Cross Sections in Deuterium at High Energies

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Recent measurements at high energies indicate that the total cross sections for collisions of both nucleons and π mesons with deuterons are noticeably smaller than the sums of the corresponding cross sections for free neutrons and protons. A formalism for calculating the cross sections of the deuteron is developed, based on the assumption that the interactions of the incident particle with the neutron and proton may individually be treated by the general methods of diffraction theory. The nonadditivity of the free-particle cross sections is shown to be due largely to "eclipses" in which either the neutron or the proton lies in the shadow cast by the other, an effect in which quantum mechanical diffraction plays an important role. Simple representations of the high-energy interactions and the ground-state wave function of the deuteron are found to lead to cross-section defects of the magnitude observed.

I. INTRODUCTION

IT has often been suggested that in collisions with incident particles of sufficiently high energy the neutron and proton comprising a deuteron may be considered as independent scatterers. For de Broglie wavelengths much smaller than the deuteron radius, it is argued, interference effects vanish, and any cross section should equal the sum of the corresponding neutron and proton cross sections measured separately. Recent measurements of nucleon attenuation at 1.4 Bev (where $\lambdabar=0.1\times10^{-13}$ cm) seem, on the contrary, to reveal a substantial lack of additivity of the neutron and proton cross sections, in deuterium.[1,2] Measurements with incident protons and incident neutrons both indicate that the deuteron cross section is less than the sum of the free-particle cross sections. The measured differences, although obviously subject to uncertainty, amount to 9 mb and 6 mb respectively, values to be compared with $\sigma(n,p)=42$ mb and $\sigma(p,p)=48$ mb.

A very similar situation has been found to arise in measurements[3] of interactions of π mesons with hydrogen and deuterium at 800 Mev (where $\lambdabar=0.2\times10^{-13}$ cm). The observations, which we shall discuss later in detail, furnish support for the hypothesis of charge symmetry and show that the deuteron cross section is once again smaller than the sum of the free-particle cross sections. The measured differences in this case are 6 mb and 4 mb, values to be compared with the magnitudes $\sigma(\pi^-,p)=41$ mb and $\sigma(\pi^+,p)=18$ mb.

Some simple considerations may be of help in indicating the nature of the effect. At these energies the attenuation of the incident amplitude by incoherent processes such as meson production may be schematically represented as due to a certain amount of absorption of the incident wave by the nucleons. Since the incident wavelengths in these cases are evidently much smaller than the ranges of interaction, the nucleons may be thought of as casting fairly well-defined shadows. It is then clear that absorption or scattering by either nucleon is reduced when it enters the shadow of the other. Astronomers have long been familiar with a time-reversed analog of this effect; the decrease in luminosity of binary star systems during eclipses.

To observe the mechanism of the effect more closely we may begin by considering an elementary model. We represent the regions of interaction surrounding the nucleons as black spheres, each of which will then have a total cross section (comprising absorption and diffraction scattering) equal to twice its absorption (i.e., geometrical) cross section. The effect of eclipsing is most easily seen on the total absorption cross section of the system. This is because absorptions by the two nucleons are mutually exclusive events, and because no interference effects are involved, conditions which are both lacking in the consideration of scattering. To find the deviation of the total absorption cross section from the sum of the two absorption cross sections, we must correct for the fact that those particles whose initial trajectories cross both regions of interaction are absorbed in the first and not the second.

The geometrical cross section of the first nucleon will be $\sigma_1/2$, where σ_1 is its total cross section. We require the probability that a straight line passing through the first region of interaction also intersects the second. We shall assume, for the moment, that the interaction ranges are small compared with the average value of $|\mathbf{r}|$, the separation of the nucleons. Then, since the probability density of the second nucleon is isotropic[4] about the first, the required probability is $\frac{1}{2}\sigma_2\langle 1/4\pi r^2\rangle_d$ which is the average solid angle subtended by the second region, as viewed from the first. The correction to the absorption cross section of the system arising from collisions in which the shadow of region 1 falls on region 2 is thus $-\frac{1}{4}\sigma_1\sigma_2\langle 1/4\pi r^2\rangle_d$. An identical correction comes from collisions in which the regions are interchanged. Hence we have

$$\sigma_{\rm abs}=\tfrac{1}{2}\sigma_1+\tfrac{1}{2}\sigma_2-(\sigma_1\sigma_2/8\pi)\langle 1/r^2\rangle_d \qquad (1)$$

[1] Coor, Hill, Hornyak, Smith, and Snow, Phys. Rev. 98, 1369 (1955).
[2] Chen, Leavitt, and Shapiro (private communication). See also Phys. Rev. 95, 663(A) (1954).
[3] Dr. O. Piccioni has kindly supplied these data.
[4] We neglect the d-state admixture and the effects of spin in the deuteron.

for the absorption cross section. The total cross section of the system is most simply found by appealing to the fact that, whatever their configuration may be, the total cross section of the black interaction regions equals twice their absorption cross section. We therefore have the expression for the total cross section

$$\sigma_d = \sigma_1 + \sigma_2 - (\sigma_1\sigma_2/4\pi)\langle 1/r^2\rangle_d. \qquad (2)$$

Assuming that the deuteron has a radius of the order of the triplet effective range $r_e = 1.7\times10^{-13}$ cm, it is easily verified that the correction term has the appropriate order of magnitude, 6 to 9 mb for nucleon scattering. We shall return to a more quantitative discussion of the effect at a later point.

