shifts. However a higher degree of experimental accuracy is needed in order to conclude whether these are the correct phase shifts. The proposal here has the advantage of simplicity and agreement with meson theory calculations. Up to 300 Mev it is not necessary to use *d*-wave phase shifts although the small *d*-waves predicted by recoil corrections to the cutoff theory are welcome. The energy dependence proposed here would arise from a pion-nucleon interaction whose interaction range is on the order of $\frac{1}{2}\hbar/\mu c$ or less.

It may be significant that at least three other approaches lead to this same conclusion about the meson nucleon range. First, that Chew and Low in analysing pion scattering and photoproduction data are led to a $k_{\max}\sim 6\mu c/\hbar$. Second, that an effective-range analysis of the energy dependence of α_{33} by Brueckner gives an effective range of $\frac{1}{2}\hbar/\mu c$.[8] Third, that the Stanford high-energy electron scattering experiments on hydrogen give an rms radius of $\sim\frac{1}{2}\hbar/\mu c$.[9]

The author wishes to acknowledge the help of Mrs. Enid Bierman with the calculations and is indebted to Professor Herbert Anderson for supplying detailed preprints.

Note added in proof (*September 18, 1955*).—The final Carnegie Tech data of Ashkin, Blaser, Feiner, and Stern has just become available ["Pion-proton scattering at 150 and 170 Mev," Phys. Rev. (to be published)]. They give 57 experimental points with total errors for each point (including charge-exchange)$\sim$5% or less. Their best-fit phase shifts ($\alpha_3=-8°$ and $\alpha_1=+10°$ at 170 Mev) agree quite well with those proposed here. Electronic computers can show whether this data is accurate enough to establish the linear extrapolation of α_1 and α_3 as the preferred solution.

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Range-Energy Relation and Masses of the New Particles*†

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The accuracy to which the masses of most of the new unstable particles can be determined is now limited principally by the uncertainty in the range-energy relations at large velocities. The extent of this uncertainty is indicated, and the available data are re-examined to try to find the best relations to use. In particular, shell corrections are applied to the Sachs-Richardson data, and the mean excitation potentials for 9 elements are determined. The evidence for Al, Cu, and emulsion indicates that the mean excitation potentials are not velocity dependent, and that they may be considerably larger than the values commonly used.

I. INTRODUCTION

MASS values for the new unstable particles generally depend upon a measurement of the range of either the particle itself or its secondaries. These determinations are now of sufficient accuracy so that it is necessary to be quite concerned about the uncertainty in the relations available for converting a measured range into energy or momentum. For instance, the range-energy curve for copper most commonly used for such mass determinations is not based on any direct experimental results.

The experimental data which do exist are correlated by using them to determine, for a given element, the mean excitation potential, I, which appears in the familiar energy-loss equation[1]:

$$-\frac{dE}{dx}=\frac{4\pi e^4 z^2 N}{mc^2\beta^2}\left\{Z\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)}-\beta^2\right]-\sum_i C_i\right\}, \quad (1)$$

where ez is the charge of the incident particle, and β its velocity relative to that of light, c; m is the electronic mass; N is the number of stopping atoms of atomic number Z per unit volume; and C_i is the correction for nonparticipating electrons of the ith shell.

II. POSSIBLE VARIATION OF I WITH ENERGY

Since I is determined by measurements of energy and either energy loss or range, which depend only logarithmically on I, it is not too surprising that there has been considerable disagreement in the values for I found in different experiments. However, as was first pointed out by Sachs and Richardson,[2] if one plots the experimental I values for a given element against the logarithm of the energy, instead of scattering badly, the points are seen to lie on a steeply sloping straight line. While an I value which is determined by an energy-loss measurement should be plotted against the incident energy, one which is determined by a range measurement should be plotted at some lower, "effective" energy, if I is not constant. In the latter case, the effective energy, ϵ, should be[2] about 0.6 of the incident

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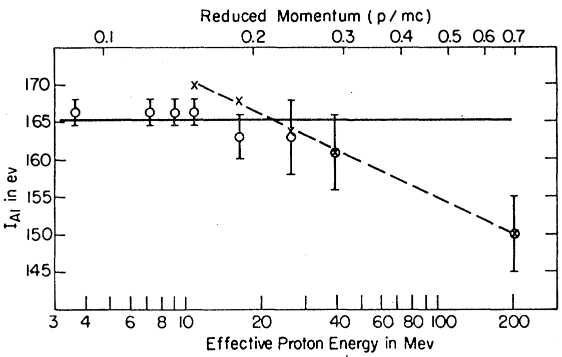

FIG. 1. Experimental values for the mean excitation potential of aluminum as a function of the "effective" proton energy. Old or uncorrected values are indicated by x's and the data available at present, by circles.

energy if $I=a-b\log\epsilon$. This plot of Al I values from reference 2 has been reproduced in Fig. 1, where the measurements available then are shown as x's and are connected by a dashed line.

