In concluding we may mention still another factor whose possible influence on the effect may be of interest. We have assumed from the outset that the complex phase changes brought about by the passage of a wave through the two regions of interaction are additive. When the interaction regions overlap, any nonlinearity in the superposition of their fields may imply nonadditivity of the phase changes. This too would contribute to the observed cross-section defect, but its analysis must clearly follow a more complete investigation of the linear effects. The author is greatly indebted to Dr. Anatole Shapiro for calling the measurements of the effect to his attention, and to C. Sommerfield and J. Bernstein for aid with some of the calculations.

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A Theory of New Particles

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A tentative scheme is developed to formulate the behavior of the new unstable particles. This scheme is a straightforward generalization of the usual charge-independent meson theory. The selection rules for isotopic spin are identical with those suggested by Gell-Mann. Owing to the particular form of the interaction assumed in this scheme, we can derive a new selection rule which seems to be of some use in interpreting the metastability of the new particles.

1. INTRODUCTION

SEVERAL attempts[1] to interpret the contradictions between the copious productions of the new particles and their metastabilities have been published. Among these attempts the "two-coupling-constant theory," due to Pais, seems most successful. In this theory the production processes are due to an interaction with a large coupling-constant while the other small coupling-constant is responsible for decay processes. Pais' recent theory[1] based on the four-dimensional ω space seems especially attractive. However there may remain, of course, other kinds of formalisms within the framework of the "two-coupling-constant theory."

In the present paper, the details of a theory of the baryons and mesons will be presented. In this theory particles will be distinguished by (a) isotopic spin I, (b) curious particle constant A, and (c) intrinsic spatial parity ϵ in addition to the usual mass, spin, and charge. The theory is constructed so that the selection rules involving I, I_z, and A are identical with those proposed by Gell-Mann.[2] In addition we will show that it is natural to introduce an additional selection rule involving the intrinsic parity ϵ. This last aspect was briefly discussed in a paper by Tobocman and the author.[2]

In the usual formalism, the neutron and the proton are described by a spinor $\psi^\alpha(\alpha=1,2)$ in a three-dimensional τ space. Similarly the charged and neutral mesons are described by a vector $\boldsymbol{\phi}$ (or a symmetric spinor of the second rank $\chi^{\alpha\beta}=\chi^{\beta\alpha}$ $(\alpha,\beta=1,2)$) in τ space. The interaction Lagrangian between the two fields in the usual charge-independent theory is

$$g\boldsymbol{\phi}\bar{\psi}\gamma_5\boldsymbol{\tau}\psi=\sum_{\alpha,\beta=1,2} g\chi^{\alpha\beta}\bar{\psi}\gamma_5\tau_{\alpha\beta}\psi. \quad (1.1)$$

Here $\tau_{\alpha\beta}$ and $\chi^{\alpha\beta}$ are defined as follows:

$$\tau_{11}=\tau_{22}{}^\dagger=i(\tau_1+i\tau_2), \quad \tau_{12}=\tau_{21}=-i\tau_3,$$

and

$$\chi^{11}=\chi^{22*}=-(i/2)(\phi_1-i\phi_2), \quad \chi^{12}=\chi^{21}=(i/2)\phi_3.$$

Now we consider a straightforward generalization of (1.1), namely

$$g\sum_n \sum_{\alpha_1\alpha_2\cdots\alpha_n=1,2} \chi^{(\alpha_1\cdots\alpha_n)}\bar{\psi}\gamma_5\cdot T_{(\alpha_1\cdots\alpha_n)}\psi. \quad (1.2)$$

Here $\chi^{(\alpha_1\cdots\alpha_n)}$ is a symmetric spinor of the nth rank in τ space and is assumed to describe a meson field with the ordinary spin 0 and τ spin $n/2$, and ψ is a wave function corresponding to some assembly of baryons with ordinary spin $\frac{1}{2}$ and various values of τ spin. $T_{(\alpha_1\cdots\alpha_n)}$ is some square matrix which is considered to be a generalization of the usual τ matrix.

In this treatment, we shall include only three kinds of meson fields: the ordinary π-meson field ($\chi_{(\pi)}{}^{\alpha\beta}$ or $\boldsymbol{\phi}$),[3] the θ-meson field with τ spin $\frac{1}{2}(\chi_{(\theta)}{}^\alpha)$, and the τ-meson field with τ spin $\frac{1}{2}$ $(\chi_{(\tau)}{}^\alpha)$. As to the baryons we shall include the nucleon field $(\psi_{(N)}{}^\alpha)$, the Λ particle with τ spin 0 $(\psi_{(\Lambda)})$, the Σ particles with τ spin 1 $(\psi_{(\Sigma)}{}^{\alpha\beta})$, and the cascade particle Y with τ spin $\frac{1}{2}$ $(\psi_{(Y)}{}^\alpha)$. Con-

[1] A. Pais, Physica **19**, 869 (1953); A. Pais, Proc. Nat. Acad. Sci. **40**, 484 (1954); M. Gell-Mann and A. Pais, Proceedings of the International Physics Conference, Glasgow, July, 1954 (Pergamon Press, London, 1955); T. Nakano and R. Utiyama, Progr. Theoret. Phys. (Japan) **11**, 411 (1954); T. Nakano and K. Nishijima, Progr. Theoret. Phys. (Japan) **10**, 581 (1953).

