Energy Dependence of the Phase Shifts in Pion-Proton Scattering*

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It is shown that all pion-proton scattering data up to 300 Mev can be fit within limits of experimental error by using the s-waves linearly extrapolated from the low-energy slopes of $\alpha_1=0.16\eta$ and $\alpha_3=-0.11\eta$. All other phase shifts are assumed zero except α_{33} which is given the energy dependence proposed by Chew and Low, $(\eta^3/\omega^*)\cot\alpha_{33}=8.05-3.8\omega^*$. The two constants in the α_{33} energy dependence fit the initial slope of $\alpha_{33}=0.235\eta^3$ as determined from low-energy experiments and make α_{33} go through 90° at 192 Mev. According to the cutoff theory this determines a $k_{\max}\approx 6\mu c/\hbar$, corresponding to a pion-nucleon interaction of much shorter range than $\hbar/\mu c$. A theoretical implication of the fit proposed here is that also the s-wave interaction range is less than $\hbar/\mu c$

A. INTRODUCTION

In the last year pion-proton scattering angular distributions have been obtained at higher energies and with increased accuracy. New results most pertinent to this paper are the 165 Mev,[1] 189 Mev[2], and 217 Mev[3] Chicago experiments and the Brookhaven 300 Mev π^+ cloud chamber results.[4]

In each case the individual experimenter has solved his data for the best fit s- and p-wave phase shifts. Figure 1 is a plot of recent best fit values of α_1, α_3, and α_{33} as a function of η, the center-of-mass momentum in units of μc.

The curve for α_{33} is the revised energy dependence proposed by Chew and Low[5] of

$$(\eta^3/\omega^*)\cot\alpha_{33}=8.05-3.8\omega^*. \tag{1}$$

ω^* is the total energy minus the proton rest energy.

The constants used here are those which fit the low-energy slope,[6] $\alpha_{33}=0.235\eta^3$, and the value $\alpha_{33}=90°$ at 192 Mev. According to Chew and Low this fit determines the two parameters in the cut-off theory of $f^2=0.08$ and $k_{\max}=6(\mu c/\hbar)$. It is seen that the best fit values of α_{33} fit this two parameter curve within the experimental errors.

The energy dependence of the two s-waves is not as clear. De Hoffmann *et al.*[7] have proposed a smoothing-out as shown by the dotted lines in Fig. 1. Glicksman has made good fits to the Chicago data by setting $\alpha_{31}=\alpha_{13}=\alpha_{11}=0$ and using values for α_1 and α_3 close to those of the dotted lines.[3] Here again α_1, α_3, and α_{33} are treated as independent variables until a best fit is obtained. He and others have observed that in the energy region above 130 Mev α_1 and α_3 may be changed with considerable freedom while still maintaining a good fit to the data.

The question arises whether there is enough freedom for α_1 and α_3 to permit their linear extrapolation from the low-energy best fit slopes of $\alpha_1=0.16\eta$ and $\alpha_3=-0.11\eta$.[6] It will be shown in the following section that the linear extrapolations do indeed give a reasonable fit to the data when used with the values of α_{33} given by Eq. (1), and all other phase shifts set equal to zero. Such an energy dependence for the s-waves is what would be expected of a simple short-range potential and should at least be given a try. It also has the advantage of being consistent with the interaction range indicated by the cutoff theory in its rather successful application to the calculation of the p-wave interaction.[5]

B. DISCUSSION

1. Reasons for Choice of Linear s-Waves

Perhaps the best justification for this linear extrapolation is curiosity and simplicity. In addition there is some theoretical basis for expecting a short-range interaction. The cutoff theory as applied by Chew and Low gives a value $k_{\max}\sim 6(\mu c/\hbar)$ which corresponds to a pion-nucleon interaction range not much greater than $1/k_{\max}$. Another indication of a short-range interaction is the effective range of $\sim\frac{1}{2}\hbar/\mu c$ obtained by Brueckner from the energy dependence of α_{33}.[8] An experimental indication of the extent of the proton virtual meson cloud is given by the Stanford high-energy electron scattering results on hydrogen which imply an rms radius of $\sim\frac{1}{2}\hbar/\mu c$.[9] Even if the virtual meson cloud were of greater extent, the main interaction of the external pion might be with the core. Of course there is the possibility that the s-wave interaction range be different from that of the p-wave. However, the γ_5-theory would suggest a shorter range for the s-wave interaction than for the p-wave.

