Note that the angular dependent factors $Y_{l_p m_p}(\Theta,\Phi)$ in the amplitude expressions do not appear in (15).

The contributions to the reaction amplitudes from a particular collision-matrix component

$$\mathfrak{U}(nj_n'l_n';dj_d'l_d';J')$$

are given by

$$\begin{aligned}\mathcal{A}(nj_n'\nu_n';dj_d'\nu_d') &= -i\pi^{\frac{1}{2}}k_n^{-1}(2l_n'+1)^{\frac{1}{2}}(j_n'l_n'\nu_n'0|J'\nu_n')\\ &\quad\times(j_d'l_d'\nu_d'\nu_n'-\nu_d'|J'\nu_n')\\ &\quad\times\mathfrak{U}(nj_n'l_n';dj_d'l_d';J')Y_{l_d'\nu_n'-\nu_d'}(\Omega_d). \qquad (16)\end{aligned}$$

By summing the absolute squares of the sum of terms from (16) and (14) over all possible ν_n', ν_d' and by dividing by the number of initial spin states, one obtains in addition to the straight pickup and resonance contributions, the interference contribution to the differential cross section which is

$$\begin{aligned}&[d\sigma_{n;d}(\theta_d)/d\Omega_d]_{\text{interference}}\\ &= \tfrac{1}{2}ifk_dk_n^{-1}(-1)^{I_f+I_t+J'}(\alpha^2+k_{pn}^2)^{-1}(2J'+1)\\ &\quad\times(2I_t+1)^{-\frac{1}{2}}\mathfrak{U}(nj_n'l_n';dj_d'l_d';J')\sum_{j_pl_p}(-1)^{j_p}\\ &\quad\times(2j_p+1)^{\frac{1}{2}}\gamma(j_pl_p)\mathcal{W}_{l_p}W(I_tl_p\tfrac{1}{2}j_d';j_pj_n')\\ &\quad\times W(\tfrac{1}{2}1j_pj_d';\tfrac{1}{2}I_f)\bar{Z}(l_n'j_n'l_d'j_d';J'l_p)\\ &\quad\times\textstyle\sum_m[(l_n'l_d'0m|l_pm)/(l_n'l_d'00|l_p0)]\\ &\quad\times[(l_d'-|m|)!(l_p-|m|)!/(l_d'+|m|)!(l_p+|m|)!]^{\frac{1}{2}}\\ &\quad\times P_{l_d'}^{|m|}(\theta_d)P_{l_p}^{|m|}(\Theta)+\text{c.c.} \qquad (17)\end{aligned}$$

The m sum extends from $-l_p$ to l_p, or from $-l_d'$ to l_d', whichever range is smaller.

The pickup reaction amplitude (14) was calculated for the deuteron ${}^1S$ wave function of the zero-range potential. For a Chew-type wave function of the form $\exp(-\alpha r)-\exp(-\beta r)$, where $\beta\approx 7\alpha$, (14), (16), and (17) should be multiplied by $\beta^{\frac{1}{2}}(\alpha+\beta)^{\frac{3}{2}}/(\beta^2+k_{pn}^2)$, and (15) should be multiplied by its square.

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Reactions $C^{12}(d,n)N^{13}{}_{\text{ground state}}$ and $C^{12}(d,t)C^{11}$ up to $E_d=20$ Mev*

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The course of the partial cross section for the direct formation of N^{13} in its ground state only in the reaction $C^{12}(d,n)N^{13}$ has been followed up to $E_d=20$ Mev by observing the N^{13} activity induced in a stack of polyethylene foils (only the ground state is stable against proton emission). The cross section falls appreciably less rapidly than would be expected for compound nucleus formation and also less rapidly than predicted by simple stripping theory. The cross section at $E_d=8$ Mev is 100 mb which is not so large relative to that for the mirror (d,p) reaction as is predicted by simple stripping theory. The reaction $C^{12}(d,t)C^{11}$ has been detected and its total cross section measured from its threshold ($E_d=14.5$ Mev) to $E_d=20$ Mev where it is 10 mb. The magnitude of this cross section indicates that this is a pickup reaction.