[2] R. Utiyama and W. Tobocman, Phys. Rev. **98**, 780 (1955). In the present paper this will be cited as U.T.

[3] Indices α, β, $\cdots$ are spinor indices in τ space. The ordinary spinor indices of baryons are omitted.

sequently the strong interaction Lagrangian (1.2) has the form

$$L_{\rm int}{}^{(S)}=g\sum_{\alpha,\beta=1,2}[\bar\psi\gamma_5\cdot T_{\alpha\beta}{}^{(\pi)}\psi\cdot\chi_{(\pi)}{}^{\alpha\beta}+\bar\psi\gamma_5\cdot T_\alpha{}^{(\theta)}\psi\cdot\chi_{(\theta)}{}^{\alpha}+\bar\psi\gamma_5\cdot T_\alpha{}^{(\tau)}\psi\cdot\chi_{(\tau)}{}^{\alpha}]+(\text{Hermitian conj.}).\quad(1.3)$$

The structure of the three kinds of T-matrices can be determined by the requirements that (1.3) be invariant under Lorentz transformations, gauge transformations and under rotations in τ space (charge independence).

On account of our special assumption that the strong coupling between the meson-family and the baryons is a γ_5 interaction, a new selection rule involving the intrinsic parities can be derived which may be of some use in interpreting the metastability of the new particles. The selection rule derived from the invariance under τ rotations is sufficient to interpret the metastability of all the known particles. However, if more new particles are discovered in future, more selection rules may be necessary to guarantee the singular metastability of the newcomers. The intrinsic parity rule mentioned above may be of use in this connection.

In the following sections, we shall give explicit expressions for the T-matrices and derive the above-mentioned new selection rule. Further we shall consider the possible types of electromagnetic and weak interactions.

2. ROTATION-INVARIANCE IN τ SPACE

The total Lagrangian density is written as

$$L=L_B+L_M+L_{\rm int}{}^{(S)},\quad(2.1)$$

$$L_B=i\bar\psi(\gamma^\mu\partial_\mu+\mathbf{M})\psi,\quad L_M=-(\partial_\mu\chi^*\cdot\partial^\mu\chi+\chi^*\mu^2\chi),$$

where χ and χ^* are defined by

$$\chi^*=(\chi_{(\pi)}{}^{*\alpha\beta},\chi_{(\theta)}{}^{*\alpha},\chi_{(\tau)}{}^{*\alpha}),\quad \chi=\begin{bmatrix}\chi_{(\pi)}{}^{\alpha\beta},\\ \chi_{(\theta)}{}^{\alpha},\\ \chi_{(\tau)}{}^{\alpha},\end{bmatrix},$$

and $\boldsymbol{\mu}^2$ stands for

$$\boldsymbol{\mu}^2=\begin{bmatrix}\mu_{(\pi)}{}^2E_3 & & 0\\ & \mu_{(\theta)}{}^2E_2 & \\ 0 & & \mu_{(\tau)}{}^2E_2\end{bmatrix},$$

where E_n is an $(n\times n)$ unit matrix.

As to the wave functions of baryons, we shall take the following representation:

$$\psi=\begin{bmatrix}\psi_{(N)}{}^\alpha\\ \psi_{(\Lambda)}\\ \psi_{(\Sigma)}{}^{\alpha\beta}\\ \psi_{(Y)}{}^\alpha\end{bmatrix},\quad \bar\psi=i\psi^*\gamma_4=(\bar\psi_{(N)}{}^\alpha,\bar\psi_{(\Lambda)},\bar\psi_{(\Sigma)}{}^{\alpha\beta},\bar\psi_{(Y)}{}^\alpha).\quad(2.2)$$

In this case, the mass operator $\mathbf{M}$ has the form

$$\mathbf{M}=\begin{bmatrix}M_{(N)}E_2 & & & 0\\ & M_{(\Lambda)}E_1 & & \\ & & M_{(\Sigma)}E_3 & \\ 0 & & & M_{(Y)}E_2\end{bmatrix}.$$

Under an infinitesimal rotation in τ space ψ and χ are assumed to be transformed in the following way:

$$\psi\to\psi'=(1+(i/2)\sum_{j,k=1}^{3}\epsilon_{jk}D^{jk})\psi$$

$$\chi\to\chi'=(1+(i/2)\sum_{j,k=1}^{3}\epsilon_{jk}\mathfrak{D}^{jk})\chi\quad(2.3)$$

$$(\epsilon_{jk}=-\epsilon_{kj},\quad j,k=1,2,3).$$

Here D and $\mathfrak{D}$ are respectively given by

$$D=\begin{bmatrix}D(\tfrac12) & & & \\ & D(0) & & \\ & & D(1) & \\ & & & D(\tfrac12)\end{bmatrix}$$

and

$$\mathfrak{D}=\begin{bmatrix}D(1) & & 0\\ & D(\tfrac12) & \\ 0 & & D(\tfrac12)\end{bmatrix},$$

where $D(l)$ ($l=0$, or $\frac12$ or 1) stands for an irreducible Hermitian representation matrix of $(2l+1)$ degree for the generator of the τ-rotation group.