The work of Lindhard and Scharff[3] gave additional support to the idea that I may be velocity dependent. They plotted stopping number per electron *vs* log $(\beta\hbar c)^2/(e^4Z)$, and found that data taken for different elements and at different energies lay on a smooth, nonlinear curve. This curve is the heavy one shown in Fig. 2, and the data on which it is based are indicated by triangles. Lindhard and Scharff were able to reproduce the general form of this curve by a calculation of stopping power based on a Fermi-Thomas model of the atom.

There is a direct correspondence[3,4] between the shape of the Lindhard-Scharff curve and the energy dependence of I for Al over the energy region shown in Fig. 1. According to the Lindhard-Scharff plot, I should have a maximum value at intermediate energies, then drop with energy about as in Fig. 1, but finally level off at higher energies. This variation with energy can be seen qualitatively by following along the curve of Fig. 2, and noting that the change in I is perpendicular to the dashed line.

A variation of I with energy would indicate a fundamental defect in the stopping theory. Also, from the point of view of the experimentalist, this variation leaves the choice of I and hence of the range-energy relation, very uncertain for high velocities. Seemingly one can choose the high value found at low energy, the low value found at high energy, or one can assume I varies logarithmically with energy, or in the manner indicated by the Lindhard-Scharff plot. As an example, this uncertainty is roughly $\pm 15\%$ in the I value for Cu at $p/mc\sim 1$, and of course gets much worse at higher velocities. Therefore it is desirable to re-examine the available data to try to determine the best range-energy relation to use at present.

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[4] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 779 (1953).

III. RE-EXAMINATION OF THE I-VALUE DATA

A. Sachs-Richardson Data

Since I is most readily and unambiguously determined from an absolute energy-loss experiment, let us consider first the only measurement of this type available in the energy region under consideration, that of Sachs and Richardson,[2,5] using 18-Mev protons. Their results, which have been amended[6] for the difference between the mean and most probable energy loss, can now be corrected for the fact that bound electrons cannot participate fully in the stopping. This correction is designated as $\sum C_i$ in Eq. (1). Due to the work of Walske, corrections for the K shell[7] and for the L shell[8] are now available. For the usually less important higher shell corrections, one can employ the form of the L-shell result,[8] which is a function of η_L and θ_L, by using for the ith shell, $\theta_i=n_i^2I_i/[(Z-\sigma_i)^2\mathrm{Ry}]$ and $\eta_i=mc^2\beta^2/[2(Z-\sigma_i)^2\mathrm{Ry}]$, where the ionization potentials, I_i, have been taken from Hill *et al.*[9]; n_i is the principal quantum number; Ry, the Rydberg constant; and σ_i, the screening constants as given by Slater.[10] The corrected I values, computed according to Eq. (1) above, are given in Table I.

B. Other Measurements for Metals

The I values (at 44 and 66 Mev for Al, and at 64 and 100 Mev for Cu) obtained from the range measurements of Bloembergen and Van Heerden[11] have been changed slightly by employing an energy-dependent multiple scattering correction, and in the case of Al, where the C_k used was known, also by introducing better shell

TABLE I. Mean excitation potentials corrected for shell effects.

Element	Uncorrected stopping power per electron	Mean excitation potential (ev)	Standard deviation (ev)	I/Z
Al	5.365	163.1[a]	3	12.6
Ni	4.538	363	19	13.0
Cu	4.486	377.5	8	13.0
Rh	3.870	656	45	14.6
Ag	3.894	659	50	14.0
Cd	3.903	654	41	13.6
Sn	3.827	708	59	14.2
Ta	3.525	962	54	13.2
Au	3.356	1136	100	14.4

[a] Obtained using an L-shell correction found by Bichsel to fit his experimental results; if instead C_L is determined from an extrapolation of Walske's calculations, I is about 0.5 ev lower.