Ranges even as large as $\hbar/\mu c$ would rule out the

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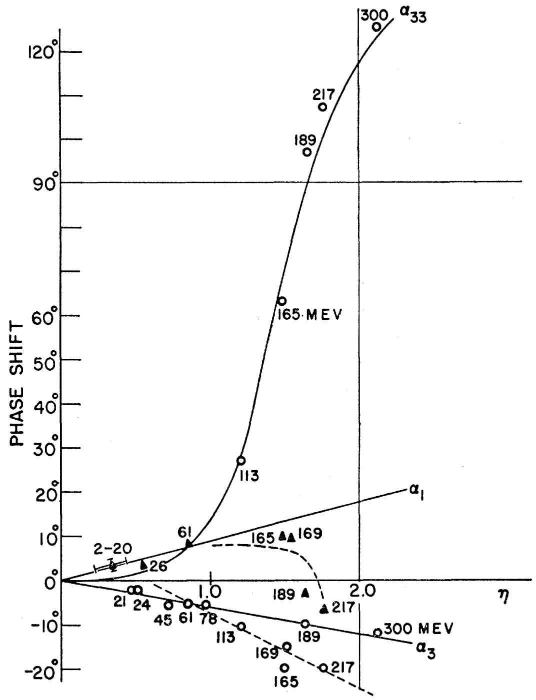

FIG. 1. Phase shifts as a function of pion momentum. The α_{33} curve is the "straight line" energy dependence proposed by Chew and Low.[5] The α_1 and α_3 solid lines are extrapolations of $\alpha_1=0.16\eta$ and $\alpha_3=-0.11\eta$. The dashed lines are proposals of de Hoffmann *et al.*[7] Triangles are best fit values of α_1 (circles for α_3) obtained by experimenters at the indicated energies.

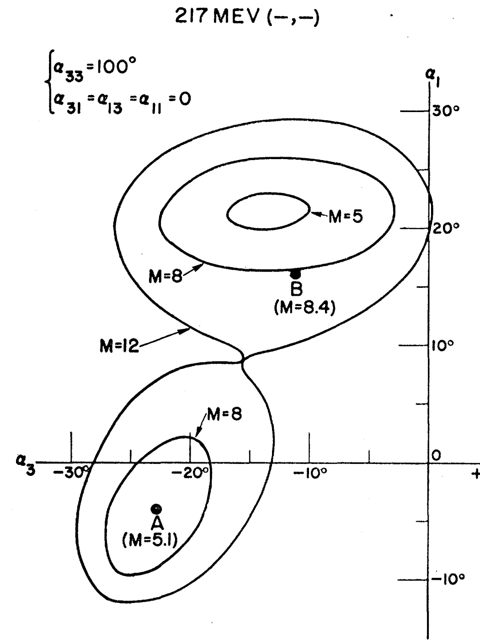

FIG. 2. M-values for 217-Mev π^- elastic scattering as a function of α_1 and α_3. α_{33} is held constant at 100°. All other phase shifts are zero. Point A is the minimum found by Glicksman in his three phase shift solution.[3] Point B is from the linear extrapolation of $\alpha_1=0.16\eta$ and $\alpha_3=-0.11\eta$ and lies near a second minimum. Contours of $M=5$, 8, and 12 are shown.

behavior of α_1 where it rather suddenly changes sign at about 180 Mev. This can be seen from the causality condition that a scattered wave cannot reach an observer before the original wave.[10] This condition requires for individual phase shifts that

$$d\alpha/dk > -r_0,$$

where r_0 is the interaction range. The Chicago values of $\alpha_1=10°$ at 165 Mev and $-2.7°$ at 189 Mev make $d\alpha/dk=-1.85\hbar/\mu c$. If α_1 truly behaved this way the s-wave interaction would have to remain strong for a distance greater than $1.85\hbar/\mu c$.