Now let us assume charge independence for the strong interaction, namely, we assume the invariance of the Lagrangian under any τ rotation. Then we get the following relation

$$\delta L_{\rm int}{}^{(S)}=\delta(g\cdot\bar\psi\gamma_5\mathbf{T}\psi\cdot\chi)=0,$$

where $\mathbf{T}$ means[4]

$$(T_{\alpha\beta}{}^{(\pi)},T_\alpha{}^{(\theta)},T_\alpha{}^{(\tau)}),$$

and $\mathbf{T}\cdot\chi$ the scalar product of $\mathbf{T}$ and χ.

Substituting (2.3) into the above equation, we get

$$\mathbf{T}\mathfrak{D}^{jk}=[D^{jk},\mathbf{T}],$$

or equivalently

$$\sum_{\alpha',\beta'=1}^{2}T_{\alpha'\beta'}{}^{(\pi)}(\alpha'\beta'|D^{jk}(1)|\alpha\beta)=[D^{jk},T_{\alpha\beta}{}^{(\pi)}],$$

$$\sum_{\alpha'=1}^{2}T_{\alpha'}{}^{(\theta)}(\alpha'|D^{jk}(1/2)|\alpha)=[D^{jk},T_\alpha{}^{(\theta)}]\quad(2.4)$$

and

$$\sum_{\alpha'=1}^{2}T_{\alpha'}{}^{(\tau)}(\alpha'|D^{jk}(1/2)|\alpha)=D^{jk},T_\alpha{}^{(\tau)}].$$

Now in our particular representation shown in (2.2), the T-matrices can be split into various rectangular submatrices in the following way:

$$T=\begin{bmatrix}(\tfrac12|T|\tfrac12) & (\tfrac12|T|0) & (\tfrac12|T|1) & (\tfrac12|T|\tfrac12)\\ (0|T|\tfrac12) & (0|T|0) & (0|T|1) & (0|T|\tfrac12)\\ (1|T|\tfrac12) & (1|T|0) & (1|T|1) & (1|T|\tfrac12)\\ (\tfrac12|T|\tfrac12) & (\tfrac12|T|0) & (\tfrac12|T|1) & (\tfrac12|T|\tfrac12)\end{bmatrix},\quad(2.5)$$

[4] α and β are not matrix suffices but spinor indices taking values 1 or 2. Each T is an 8×8 matrix whose matrix indices are omitted.

where $(l|T|l')$ is a $(2l+1)\times(2l'+1)$ rectangular matrix.

By virtue of the relations (2.4) the zero submatrices of T's are given by

$$\begin{aligned}(l|T^{(\pi)}|l')&=0 \quad \text{for} \quad l-l'\neq\pm1 \text{ or } 0,\\ (l|T^{(\theta)}|l')&=0 \quad \text{for} \quad l-l'\neq\pm\tfrac{1}{2},\\ (l|T^{(\tau)}|l')&=0 \quad \text{for} \quad l-l'=\pm\tfrac{1}{2}\end{aligned} \tag{2.6}$$

(see Appendix).

Since L_B and L_M are invariant under transformation (2.3), we get the conservation-law of τ angular momentum:

$$\frac{\partial}{\partial x^\mu}[\bar{\psi}\gamma^\mu D^{ik}\psi - i\{\chi^*\mathfrak{D}^{ik}\partial^\mu\chi - \partial^\mu\chi^*\cdot\mathfrak{D}^{ik}\chi\}]=0. \tag{2.7}$$

In particular, we have

$$I_Z=\int\left[\bar{\psi}\gamma^0 D_Z\psi + i\left(\chi^*\mathfrak{D}_Z\frac{\partial\chi}{\partial t}-\frac{\partial\chi^*}{\partial t}\mathfrak{D}_Z\chi\right)\right]dx$$

$$=\text{const.} \qquad D_Z=D^{12}, \quad \mathfrak{D}_Z=\mathfrak{D}^{12}. \tag{2.8}$$

3. GAUGE INVARIANCE AND ELECTROMAGNETIC INTERACTION

To introduce the electromagnetic interaction, (2.1) must be replaced by the following expressions:

$$\begin{aligned}L_B &\to L_B'=i\bar{\psi}\{\gamma^\mu(\partial_\mu - ieCA_\mu)+\mathbf{M}\}\psi,\\ L_M &\to L_M'=-\{(\partial_\mu-ieC'A_\mu)\chi\}^*\\ &\qquad\times\{(\partial^\mu-ieC'A^\mu)\chi\}-\chi^*\mathfrak{u}^2\chi.\end{aligned} \tag{3.1}$$

L_B' and L_M' are invariant under the following infinitesimal gauge transformation:

$$\begin{aligned}\psi&\to\psi'=[1+ie\lambda(x)C]\psi,\\ \chi&\to\chi'=[1+ie\lambda(x)C']\chi.\end{aligned} \tag{3.2}$$