INTRODUCTION

THE many measurements of "stripping" angular distributions in recent years have given ample grounds for believing that (d,p) and (d,n) reactions induced by deuterons of energy greater than three or four Mev do not as a rule involve the strong formation of a compound nucleus but that the reactions proceed most frequently by a direct interaction in which a nucleon of the impinging particle simply severs its "deuteron bond" at the nuclear surface and attaches itself to the existing (and undisturbed) structure of the target nucleus thereby forming one or other of those states of the residual nucleus of which the target nucleus is a parent.[1] Although fair to good qualitative agreement between experimental and theoretical stripping patterns can usually be obtained (albeit by a somewhat cavalier approach to the problem of the nuclear radius), it is abundantly clear that neither the details of the patterns predicted by simple stripping theory nor the theoretical absolute cross sections are reproduced by experiment and that considerable refinements to the theory are needed. Examples of such refinements are the taking account of Coulomb effects, scattering of the incident deuteron wave, exchange effects, and boundary conditions for the outgoing particle.[2] As soon as such refinements are introduced the fit with experiment may, of course, be much improved since it is not usually clear which of many alternative procedures should be followed at each stage, and advantage may be taken of this uncertainty. Thus, although some empirical working recipe may emerge for the fitting of experimental angular distributions and reduced widths, we cannot feel confident that the particular constellation of param-

* Performed under the auspices of the U. S. Atomic Energy Commission.
† On leave from Cavendish Laboratory, Cambridge, England.
[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).
[2] See, for example, W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).

eters represented by that recipe necessarily reflects the physical conditions obtaining in the reaction. We must, therefore, look around for some other aspect of the experimental situation that will, in the first place, distinguish between reactions proceeding via a compound nucleus and via direct interaction and, secondly, provide another datum against which the refinements of stripping theory, adduced to explain angular distributions and reduced widths, may be tested.

When we consider the variation with deuteron energy of the partial cross section for the formation of the residual nucleus in a particular state, another way of distinguishing between compound nucleus formation and direct interaction becomes apparent since the considerations governing the energy dependence of the partial cross sections are entirely different in the two cases. If we have compound nucleus formation, a given residual state will be populated as the result of statistical competition between that state and all other energetically-accessible states of the residual nucleus, and so the energy-dependence of the partial cross section will be governed largely by more and more competing states becoming available as the excitation increases. In the case of stripping, there is a fixed set of really or potentially available residual states, namely those states which have appreciable fractional parentage coefficients for the target nucleus; whether a particular state is energetically accessible or not does not affect its potency as a "competitor" of the particular residual state in which we are interested. The various partial cross sections to the various residual states are now independent of each other. The "competition" between the states is fixed once and for all, in a manner essentially independent of the energy of the deuteron, by the fractional parentage coefficients that describe the relationship between the target and residual nuclei. The statistical aspect of the competition that characterizes decay following compound nucleus formation is wholly absent. In the stripping reaction the energy dependence of the partial (differential) cross section is governed largely by the instantaneous availability in the incident deuteron of a nucleon with the correct momentum for tacking onto the target nucleus so as to leave the residual nucleus in the desired state when the emergent nucleon leaves in a particular direction. (To compare with compound nucleus theory, we of course integrate the theoretical differential cross section over all angles.) Since the mechanisms governing the energy dependence of the partial cross sections are so different in the two cases, we may hope to distinguish quite sharply between the two possibilities, particularly when we bear in mind that the theoretical energy dependence derived from compound nucleus theory must be normalized to experiment at low deutron energies, since it is certainly there that the theory is valid if anywhere, while the normalization between stripping theory and experiment should be made rather in the region of high deuteron energy.

The reason why no such test has been made heretofore is probably that the taking of angular distributions on a particular group of product particles over a wide range of deuteron energies would be a most tedious business. An alternative approach is to study the excitation function for the production of a radioactive product nucleus. This as a rule is uninformative since the observed radioactive ground state may be reached not only directly in the particle transition but also by gamma-ray transitions from excited residual states. We do not know the relative populations of the several residual states to which the actual particle transitions take place; and these relative populations depend on the energy of the deuterons.