We rely mainly on this argument to discard the α_1 energy dependence proposed by de Hoffmann *et al.*[7] Furthermore, it will be shown in the next section that there are two equally good Fermi-type solutions to the 217-Mev data: the solution given by Glicksman where α_1 is negative, and a solution where $\alpha_1 \sim +20°$ and $\alpha_3 \sim -10°$ (see Fig. 2).

Choosing an energy dependence for α_3 is a more difficult problem. De Hoffmann *et al.*[7] and Dyson *et al.*[11] have proposed a slope $\alpha_3=-0.3\eta$ for the region above 100 Mev as shown by the dashed line in Fig. 1. In order not to violate low-energy data it is necessary for α_3 to start off with a slope not nearly so steep. One way of overcoming this difficulty is to propose a short-range repulsive potential with a longer-range attractive part.[12] Then one would expect to observe a change in slope in α_3 between zero and 80 Mev. However the recent low-energy results at (2.5–20) Mev π^-,[13] 21 Mev π^+,[14] 24 Mev π^+,[15] and the pi-mesonic atom K-level shifts[16] give no indication of requiring a change in slope. They confirm the earlier choice of $\alpha_1=0.16\eta$ and $\alpha_3=-0.11\eta$ with greater accuracy.

The main evidence for the steeper slope of $\alpha_3=-0.3\eta$ consists of some of the pion-proton scattering experiments above 120 Mev. In the half-dozen or so experiments which have been performed between 120 and 165 Mev, all of them give best fit values for α_3 below $-10°$. However the most recent α_3 best-fit values at 189 Mev[2] and 300 Mev[4] fall rather close to the curve $\alpha_3=-0.11\eta$. Perhaps a choice of a linear α_3 with a somewhat steeper slope might give a slightly better fit, although with the existing data it is seen that the choice made here does quite well.

One possible effect to explain low α_3 best-fit values would be the presence of some d-wave. It has been

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[14] S. Whetstone and D. Stork, Phys. Rev. **99**, 673(A) (1955).
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[16] Sterns, Stearns, De Benedetti, and Leipuner, Phys. Rev. **97**, 240 (1955).

TABLE I. M-values for the Chicago 165, 189, and 217-Mev experiments using phase shifts proposed in Fig. 1 or Eq. (2) The mean M-value (number of experimental points) is given in the next to last column.

Experiment	α_1	α_3	α_{33}	$\bar{M}$	M
165(+,+)	13.5°	−9.3°	70°	5	3.0
165(−,−)	13.5	−9.3	70	5	4.7
165(−,0)	13.5	−9.3	70	5	0.3
189(+,+)	15	−10.3	89	5	4.1
189(−,−)	15	−10.3	89	6	7.4
189(−,0)	15	−10.3	89	6	3.8
217(−,−)	16	−11	101	6	8.4
			Total	38	31.7

observed that if a negative d-wave phase shift ($T=3/2$, $J=5/2$) is present at 165 Mev, the best fit solution of α_3 will be shifted more negative when the analysis is made ignoring the d-wave.[17] Certainly a small negative D_{35} phase shift would be helpful in promoting the linear extrapolation of α_3 with slope -0.11η. Indeed Henley and Ruderman[18] have shown that just such a d-wave phase shift is a consequence of the coupling used by Chew and Low when recoil effects are considered. Their calculations give $D_{35}=-2.1°$ at 165 Mev and $-7°$ at 300 Mev.[18] Their other d-wave phase shifts are smaller and such as to give no effect in the π^- elastic scattering. Their values should be reduced about a factor two if the revised coupling constant of Chew and Low is used.