Here C and C' are defined as follows:

$$C=D_Z+A, \quad C'=\mathfrak{D}_Z+A',$$

and in that particular representation in which D_Z and $\mathfrak{D}_Z$ are diagonal, C, C', A, and A' have the forms[5]

$$C=\begin{bmatrix}1&&&&&&&\\ &0,&&&&&&\\ &&0,&&&&&\\ &&&1&&&&\\ &&&&0&&&\\ &&&&&-1,&&\\ &&&&&&0&\\ &&&&&&&-1\end{bmatrix}=D_Z+\tfrac{1}{2}\begin{bmatrix}E_2&&&\\ &0_1&&\\ &&0_3&\\ &&&-E_2\end{bmatrix},$$

$$C'=\begin{bmatrix}1&&&&&&\\ &0&&&&&\\ &&-1,&&&&\\ &&&1&&&\\ &&&&0,&&\\ &&&&&1&\\ &&&&&&0\end{bmatrix}=\mathfrak{D}_Z+\begin{bmatrix}0_3&&\\ &\tfrac{1}{2}E_2&\\ &&\tfrac{1}{2}E_2\end{bmatrix}. \tag{3.3}$$

Now $L_{\text{int}}^{(S)}$ must be invariant under the transformation (3.2). From this requirement, we have

$$\mathbf{T}C'=[C\cdot\mathbf{T}],$$

or

$$[A\cdot\mathbf{T}]-\mathbf{T}A'=\mathbf{T}\cdot\mathfrak{D}_Z-[D_Z,\mathbf{T}]=0$$

on account of (2.4). Thus, we have the following representation for T-matrices:

$$T^{(\pi)}=\begin{bmatrix}N_2&&0\\ &N_4&\\ 0&&N_2'\end{bmatrix},$$

$$T^{(\theta)}=T^{(\tau)}=\begin{bmatrix}0_2&N_{2\times4}&0_2\\ &0_4&N_{4\times2}\\ 0_2&&0_2\end{bmatrix}, \tag{3.4}$$

where N_i and $N_{i\times k}$ stand for $i\times i$ and $i\times k$ submatrices having nonvanishing elements respectively.

The conservation of charge is now expressed by

$$Q=e\int\left[\bar{\psi}\gamma^0 C\psi + i\chi^* C'\left(\frac{\partial}{\partial t}-ieC'A_0\right)\chi - i\left\{\left(\frac{\partial}{\partial t}-ieC'A_0\right)\chi\right\}^* C'\chi\right]dx=\text{const.} \tag{3.5}$$

If we omit the electromagnetic interaction, (3.5) becomes

$$Q=\int\left[\bar{\psi}\gamma^0 C\psi + i\left\{\chi^* C'\frac{\partial\chi}{\partial t}-\frac{\partial\chi^*}{\partial t}C'\chi\right\}\right]dx=\text{const.} \tag{3.6}$$

From (3.6) and (2.8), we have the law of conservation of A for the strong coupling:

$$\int\left[\bar{\psi}\gamma^0 A\psi + i\left\{\chi^* A'\frac{\partial\chi}{\partial t}-\frac{\partial\chi^*}{\partial t}A'\chi\right\}\right]dx=\text{const.} \tag{3.7}$$

Since every transition process must conserve the total charge Q, the conservation of A for the strong coupling is synonymous with that of I_Z for such a coupling.

Besides the gauge transformation, our Lagrangian is invariant under the phase transformation of baryon-fields

$$\psi\to\psi e^{i\alpha}, \quad \bar{\psi}\to\bar{\psi}e^{-i\alpha}, \quad \alpha=\text{const.}$$

Here it must be noted that all the baryon fields are assumed to be complex functions. From this invariance, we get the conservation of the number of baryons.

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[5] These representations of C and C' correspond to the following representations of ψ and χ:

$$\psi=\begin{bmatrix}\text{proton}\\ \text{neutron}\\ \hline \Lambda^0\\ \hline \Sigma^+\\ \Sigma^0\\ \Sigma^-\\ \hline Y^0\\ Y^-\end{bmatrix}, \qquad \chi=\begin{bmatrix}\pi^+\\ \pi^0\\ \pi^-\\ \hline \theta^+\\ \theta^0\\ \hline \tau^+\\ \tau^0\end{bmatrix}.$$

4. INVERSION-INVARIANCE AND THE SELECTION-RULE

It is easily seen that our Lagrangian is invariant under the proper Lorentz group. As to spatial inversion, however, some care is needed. As Yang and Tiomno[6] pointed out, spinor fields have four possible transformation characters under inversion in ordinary space:

$$\psi(x) \rightarrow \epsilon\gamma_4\psi(-x),$$

$$(A)\ \epsilon=1,\quad (B)\ \epsilon=-1,\quad (C)\ \epsilon=i,\quad (D)\ \epsilon=-i. \tag{4.1}$$

Since the θ meson is assumed in the present paper to be a scalar field owing to its decay mode, and since we assume that the strong coupling interaction is a γ_5 interaction, we are forced to introduce baryon fields with various intrinsic parities.

Now let us consider any element of the S-matrix

$$\langle f|S|i\rangle=(\Psi_f{}^*\cdot S\Psi_i)$$

which is rewritten under inversion as

$$\langle f|S|i\rangle=[(I\Psi_f)^*\cdot ISI^{-1}\cdot(I\Psi_i)].$$

Here I is a unitary operator representing inversion. Now suppose that Ψ_i and Ψ_f are the eigenstates of I, i.e.,

$$I\Psi_i=\eta_i\Psi_i,\quad I\Psi_f=\eta_f\Psi_f,\quad \eta=+1 \text{ or } -1,$$