[5] D. C. Sachs and J. R. Richardson, Phys. Rev. **83**, 834 (1951).
[6] D. O. Caldwell and J. R. Richardson, Phys. Rev. **94**, 79 (1954).
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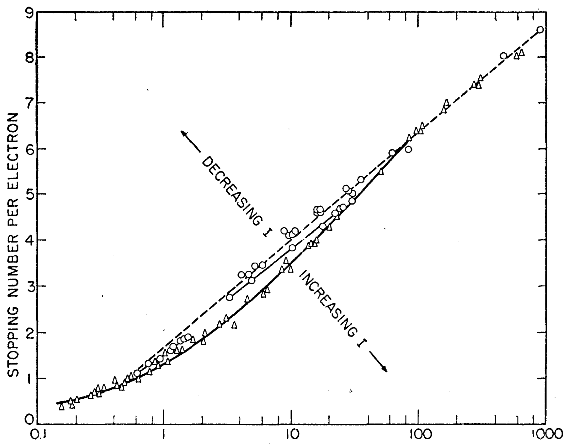

FIG. 2. Experimental stopping power per electron as a function of $(\beta\hbar c)^2/(e^4 Z)$. The old data (indicated by triangles) and the solid curve have been taken from reference 3, while the new data (circles) have been taken mainly from plots in references 4 and 18.

corrections. In addition, some excellent range measurements have been made recently by Bichsel and Mozley[12] for 6-, 12-, 15-, and 18-Mev protons in Al and 10- and 18-Mev protons in Cu.

The data available now for Al and Cu appear as circles in Figs. 1 and 3, plotted as a function of the "effective"[2] energy. It is apparent that, except for the 340-Mev range measurements of Mather and Segrè,[13] there is good agreement among the data and no longer any need for assuming that I is energy dependent.

Furthermore, if one adds new or previously unused stopping power measurements[11,14–19] (the circles in Fig. 2), the shape of the Lindhard-Scharff plot is entirely changed. The two sets of circles connected by straight lines represent absolute stopping power data,[17,19] while the other circles are for stopping powers relative to Al. For each set of relative data the Al point has been placed arbitrarily on the dashed straight line representing the Bloch equation,[20] $I \propto Z$. While the ordinate for the relative points from any one experiment can be multiplied by a constant, this does not alter the conclusion that these additional data are represented better by a line of the same *slope* as the dashed line shown than they are by the original (solid) curve. A straight line of this slope represents I values which are independent of velocity and proportional to Z. Actually, since shell corrections are not made for this plot, one should not expect the data to lie strictly on such a line.

The one disturbing feature remaining is that the only absolute high-energy range measurement[13] does not agree with the lower energy data. In order to obtain agreement, Mather's measurement of the angle of Čerenkov radiation would have to be altered by about 25 minutes, which is much in excess of the stated error. Although there is no reason to expect any discrepancy, it is interesting to note that the theoretical Čerenkov relation apparently has otherwise never been checked experimentally to better than $1-2°$.[21]

C. $K_{\mu 2}$ Mass Measurements

Some evidence favoring a larger I value for Cu than that found by Mather and Segrè,[13] and at even higher velocities ($p/mc \gtrsim 1$), is provided by the measurements of the mass of the $K_{\mu 2}$ meson made with magnet and multiplate cloud chambers by the Ecole Polytechnique group. By determining the momentum and range in Cu of the primary $K_{\mu 2}$, and by using a range-energy curve[22] computed for $I_{Cu}=333.5$ ev, they[23] found a mass of 921 ± 16 m.[24] A second mass determination was made by measuring the range in Cu of the μ-meson secondary. An average of the secondary ranges obtained at E.P. and M.I.T. (which are in excellent agreement) gives a mass of 942 ± 8 m,[25] when using the same range-energy curve. Since these two mass measurements are affected oppositely by changing I_{Cu}, it is possible to bring them into agreement. If one assumes only that I_{Cu} is constant from $p/mc \sim 1$ to 2.2, one can solve for that value of I

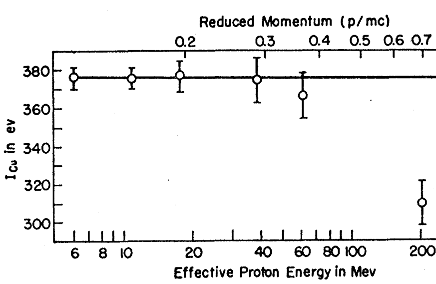

FIG. 3. Experimental values for the mean excitation potential of copper as a function of the "effective" proton energy (0.6 of the incident energy for range measurements, or the incident energy for energy-loss determinations).

[12] H. Bichsel and R. F. Mozley, Phys. Rev. **94**, 764(A) (1954), and private communications. Note that their I values and errors as used here are still preliminary, but it is not expected that their results will change significantly.