There is, however, one case at least, though possibly it is unique, where the residual radioactivity measures transitions to one state only of the product nucleus, and that is in the reaction $C^{12}(d,n)N^{13}$. N^{13} is a positron emitter of half-life 10 minutes, and since *all* excited states of N^{13} are energetically unstable against proton emission, we may be sure that the N^{13} positron activity measures only those neutron transitions that lead directly to the ground state except for the negligible fraction of cases where gamma-ray emission to the ground state of N^{13} successfully competes with proton emission following a neutron transition to an excited state.

It was therefore thought profitable to study the production of N^{13} in the deuteron bombardment of carbon over as wide a range of deuteron energy as possible. Since detailed measurements already exist up to $E_d = 3$ Mev,[3] particular attention has been given to the region of higher deuteron energy up to 20 Mev available with the Brookhaven 60-in. cyclotron. It is known from isolated measurements of angular distributions in the reaction $C^{12}(d,p)C^{13}$ at 3.3 Mev[4] and 8 Mev[5] that the stripping process predominates in the region of present investigation, and it was therefore hoped that our results would display a clear discrimination against the form of cross-section variation with deuteron energy to be expected on the compound nucleus model. It was also hoped that the results would show deviations from expectations based on the simple form of stripping theory that neglects the refinements detailed above, so that the results might provide a test of the applicability of those refinements.

The reaction $C^{12}(d,n)N^{13}$ is endothermic by only 281 kev and so proceeds almost throughout our range of deuteron energy; but a second reaction that produces a radioactive end product becomes possible above a deuteron energy of 14.5 Mev, to wit $C^{12}(d,t)C^{11}$ which has a Q value of -12.45 Mev. This reaction has not been reported in the literature and it was thought to be of interest to see whether it could be detected, since if it

[3] See F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[4] Holmgren, Blair, Simmons, Stratton, and Stuart, Phys. Rev. **95**, 1544 (1954).

[5] J. Rotblat, Nature **167**, 1027 (1951).

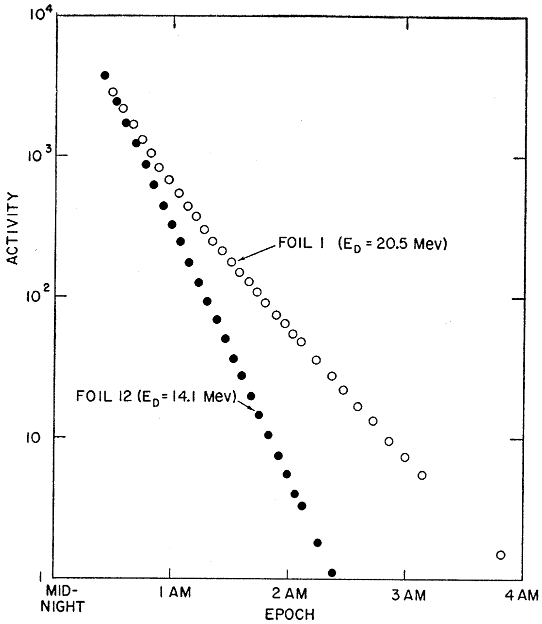

FIG. 1. Decay curves of polyethylene foils irradiated with deuterons of 14.1 Mev and 20.5 Mev. The former shows a pure exponential decay characteristic of N^{13} formed in the reaction $C^{12}(d,n)N^{13}$; the latter shows mixed N^{13} and C^{11} decay, the C^{11} coming from the reaction $C^{12}(d,t)C^{11}$ which has its threshold at deuteron energy of 14.5 Mev.

proceeds by a direct or pickup mechanism it provides a measure of the probability that a deuteron on coming near a nucleus should snatch out a nucleon from that nucleus rather than lose a nucleon to it. It was also hoped that in this case also the results might discriminate between a mechanism involving compound nucleus formation and one of direct interaction—in this case pickup. The situation is not so clear-cut as in the (d,n) reaction, because several excited states of C^{11} are stable against heavy particle breakup and by $E_d=20$ Mev three of them have become energetically available. C^{11} is a positron emitter of half-life 20 minutes, and so its separation from the 10 minute N^{13} would not be possible unless the (d,t) reaction possessed a fair cross section.