and that S is invariant under inversion (no external field is present), namely,

$$ISI^{-1}=S. \tag{4.2}$$

Then we get

$$\langle f|S|i\rangle=\eta_i\eta_f\langle f|S|i\rangle$$

or

$$\eta_i=\eta_f. \tag{4.3}$$

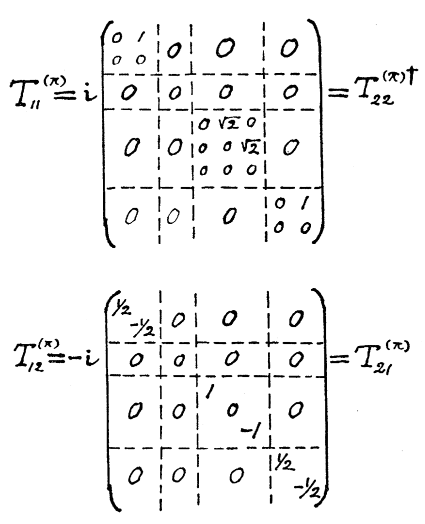

FIG. 1. Representation of $T^{(\pi)}$.

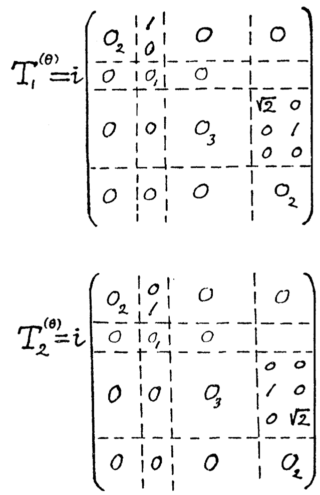

FIG. 2. Representation of $T^{(\theta)}$.

This is nothing but the usual conservation law of parity.[7] (4.2) is a trivial relation in ordinary meson theory. However, in our case this requirement gives rise to some restrictions on the form of the interaction.

By using (3.4), $L_{\text{int}}{}^{(S)}$ can be written in the following way:

$$L_{\text{int}}{}^{(S)}=g\sum_{a,b,b'}\chi_{(a)}\bar{\psi}_{(b)}\gamma_5 T_{(a,b,b')}\cdot\psi_{(b')}, \tag{4.4}$$

where $\chi_{(a)}$ means some kind of meson, $\psi_{(b')}$ and $\bar{\psi}_{(b)}$ stand for some kinds of baryons, and finally $T_{(a,b,b')}$ stands for some rectangular submatrix shown in (3.4).

Now (4.2) is equivalent to

$$\int IL_{\text{int}}{}^{(S)}(\chi)I^{-1}d^4x=\int L_{\text{int}}{}^{(S)}(\chi)d^4x,$$

and from this relation, we get

$$-\epsilon_a\epsilon_b{}^*\epsilon_{b'}=1. \tag{4.5}$$

Here ϵ_a is equal to ± 1 and means the intrinsic parity of $\chi_{(a)}$, ϵ_b and $\epsilon_{b'}$, (ϵ^* is a complex conjugate of ϵ) are the intrinsic parities of baryons introduced in (4.1).[8] If the ordinary nucleon is assumed to be a type-A spinor, the spinors of type-C and -D can be excluded from consideration on account of (4.5). From (4.4) and (4.5), we see that some of the submatrices in (3.4) must be put equal to zero. If we assume the parity assignment as shown in the table which was given in the paper of U. T., the representation of T-matrices is given by

[6] C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950); S. Watanabe, Sci. Papers Inst. Phys. Chem. Research (Tokyo) **39**, 157 (1941); Wick, Wightman, and Wigner, Phys. Rev. **88**, 101 (1952).

[7] Of course in η the parity due to the orbital angular momentum is included.

[8] In the present paper the intrinsic parity is defined in the following way; (i) spinor field is transformed under inversion as

$$\psi(x)\rightarrow\epsilon\gamma_4\psi(-x),$$

(ii) tensor or scalar field is transformed as

$$A_{\mu\nu\cdots\lambda}(x)\rightarrow\epsilon(-1)^n A_{\mu\nu\cdots\lambda}(-x);$$

n=the number of suffices which are not equal to 4 among μ, ν, $\cdots\lambda$. In both cases, ϵ is called the intrinsic parity of the field considered.

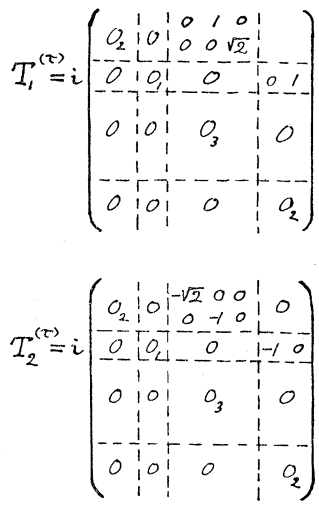

FIG. 3. Representation $T^{(\tau)}$.

Figs. 1, 2, and 3. In obtaining these representations, some use has been made of the assumed invariance under charge conjugation.

Since $L_{\text{int}}^{(S)}$ satisfies relation (4.5), we have the following relation for any transition exclusively depending on $L_{\text{int}}^{(S)}$:

$$\epsilon_i(-1)^{n_i}=\epsilon_f(-1)^{n_f}. \qquad (4.6)$$

Here ϵ is the product of the intrinsic parities of all the particles present, n is the total number of all the kinds of mesons present, and i and f indicate the initial and final quantities respectively. The relation (4.6) is easily verified by considering any particular Dyson-Feynman diagram and supposing that at every vertex of the diagram the relation (4.5) is valid.