The other p-waves α_{31}, α_{13}, and α_{11} have been set equal to zero in all the attempts to fit the data. Experimentally the best-fit values for these phase shifts are always small with larger errors. Also no trend for any of them in either magnitude or sign has shown up.

Chew has calculated values for these 3 p-wave phase shifts using the same coupling which gives his α_{33} results. He finds that they must all be small. He believes α_{13} and α_{31} are equal, but is not sure of their sign or magnitude. He believes α_{11} should be small and negative but is not sure of its magnitude.[19]

2. Comparison with Experiment

If the proposed phase shifts are to make sense, they should not give a large M-value or least squares deviation from any pion-proton scattering experiment which has been performed.

$$M=\sum_{i=1}^{N}\left[\frac{y_i-\bar{y}_i(\alpha_1,\alpha_3,\alpha_{33})}{\Delta y_i}\right]^2.$$

y_i= experimental cross section at angle x_i.

$\bar{y}_i(\alpha_1,\alpha_3,\alpha_{33})$ = cross section calculated from Eq. (2).

Δy_i= standard deviation given by experimenter for his determination of y_i.

If the phase shifts used are the correct ones and all others are zero, the mean expected value of M would be N, the total number of experimental points.[20] Comparisons are given in Table I. The mean M value or number of experimental points is given in the next to last column. The final column is the M value obtained when using the phase shifts given by the solid curves in Fig. 1 which are

$$\tan\alpha_1=0.16\eta,\quad \tan\alpha_3=-0.11\eta,$$
$$(\eta^3/\omega^*)\cot\alpha_{33}=8.05-3.8\omega^*, \qquad (2)$$
$$\text{all other phase shifts}=0.$$

For reference the numerical values used for the phase shifts are shown in the first three columns.

All the data above 140 Mev which is presently available for calculation of M values was used. These are the following Chicago experiments: 165(++),[1] 165(−−),[1] 165(−0),[1] 189(++),[2] 189(−0),[2] 189(−−),[2] and 217(−−).[3] The 217(−0) data require additional information about γ-ray detector efficiency *vs* energy for calculation of M values. The experimental errors for these charge-exchange experiments have recently been increased by such an amount that this data is now not very useful in narrowing down the phase shifts.

It is seen from Table I that all the M-values are statistically reasonable. This is most surprising for the 217-Mev data. Here the best-fit phase shifts depart appreciably from the s-wave lines in Fig. 1. The explanation is the existence of a Fermi-type solution other than that used by Glicksman. The s-waves proposed here happen to fall into the region of this equally good solution as can be seen from Fig. 2. Figure 2 is a plot of M-values obtained from the 217(−−) data as a function of α_1 and α_3. α_{33} is held constant at 100°. Glicksman's three-phase shift solution is at point A. The fit to the 217(−0) data should not be much different from that obtained by Glicksman since the quantity $(\alpha_1-\alpha_3)$ is not very different in the two cases.

Other experiments not listed in Table I tend to fit the proposed phase shifts in the sense that the proposed phase shifts are usually within the errors of the experimental phase shifts whenever such errors are given.

C. CONCLUSIONS

One concludes that the energy dependence of phase shifts proposed in Eq. (2) consistently fits all present pion-nucleon experimental knowledge almost as well as any other possible proposal for pion-nucleon phase

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[18] E. M. Henley and M. A. Ruderman, Phys. Rev. **90**, 719 (1953).

[19] G. Chew (private communication).