The production of C^{11} in the bombardment of carbon with deuterons may be safely attributed to the (d,t) reaction below $E_d=21.8$ Mev, at which energy the $C^{12}(d,dn)C^{11}$ reaction becomes possible. The production of C^{11} by several contaminant reactions such as $N^{14}(p,\alpha)C^{11}$ was also considered. Chemical analysis of the foils together with the known proton (H_2^+) content of the deuteron beam showed them to be wholly negligible.

EXPERIMENTAL METHOD

The experiment was performed by bombarding stacks of 4-mil polyethylene foils with about 1/100 microampere of 20-Mev deuterons for periods of the order of 30 minutes, during which time the beam current was held constant to better than 10%. The diameter of the beam was about 4 mm. After the bombardment the individual foils were trapped between aluminum sheets of thickness adequate to absorb the positrons of N^{13} and C^{11}, and the decay was followed over many half-lives by counting the annihilation quanta with an NaI(Tl) crystal. (Neither body gives nuclear gamma rays.)

Those foils for which the deuteron energy was below the threshold for C^{11} production showed an accurately exponential decay with a half-life of 10 minutes; others, above the threshold, showed an initially nonexponential decay, rapid at first but leveling off to a half-life of 20 minutes. Examples of such decays are shown in Fig. 1. The half-life displayed by the foil bombarded with deutrons of mean energy 14.1 Mev[6] is 10.08 ± 0.04 minutes, which figure accords well with the best recent value for N^{13} of 10.05 ± 0.03 min.[7] The foil bombarded with deutrons of mean energy 20.5 Mev clearly shows the mixed periods and eventually decays with a half-life of about 20 minutes; the values in the literature for for C^{11} are about 20.5 ± 0.1 minutes.[3]

All decay curves were analyzed to find the relative amounts of N^{13} and C^{11} present in each foil at the end of the irradiations, and these quantities were in turn

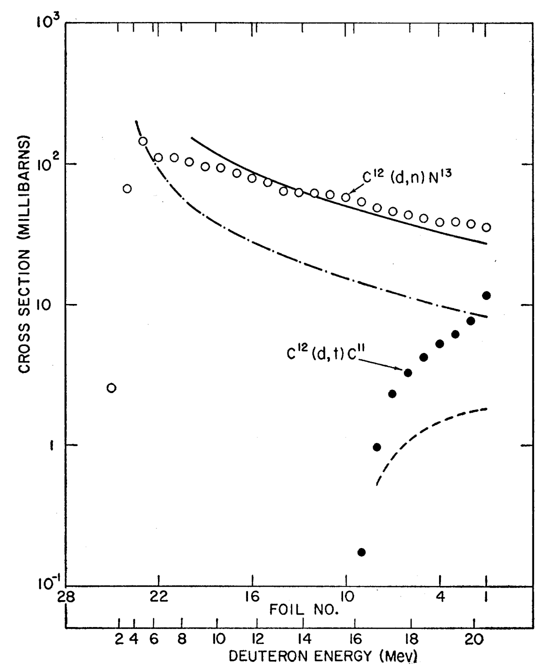

FIG. 2. Excitation functions for the reactions $C^{12}(d,n)N^{13}$ and $C^{12}(d,t)C^{11}$. The dashed-dotted curve shows the expected course of the (d,n) reaction if the reaction proceeds via compound nucleus formation (normalized at $E_d=5$ Mev); the full curve is the prediction of simple stripping theory (normalized at $E_d=14$ Mev). The dashed curve is the prediction based on compound nucleus formation for the reaction $C^{12}(d,t)C^{11}$ under the assumption that the reduced width for triton emission is as great as that for nucleon emission.

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[6] We have used the range-energy relations of M. Rich and R. Madey, University of California Radiation Laboratory Report UCRL 2301 (unpublished).

[7] Churchill, Jones, and Hunt, Nature **172**, 460 (1953).

corrected for the finite duration of the bombardments. The surface densities of the polyethylene foils were determined by weighing small triangles cut from the bombarded region of each foil; they were constant to within 2 percent from foil to foil and no correction was necessary on account of variations in surface density. With these data, relative cross sections for the (d,n) and (d,t) reactions were computed. The absolute sensitivity of the detecting apparatus was computed and this enabled an absolute cross section scale to be established. This scale, however, was not set up with great care and may be in error by as much as 50 percent; the relative cross sections are established with very much better accuracy.

