\rho_1=0.11$ for $|\bar{H}|^2=1$,

(ii) $\rho_2=0.31$ for $|\bar{H}|^2=\cos^2\bar\theta$,

(iii) $\rho_3=0.21$ for $|\bar{H}|^2=\bar\gamma^2-1$,

(iv) $\rho_4=0.63$ for $|\bar{H}|^2=(\bar\gamma^2-1)\cos^2\bar\theta$.

Only ρ_4 appears to be included within the experimental limits. However, ρ is very sensitive to the kinetic energy available in the c.m. system. The calculations were repeated for reaction (2) in which the K meson is created together with a Λ^0, and a bombarding energy of 3 Bev, which is probably closer to the experimental value, was used. Both the larger energy and smaller rest mass of the Λ^0 act to increase the value of ρ. Recalculating using these assumptions and the same choice of matrix elements as above, we find: $\rho_1=0.18$, $\rho_2=0.44$, $\rho_3=0.39$, and $\rho_4=0.98$. Thus we conclude that $|\bar{H}|^2=1$ (isotropic and energy independent) appears to be ruled out by experiment, whereas the other choices of matrix elements are probably not inconsistent with the observations, considering the limited statistics of the experiment and the approximations made in the calculations. In this connection, we note that some of the K mesons emitted from a heavy nucleus may have been produced in a secondary reaction, i.e., pions were formed which subsequently collided with a nucleon within the same nucleus to produce a K particle. It is also possible that some of the K mesons emerging at 45° or 90° were produced near the forward direction but were scattered through large angles by collision with nucleons within the target nucleus.

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High-Energy Electromagnetic Phenomena in Cosmic Radiation*

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Isolated high-energy electron showers in photographic emulsion have been investigated and have yielded the following conclusions: (1) out of 16 cases of isolated electron showers observed to originate from single electron pairs of energy greater than 1 Bev, 2 cases have been found to be anomalous in the sense that they seem to have been initiated by more than 2 photons; one of the two has been analyzed in detail. (2) The discrepancy between the experimental observations and theoretical predictions on the trident process found in a previous work has been obtained again with the additional experimental data of this experiment.

1. INTRODUCTION

SINCE the work of Schein *et al.*[1] in 1954 on an anomalous electron shower pointed to some possible difficulties in high-energy cascade showers, it has been felt that this field requires a more detailed survey. The purpose of this work is twofold; to check the results obtained previously[2] pertaining to the trident process with additional experimental data and to investigate the high-energy cascade shower development in emulsion under the more favorable experimental conditions of single γ-ray initiated showers to see whether there actually exists some anomaly or not. It seems pertinent, however, to give some general comments on the difficulties inherent in this field before the presentation and discussion of this experiment.

First of all, it must be emphasized that in the development of individual electron-photon cascades, the fluctuations in the numbers of electrons and photons (referred to as number fluctuations) from the average value can be enormously large. In fact, the number fluctuations in the cascade process were, under some simplified assumptions, shown to be similar to that of the Polya distribution, instead of the familiar Poisson distribution of random events. This means that the number fluctuation from the average can be as large as the average itself. In order to illustrate the situation more clearly the results of a Monte Carlo calculation[3] on cascade showers have been given in the Appendix. The original results on 100 showers with a single photon primary were obtained by one of us (M.F.K.) in collaboration with D. M. Ritson, using the cross sections of Approximation A of Rossi and Griesen.[4] The other results with different initial conditions were derived from the original results by a change of shower origin or by a superposition of different initial conditions. As can be seen from these results the number fluctuations are quite large compared with those encountered in random processes.

There is another difficulty which arises when we attempt to measure electron energies. The available methods of energy measurement for electrons or photons, by determination of the multiple Coulomb scattering or of the opening angle of a converted electron pair, have, when applied to high-energy electrons or photons, some defects which usually lead to an underestimation of the energy. That is, in the conventional method of multiple Coulomb scattering, no account is taken of the bremsstrahlung energy loss of the electron. Also, in the energy estimation of a γ ray from the opening angle of its converted electron-positron pair, some care must be

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