[20] In this case where there are no free parameters, the least squares theory does not require that the cross sections be linear in the phase shifts over the region involved. All we are doing here is comparing experiment with a 'known' curve. In the cases where 3 or 6 phase shifts are adjusted to minimize M, the mean M value should be considerably less than the number of experimental points.

shifts. However a higher degree of experimental accuracy is needed in order to conclude whether these are the correct phase shifts. The proposal here has the advantage of simplicity and agreement with meson theory calculations. Up to 300 Mev it is not necessary to use *d*-wave phase shifts although the small *d*-waves predicted by recoil corrections to the cutoff theory are welcome. The energy dependence proposed here would arise from a pion-nucleon interaction whose interaction range is on the order of $\frac{1}{2}\hbar/\mu c$ or less.

It may be significant that at least three other approaches lead to this same conclusion about the meson nucleon range. First, that Chew and Low in analysing pion scattering and photoproduction data are led to a $k_{\max}\sim 6\mu c/\hbar$. Second, that an effective-range analysis of the energy dependence of α_{33} by Brueckner gives an effective range of $\frac{1}{2}\hbar/\mu c$.[8] Third, that the Stanford high-energy electron scattering experiments on hydrogen give an rms radius of $\sim\frac{1}{2}\hbar/\mu c$.[9]

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Note added in proof (*September 18, 1955*).—The final Carnegie Tech data of Ashkin, Blaser, Feiner, and Stern has just become available ["Pion-proton scattering at 150 and 170 Mev," Phys. Rev. (to be published)]. They give 57 experimental points with total errors for each point (including charge-exchange)$\sim$5% or less. Their best-fit phase shifts ($\alpha_3=-8°$ and $\alpha_1=+10°$ at 170 Mev) agree quite well with those proposed here. Electronic computers can show whether this data is accurate enough to establish the linear extrapolation of α_1 and α_3 as the preferred solution.

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Range-Energy Relation and Masses of the New Particles*†

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The accuracy to which the masses of most of the new unstable particles can be determined is now limited principally by the uncertainty in the range-energy relations at large velocities. The extent of this uncertainty is indicated, and the available data are re-examined to try to find the best relations to use. In particular, shell corrections are applied to the Sachs-Richardson data, and the mean excitation potentials for 9 elements are determined. The evidence for Al, Cu, and emulsion indicates that the mean excitation potentials are not velocity dependent, and that they may be considerably larger than the values commonly used.

I. INTRODUCTION

MASS values for the new unstable particles generally depend upon a measurement of the range of either the particle itself or its secondaries. These determinations are now of sufficient accuracy so that it is necessary to be quite concerned about the uncertainty in the relations available for converting a measured range into energy or momentum. For instance, the range-energy curve for copper most commonly used for such mass determinations is not based on any direct experimental results.

The experimental data which do exist are correlated by using them to determine, for a given element, the mean excitation potential, I, which appears in the familiar energy-loss equation[1]:

$$-\frac{dE}{dx}=\frac{4\pi e^4z^2N}{mc^2\beta^2}\left\{Z\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)}-\beta^2\right]-\sum_i C_i\right\}, \quad (1)$$

where ez is the charge of the incident particle, and β its velocity relative to that of light, c; m is the electronic mass; N is the number of stopping atoms of atomic number Z per unit volume; and C_i is the correction for nonparticipating electrons of the ith shell.

II. POSSIBLE VARIATION OF I WITH ENERGY

Since I is determined by measurements of energy and either energy loss or range, which depend only logarithmically on I, it is not too surprising that there has been considerable disagreement in the values for I found in different experiments. However, as was first pointed out by Sachs and Richardson,[2] if one plots the experimental I values for a given element against the logarithm of the energy, instead of scattering badly, the points are seen to lie on a steeply sloping straight line. While an I value which is determined by an energy-loss measurement should be plotted against the incident energy, one which is determined by a range measurement should be plotted at some lower, "effective" energy, if I is not constant. In the latter case, the effective energy, ϵ, should be[2] about 0.6 of the incident

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† A shortened version of some of this work has appeared in Nuovo cimento 2, 183 (1955).
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