The results for the energy dependence of the two cross sections are displayed in Fig. 2. Little attention should be given to the two points of lowest energy for the (d,n) cross section since straggling is undoubtedly of importance here at the end of the deuteron range.

DISCUSSION

The chief feature of both cross sections is that they change smoothly with deuteron energy. It is known that the (d,n) cross section for low deuteron energies shows strong resonances[3]; none is revealed at high energies, although even for our higher energies the thickness of a foil represents about 500 kev of deuteron energy so the energy resolution is always very poor.

The most interesting feature of the cross section for the (d,n) reaction is that it varies so little with deuteron energy. The experimental variation has been compared with what may be expected on the basis of compound nucleus theory and simple stripping theory.

Although N^{14} is too light a nucleus for us to attempt to describe realistically by the usual thermodynamic methods, it is possible to construct an expression that represents fairly well the empirically observed level densities in odd-mass nuclei in this region; this is:

$$w(E)=0.3\exp[2(0.3E)^{\frac{1}{2}}],$$

where $w(E)$ is the level density (per Mev) at an excitation of E Mev. We use this expression and the usual formalism, and assume that the total cross section for compound nucleus formation is independent of deuteron energy in order to compute the variation with deuteron energy of the cross section for formation of N^{13} in its ground state. A fit was made at low deuteron energy for the reason given in the Introduction, and the result is shown as the dashed-dotted line of Fig. 2. It is clear that the theoretical falloff with deuteron energy is initially much faster than indicated by experiment.

The energy variation expected on stripping theory has been computed using the simple Born approximation formulation[8]; that is, we have taken

$$\sigma(\theta)=(k_p/k_d)G^2 j_1^2(kR).$$

Here the symbols have their usual meaning as used in stripping theory; we use j_1 since we work entirely in the p shell. The slightly more complicated Butler formula[1] gives essentially similar predictions. The prediction of the above formula, after integration over angle, is compared with experiment as the full line of Fig. 2. Normalization has been performed in the higher-energy region since we expect stripping theory more nearly to hold for high deuteron energies. It is seen that the prediction of the simple stripping theory fits experiment fairly well over a rather wide range of deuteron energy, whereas the deviation of the prediction of compound nucleus theory from experiment is immediate.

However, the experimental cross section seems to fall less rapidly than simple stripping theory would suggest. This may be due in part to the influence of Coulomb effects which will oppose the entry of the proton into the nucleus at low energies but allow it free entry at high. It is clear that refinement to stripping theory is needed to explain the detailed course of the present cross section and, as remarked above, the same refinements must suffice for this task as are postulated in order to improve agreement between theory and experiment in other aspects of the stripping process.

It is interesting to compare the absolute cross section for the (d,n) reaction with that for the mirror reaction $C^{12}(d,p)C^{13}$ which has been measured by Rotblat[5] at $E_d=8$ Mev. Under the assumption of the charge symmetry of nuclear forces we may use the comparison of these two "mirror cross sections" as a further test of simple stripping theory. The total cross section for the (d,p) reaction was computed from Rotblat's data (which run from $\theta=15°$ to $\theta=160°$) by using simple stripping theory to extrapolate his differential cross section to small angles and assuming a constant cross section between $\theta=160°$ and $\theta=180°$. We then find that our measured (d,n) cross section at $E_d=8$ Mev is greater than the (d,p) cross section by a factor of 1.2. The ratio predicted by simple stripping theory is 2.0. In view of the possible error of about 50% in the (d,n) cross section, this discrepancy cannot be said to be very great. It may, however, be due in part to the presence of a "compound nucleus" component in the reactions. If we interpret the differential cross section at backward angles in Rotblat's data as due to such compound nucleus formation and assume a consonant contribution in the (d,n) case, the experimental stripping cross section ratio becomes 1.5 instead of 1.2. Another possible explanation is in terms of the differing importance for the two mirror reactions of the refinements to stripping theory referred to above. It might be naively argued that the (d,n) to (d,p) ratio predicted by simple stripping theory should be diminished owing to the opposition of the Coulomb barrier to the entry of the proton into the nucleus in the (d,n) case, and this change is in the sense indicated by experiment. If a suitably refined stripping theory correctly accounts for the form of the variation of cross section with energy, and if

[8] Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952).

after these corrections and that for compound nucleus contribution a similar discrepancy yet remains, it is in the sense to correspond to a greater reduced width for neutrons (in C^{13}) than for protons (in N^{13}); i.e., "the neutrons stick out further than the protons." Such an effect has been suggested for heavier nuclei, though it would be very surprising to find it holding for so light a nucleus as $A=13$.