This new rule is essentially due to the particular choice of the interaction Lagrangian and, of course, does not contradict the validity of the usual parity law (4.3).

5. ELECTROMAGNETIC INTERACTION AND WEAK INTERACTION

The Lagrangian of the electromagnetic interaction introduced in Sec. 3 is

$$L^{(1)}{}_{\text{el. mag}}=eA_\mu\bar{\psi}\gamma^\mu C\psi+ieA_\mu\{-\chi^*C'\partial^\mu\chi+\partial^\mu\chi^*C'\chi\} -e^2A_\mu A^\mu\chi^*C'^2\chi. \qquad (5.1)$$

The matrices C and C' are diagonal in the particular representation in which χ and ψ have the representations shown in footnote 5. Therefore transitions due to (5.1) do not give rise to any change of characters of baryons and mesons. In other words, the conservation of A and the ϵ rule (4.6) are also valid for these transitions.

In the usual meson theory, besides (5.1), we have another kind of electromagnetic interaction, namely, the so-called "Pauli-type interaction":

$$\tfrac{1}{4}\mu\bar{\psi}[\gamma^\mu\gamma^\nu]\psi F_{\mu\nu}, \quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,$$

μ = the magnetic moment of nucleons.

Similarly, we can introduce in the present framework the following gauge invariant electromagnetic interaction:

$$L^{(2)}{}_{\text{el. mag}}=\tfrac{1}{4}\mu\bar{\psi}[\gamma^\mu\gamma^\nu]U\psi F_{\mu\nu}, \qquad (5.2)$$

where the matrix U is an (8×8)-matrix and plays a similar role to that of our T-matrices.

This U-matrix is not necessarily diagonal but the requirements of the conservation of A and the ϵ rule (4.6) lead to the following representation of U:

(i) if Σ^0 and Λ^0 have the same intrinsic parity,

$$U=\begin{pmatrix} \mu_1 & 0 & & & & & & 0\\ 0 & \mu_2 & & & & & & \\ & & \mu_3 & 0 & a & 0 & & \\ & & 0 & \mu_4 & 0 & 0 & & \\ & & a & 0 & \mu_5 & 0 & & \\ & & 0 & 0 & 0 & \mu_6 & & \\ 0 & & & & & & \mu_7 & 0\\ & & & & & & 0 & \mu_8 \end{pmatrix},$$

where the μ_i's and a are some real constants;

(ii) if Σ^0 and Λ^0 have the opposite parities, U is diagonal.

In the case (i) a fast γ decay of Σ^0 to Λ^0 is possible if the value of a is suitably chosen.

So far, we have discussed only the strong and electromagnetic interactions. For both types of interactions the ϵ rule (4.6) and the conservation of A-values and of the number of baryons are always valid.

Now let us discuss the possible types of weak interactions. These we define in the following way; a weak interaction is one which violates the ϵ rule or the conservation of A-values or both. However, the requirements of gauge-invariance and of the conservation of the number of baryons should be also satisfied, for this interaction.

We shall discuss separately the following three possible cases.

(i) A is conserved but ϵ rule is violated.

The interaction Lagrangian is

$$L_1^{(W)}=f[\bar{\psi}\mathbf{V}\psi\cdot\chi+\text{Hermitian conj.}], \qquad (5.3)$$

where

$$\mathbf{V}=(V_{\alpha\beta}{}^{(\pi)},V_\alpha{}^{(\theta)},V_\alpha{}^{(\tau)}),$$

and

$$f\ll g.$$

Each V-matrix can be determined (to some extent) by the requirements of the gauge invariance and of the conservation of A. Namely, by using the notation of Sec. 3, these two requirements lead to

$$\mathbf{V}C'=[C,\mathbf{V}] \quad \text{(gauge invariance)} \qquad (5.4)$$

TABLE I. Assignments of τ spin and intrinsic parity.

Particle	P, N	Λ^0	$\Sigma^+, \Sigma^0, \Sigma^-$	Y^0, Y^-	π^+, π^0, π^-	θ^+, θ^0	τ^+, τ^0
Spatial spin	1/2	1/2	1/2	1/2	0	0	0
Spatial parity	A (+)	B (−)	A (+)	B (−)	P.S. (−)	S. (+)	P.S. (−)
τ-spin	1/2	0	1	1/2	1	1/2	1/2
A-value	1/2	0	0	−1/2	0	1/2	1/2
Wave function	$\psi_{(N)}{}^{\alpha}$	$\psi_{(\Lambda)}$	$\psi_{(\Sigma)}{}^{\alpha\beta}$	$\psi_{(Y)}{}^{\alpha}$	$\chi_{(\pi)}{}^{\alpha\beta}$	$\chi_{(\theta)}{}^{\alpha}$	$\chi_{(\tau)}{}^{\alpha}$
	ψ				χ		

and

$$\mathbf{V}A'=[A,\mathbf{V}] \quad \text{(conservation of } A). \qquad (5.5)$$

Furthermore since (5.3) is a non-γ_5 interaction, $\mathbf{V}$ has the following property:

$$\begin{aligned}(A|V^{(\pi)}|A)&=(B|V^{(\pi)}|B)=0,\\(A|V^{(\tau)}|A)&=(B|V^{(\tau)}|B)=0,\\(A|V^{(\theta)}|B)&=(B|V^{(\theta)}|A)=0.\end{aligned} \qquad (5.6)$$

These conditions (5.4)–(5.6) lead to a concrete representation for the $\mathbf{V}$-matrix. For example, in our particular case, as shown in Table I, the representation of $V^{(\pi)}$ is given in Fig. 4, where a and b are some complex numbers while c must be a pure imaginary constant.