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[23] Armenteros, Gregory, Hendel, Lagarrigue, Leprince-Ringuet, Muller, and Peyrou, Nuovo cimento **1**, 915 (1955).

[24] This value is obtained for those K's which have secondaries of range greater than those of a τ, and which do not have an associated electron cascade.

[25] Armenteros, Gregory, Hendel, Lagarrigue, Leprince-Ringuet, Muller, Peyrou, Bridge, DeStaebler, Rossi, and Sreekantan, Nuovo cimento (to be published).

which makes equal the mass values determined by the two methods. One gets $I_{Cu}=383\pm43$ ev, which gives a unique mass of 933 m. This I value is in excellent agreement with the low-energy measurements, but nearly two standard deviations from the value determined in reference 13 at a somewhat lower p/mc. Of course, if there were some large systematic error in the magnetic field, this conclusion could be invalidated.

D. Emulsion Data

Additional evidence that I is not velocity dependent is furnished by the work of Vigneron,[26,27] who fitted essentially all the emulsion range data from 1 to 40 Mev, using a constant I value. In his earlier work,[26] Vigneron took the emulsion composition as given by the manufacturer and adjusted I to get the best fit to the data. The value he found, $\sim$396 ev, is in good agreement with that which one obtains for the same composition by using I values similar to those given in Table I (i.e., assuming $I\sim 13Z$ for the medium and heavy elements). In order to lower I to correspond to values then available, he assumed in his later work[27] that the emulsion composition was in error by 3% (the maximum amount he thought possible), giving a fit to the data with $I=332$ ev. Even this lower I value is not consistent with the determinations of reference 13.

The range-energy relation at high velocities for emulsion is much more in doubt than is that for Al or Cu. Not only do uncertainties in the composition and density (particularly as affected by humidity) of the emulsion cause additional difficulties, but also the only high-energy emulsion range or energy-loss measurements[28,29] have been made relative to Cu. The energy values associated with these measurements are based on the Mather-Segrè[13] I value for Cu of 310 ev. If instead we take the value of 377.5 ev from Table I, then Heinz's range measurement[28] would be for 335.9 Mev (instead of 342.5) and that of deCarvalho and Friedman[29] would be for 203.9 Mev (instead of 208).

Various attempts have been made to reconcile the low-energy data with Heinz's point, when the latter is assumed to be at 342.5 Mev. These generally involve some unrealistic "tilting" of the range-energy curve, and since different procedures have been used by different laboratories, it is not unexpected that mass values for the new particles based on secondary ranges have been rather inconsistent. Indeed, if the high I values found for lower energies are the correct ones to use, then the mass values from secondary emulsion ranges, as determined at various laboratories, may be in error by roughly the following amounts: τ, 2−8 m; $K_{\pi2}$, 5−12 m; $K_{\mu2}$, 5−25 m.

IV. CONCLUSIONS

Clearly, there is great need for further absolute range or, preferably, energy-loss measurements at high velocities. The use of μ mesons for this purpose seems attractive, first, because the nuclear collision loss for these particles would be small. Secondly, since for a given velocity a μ meson has a much lower momentum than a proton, it would be more practicable to determine the momentum in a magnetic field.

Concerning the information available at present, the results in Al, Cu, and emulsion all seem to indicate that each I is constant (and of much higher value than that commonly used) over more than a decade in energy. If the empirical I were suddenly to start dropping off above, say, $p/mc\sim 0.5$, this would indicate something radically wrong with the energy loss theory. There is some evidence against this unlikely occurrence furnished by the measurements of the $K_{\mu2}$ mass.[30] It therefore seems safest at the present time to take I as determined by the low-energy data, and use range-energy relations based on that result. The masses of some of the K particles computed on this basis have been presented elsewhere.[25]

V. ACKNOWLEDGMENTS

The author wishes to thank Professor Bruno Rossi and Dr. Bernard Gregory for many helpful discussions, and Dr. Herbert Bridge for constant aid and encouragement. He is also grateful to Dr. M. C. Walske and Dr. Hans Bichsel for not only providing their results prior to publication, but also for supplying much advice.

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[30] *Note added in proof.*—Stronger evidence is furnished by the measurements of energy loss of 15.7-Mev electrons reported by Goldwasser, Mills, and Robillard in Phys. Rev. **98**, 1763 (1955). At very high velocity ($p/mc=32$), they find $I/Z\sim 14$ for Z of 8–10, which is consistent with the values given in Table I.