An estimate of the course of the cross section for the reaction $C^{12}(d,t)C^{11}$ was made on the basis of compound nucleus formation by assuming, as before, that the whole of the cross section for $C^{12}(d,n)N^{13}$ at low deuteron energies involves compound nucleus formation. On the assumption that the reduced width for triton emission is as great as that for neutron emission (the assumption of "preformed" tritons), we predict the dashed line of Fig. 2—in which the coming into play of successive residual states of C^{11} has been allowed for and the associated irregularities smoothed out. It is seen that even under the very unplausible assumption of the existence of preformed tritons, compound nucleus theory fails by an order of magnitude to explain the observed C^{11} formation. We are forced then to assume that this (d,t) reaction proceeds by some pickup mechanism and that we are indeed measuring the relative probability of the deuteron's losing a nucleon to the nucleus and removing one from it. As yet no sufficiently reliable theory of (d,t) pickup exists to warrant a comparison being made with these results. It is interesting to note that, at $E_d=3.3$ Mev, the angular distribution of the reaction $C^{13}(d,t)C^{12}$ is such as to suggest that a direct mechanism already predominates.[4]

It is interesting to compare these results with those of Cohen and Handley[9] on (p,t) reactions. These authors suggest that triton emission from a compound nucleus state has an inherent probability comparable with that for single nucleon emission. They base this argument on the rather flat angular distributions sometimes obtained which, they remark, tell against a pickup process. However, this conclusion is no longer valid when the energy of one or both the charged particles concerned is of the order of or below the Coulomb barrier; here a direct mechanism can give a sensibly isotropic angular distribution. It appears that considerable interest attaches to the resolution of this question of the mechanism by which tritons and similar complicated particles are emitted from nuclei in events of moderate to high energy.

[9] B. L. Cohen and T. H. Handley, Phys. Rev. 93, 514 (1954)

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Many-Body Problem for Strongly Interacting Particles. II. Linked Cluster Expansion*

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An approximation method developed previously to deal with many particles in strong interaction is examined in further detail. It is shown that the series giving the interaction energy is a development in a sequence of linked or irreducible cluster terms each of which gives a contribution to the energy proportional to the total number of particles. Consequently the convergence of the expansion is independent of the total number of particles. The origin of this simple feature is illustrated by showing that a similar situation exists in the expansion of standard perturbation theory. The numerical convergence of the expansion is quantitatively discussed for the nuclear problem where it is shown that the correction arising from the first cluster term involving three particles is less than the leading term by a factor of about 10^{-4}. The smallness of the correction is largely a result of the action of the exclusion principle.

I. INTRODUCTION

In a previous paper[1] (to be referred to as I) we have given a method for reducing approximately the many body problem for strongly interacting particles to a problem of self-consistent fields. Some of the physical content and origin of the method were discussed there and the nature of certain correction terms which can be neglected for very many particles was discussed. We shall in this paper examine the structure of another type of correction term which arises from interaction of clusters of particles and in so doing exhibit the general structure of the expansion involved. This will also allow us to draw some general conclusions about the convergence and accuracy of the method.

In Sec. II, we shall briefly summarize the relevant formulas from I and describe some difficulties which appear in high-order terms in the expansion for the energy which can be removed by a simple modification of the many-body propagation function. In Sec. III, we show how similar terms appear to arise in the usual perturbation theory but that they cancel identically, in a manner simply related to the cancellation discussed in Sec. II. In Sec. IV, we summarize these results and show how they may be generalized into a simple pre-

* Supported in part by a grant from the National Science Foundation.
[1] K. A. Brueckner and C. A. Levinson, Phys. Rev. 97, 1344 (1955).