(ii) ϵ rule is valid but A value is not conserved.

The interaction Lagrangian is

$$L_2{}^{(W)}=f[\bar{\psi}\gamma_5\mathbf{W}\psi\cdot\chi+\text{Hermitian conj.}]. \qquad (5.7)$$

In this case, $\mathbf{W}$ must satisfy the following relations:

$$\mathbf{W}C'=[C,\mathbf{W}], \qquad (5.8)$$

$$\mathbf{W}A'\neq[A,\mathbf{W}], \qquad (5.9)$$

$$\begin{aligned}(A|W^{(\pi)}|B)&=(B|W^{(\pi)}|A)=0,\\(A|W^{(\tau)}|B)&=(B|W^{(\tau)}|A)=0,\\(A|W^{(\theta)}|A)&=(B|W^{(\theta)}|B)=0.\end{aligned} \qquad (5.10)$$

(5.9) is just the formal expression of the following postulate: we must put equal to zero all those matrix elements which allow transitions conserving the A-value.

(iii) ϵ rule and conservation of A are violated.

In this final case, the Lagrangian is

$$L_3{}^{(W)}=f[\bar{\psi}\mathbf{Y}\psi\cdot\chi+\text{Hermitian conj.}]. \qquad (5.11)$$

The $\mathbf{Y}$-matrices must have the following properties:

$$\begin{aligned}\mathbf{Y}C'&=[C,\mathbf{Y}],\\\mathbf{Y}A'&\neq[A,\mathbf{Y}],\\(A|Y^{(\pi)}|A)&=(B|Y^{(\pi)}|B)=0,\\(A|Y^{(\tau)}|A)&=(B|Y^{(\tau)}|B)=0,\\(A|Y^{(\theta)}|B)&=(B|Y^{(\theta)}|A)=0.\end{aligned} \qquad (5.12)$$

From (5.8)–(5.10) and (5.12), we can derive concrete representations for $\mathbf{W}$ and $\mathbf{Y}$.

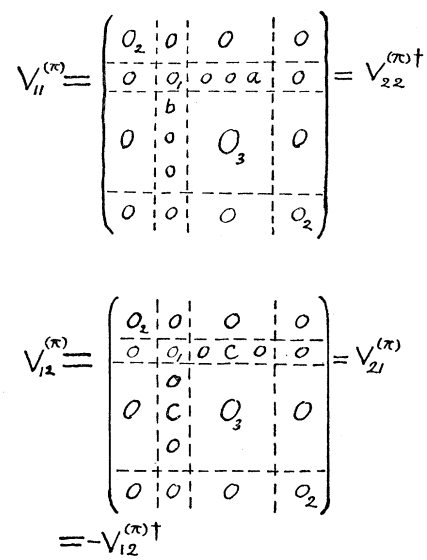

FIG. 4. Representation of $V^{(\pi)}$.

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APPENDIX[9]

We shall explain briefly how to get the representation of T-matrices. As an example, let us consider the second equation of (2.4):

$$\sum_{\alpha'=1}^{2} T_{\alpha'}{}^{(\theta)}(\alpha'|D^{ik}(\tfrac{1}{2})|\alpha)=[D^{ik},T_{\alpha}{}^{(\theta)}]. \qquad (2.4)$$

Now let us take a particular representation for $D^{(ik)}$ as given in Sec. 3. In such a representation the D's are

[9] The method here developed is quite similar to what was presented by Bhabha in his paper: H. J. Bhabha, Revs. Modern Phys. **17**, 200 (1945).

given by the following expressions: put

$$D_P=D^{23}+iD^{31},\quad D_q=D^{23}-iD^{31},\quad D_Z=D^{12};$$

then

$$(l,m|D_P|l,m-1)=(l,m-1|D_q|l,m)=[(l+m)(l-m+1)]^{\frac{1}{2}},\quad \text{(A.1)}$$

$$(l,m|D_Z|l,m)=m,\quad m=l,l-1,\cdots-(l-1),-l,$$

where $(l,m|D|l',m')$ means an (m,m')-element of a $(2l+1)\times(2l'+1)$-rectangular-submatrix of D. Furthermore, $(\alpha'|D(\frac{1}{2})|\alpha)$ is represented in the following way:

$$(\alpha|D^{23}(\tfrac{1}{2})|\beta)=(\alpha|\tau_1|\beta),\quad \text{etc.},$$

where the τ's are defined by

$$\tau_1=\begin{pmatrix}0 & 1/2\\ 1/2 & 0\end{pmatrix},\quad \tau_2=\begin{pmatrix}0 & -i/2\\ i/2 & 0\end{pmatrix},\quad \tau_3=\begin{pmatrix}1/2 & 0\\ 0 & -1/2\end{pmatrix}.\quad \text{(A.1)}'$$

For the sake of convenience, let us rewrite (2.4) as follows:

$$[D_P\cdot T_1]=0,\quad \text{(A.2)},\qquad [D_P,T_2]=T_1,\quad \text{(A.3)}$$

$$[D_q\cdot T_1]=T_2,\quad \text{(A.4)},\qquad [D_q,T_2]=0,\quad \text{(A.5)}$$

$$[D_Z\cdot T_1]=\tfrac{1}{2}T_1,\quad \text{(A.6)},\qquad [D_Z,T_2]=-\tfrac{1}{2}T_2,\quad \text{(A.7)}$$

where the representation (A.1)′ has been used. Substituting (A.1) into (A.6) and (A.7), we get

$$(m-m'-\tfrac{1}{2}),\ (l,m|T_1|l',m')=0,$$
$$(m-m'+\tfrac{1}{2}),\ (l,m|T_2|l',m')=0.$$

Therefore the nonvanishing elements of T_1 and T_2 have the forms

$$(l,m|T_1|l',m-\tfrac{1}{2})\neq 0,\qquad (l,m|T_2|l',m+\tfrac{1}{2})\neq 0.\quad \text{(A.8)}$$

From (A.2) we have

$$[(l+m)(l-m+1)]^{\frac{1}{2}}\cdot(l,m-1|T_1|l',m-\tfrac{3}{2})=[(l'+m-\tfrac{1}{2})(l'-m+\tfrac{3}{2})]^{\frac{1}{2}}\cdot(l,m|T_1|l',m-\tfrac{1}{2}).\quad \text{(A.9)}$$

By putting $m=-l$ or $l'+\frac{3}{2}$ in (A.9), this equation becomes

$$[(l'-l-\tfrac{1}{2})(l'+l+\tfrac{3}{2})]^{\frac{1}{2}}\cdot(l,-l|T_1|l',-l-\tfrac{1}{2})=0$$

or

$$[(l+l'+\tfrac{3}{2})(l-l'-\tfrac{1}{2})]^{\frac{1}{2}}\cdot(l,l'+\tfrac{1}{2}|T_1|l',l')=0.$$

Thus, if

$$l'=l\pm\tfrac{1}{2},\quad \text{(A.10)}$$

we have

$$(l,-l|T_1|l+\tfrac{1}{2},-l-\tfrac{1}{2})\neq 0,$$
$$(l,l|T_1|l-\tfrac{1}{2},l-\tfrac{1}{2})\neq 0.$$

Similarly, from (A.5) we can derive the relation (A.10). Accordingly, the nonvanishing elements of T-matrices take the forms

$$f_1^{\pm}(l,m)\equiv(l,m|T_1|l\pm\tfrac{1}{2},m-\tfrac{1}{2}),\qquad f_2^{\pm}(l,m)\equiv(l,m|T_2|l\pm\tfrac{1}{2},m+\tfrac{1}{2}).\quad \text{(A.11)}$$

By substituting (A.11) into (A.2) and (A.5), the following recursion formula can be derived:

$$(l-m+1)^{\frac{1}{2}}\cdot f_1^{+}(l,m-1)=(l-m+2)^{\frac{1}{2}}\cdot f_1^{+}(l,m),$$
$$(l+m)^{\frac{1}{2}}\cdot f_1^{-}(l,m-1)=(l+m-1)^{\frac{1}{2}}\cdot f_1^{-}(l,m),$$
$$(l+m)^{\frac{1}{2}}\cdot f_2^{+}(l,m)=(l+m+1)^{\frac{1}{2}}\cdot f_2^{+}(l,m-1),$$
$$(l-m+1)^{\frac{1}{2}}\cdot f_2^{-}(l,m)=(l-m)^{\frac{1}{2}}\cdot f_2^{-}(l,m-1).$$

Therefore, we have the solution

$$f_1^{+}(l,m)\propto(l-m+1)^{\frac{1}{2}},$$
$$f_1^{-}(l,m)\propto(l+m)^{\frac{1}{2}},$$
$$f_2^{+}(l,m)\propto(l+m+1)^{\frac{1}{2}},$$
$$f_2^{-}(l,m)\propto(l-m)^{\frac{1}{2}}.$$

Substituting these into (A.3) and (A.4), we get the final solution

$$f_1^{+}(l,m)=a_l(l-m+1)^{\frac{1}{2}},$$
$$f_1^{-}(l,m)=b_l(l+m)^{\frac{1}{2}},$$
$$f_2^{+}(l,m)=-a_l(l+m+1)^{\frac{1}{2}},$$
$$f_2^{-}(l,m)=b_l(l-m)^{\frac{1}{2}},$$

where a_l and b_l are arbitrary complex constants only depending on l. For example, in our case we have

$$T_1^{(\theta)}=\begin{pmatrix}0&0&b&0&a&0&0&0\\0&0&0&0&0&a\sqrt{2}&0&0\\0&c&0&0&0&0&0&d\\e\sqrt{2}&0&0&0&0&0&f\sqrt{2}&0\\0&e&0&0&0&0&0&f\\0&0&0&0&0&0&0&0\\0&0&g&0&h&0&0&0\\0&0&0&0&0&h\sqrt{2}&0&0\end{pmatrix}.$$

On account of the gauge invariance, c, e, g, and h must vanish, Further, the ϵ rule makes a and d vanish because of our parity assignments.