Although the representation of the interaction regions as black spheres has been helpful in the foregoing derivation, it must be emphasized that the evidence available to date is not sufficiently detailed to support such a model firmly. While meson production may indeed represent a considerable absorption, the spatial distributions of opacity remain unknown within wide limits. Nor can we exclude the possibility of coherent scattering taking place within the interaction regions. For these reasons then, in examining the effect further, we shall avoid placing any strong restrictions on the nature of the interactions involved.

II. CALCULATION OF THE TOTAL CROSS SECTION

In the energy region of interest, incident particles have wavelengths considerably smaller than their ranges of interaction with nucleons. If it may be assumed that the energies of interaction of the incident particles with the nucleons are smaller than the incident kinetic energies, the collisions may be described by methods which are essentially those of diffraction theory.[5] In the diffraction approximation we assume that the incident plane wave sweeps, virtually undeviated, through the region of interaction, and emerges suffering only a position-dependent change of phase and amplitude.

If we let $\mathbf{b}$ be an impact-parameter vector in the plane perpendicular to the direction of incidence, the scattering process may be characterized by a function $\chi(\mathbf{b})$ which represents the change of phase, at a point $\mathbf{b}$ of the emerging wave front caused by passage through the region of interaction. The decrease of amplitude due to incoherent processes is represented by allowing $\chi(\mathbf{b})$ to assume complex values ($\mathrm{Im}\chi<0$). The coherent scattering at small angles is then given by[5,6]

$$f(\mathbf{k}',\mathbf{k}) = \frac{k}{2\pi i}\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}\{e^{-i\chi(\mathbf{b})}-1\}d^{(2)}\mathbf{b}, \qquad (3)$$

in which $\mathbf{k}$ is the initial propagation vector and $\mathbf{k}'$ the deflected one. The integration is carried out over a plane perpendicular to the direction of incidence. We shall find it convenient to employ the abbreviation

$$\Gamma(\mathbf{b}) = 1 - e^{-i\chi(\mathbf{b})}. \qquad (4)$$

Then for axially symmetric regions of interaction centered at $\mathbf{b}=0$, the scattering amplitude reduces to

$$f(\mathbf{k}',\mathbf{k}) = ik\int_0^\infty J_0(|\mathbf{k}-\mathbf{k}'|b)\Gamma(b)bdb. \qquad (5)$$

We shall assume that the interactions of the incident particles with nucleons are completely characterized by functions $\chi(\mathbf{b})$ or $\Gamma(\mathbf{b})$ defined separately for the neutron and proton. In the work that follows, we make use of the scattering amplitudes only at or near the forward direction $\mathbf{k}'=\mathbf{k}$. It is this restriction which justifies the neglect of recoil effects implicit in the use of (3) and (5) for the description of scattering by isolated nucleons.[7]

In discussing scattering by deuterons and more complicated systems, the possibility of excitation of the system requires explicit treatment of its internal degrees of freedom. For this purpose a generalization of the diffraction procedures already outlined has been developed. While the application of the method is sufficiently direct in the present context, we shall return to a fuller discussion of its mathematical basis along with certain more general procedures in future work.

We may formulate the problem in a general way by considering the scattering by a bound system of nucleons whose individual velocities are small compared with the velocity of the incident particle. The nucleons may then be considered frozen in their instantaneous positions, say $\mathbf{r}_1\cdots\mathbf{r}_A$, during the particle's passage through the system. Assuming that the incident particle interacts with the nucleons through two-body forces, the total phase change of the emerging wave, $\chi_{\mathrm{tot}}(\mathbf{b})$, will be the sum of the phase changes produced by the individual nucleons. If the projections of the coordinates $\mathbf{r}_1\cdots\mathbf{r}_A$ on a plane perpendicular to the direction of incidence are $\mathbf{q}_1\cdots\mathbf{q}_A$, we may write

$$\chi_{\mathrm{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A) = \sum_{j=1}^{A}\chi_j(\mathbf{b}-\mathbf{q}_j). \qquad (6)$$

The function Γ analogous to (2) is then

$$\Gamma_{\mathrm{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A) = 1-\exp[-i\textstyle\sum_j\chi_j(\mathbf{b}-\mathbf{q}_j)]. \qquad (7)$$

[5] G. Molière, Z. Naturforsch. **2a**, 133 (1947); Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[6] If the interaction is due to a potential $V(r)$, the phase function $\chi(\mathbf{b})$ is given by $(\hbar v)^{-1}\int_{-\infty}^{\infty}V(\mathbf{z}+\mathbf{b})dz$, where $\mathbf{z}$ is the component of $\mathbf{r}$ in the direction of propagation and v is the incident velocity.

[7] Recoil effects are in fact easily included in a somewhat more general formulation of the method outlined in the succeeding paragraphs, but the corrections they represent for $\mathbf{k}'$ different from $\mathbf{k}$ are characteristically quite small. This is because the assumptions inherent in the diffraction approximation restrict the scattering predominantly to small angles. The forwardness of the scattering does not, however, exclude the possibility of internal excitation of scatterers containing more than one nucleon. To find the coherent scattering amplitude we must exclude the latter explicitly.

If the nucleons were rigidly fixed in their instantaneous positions, substitution of this expression in (3) would yield the scattering amplitude. Since the nucleons are not fixed in position the quantity $\Gamma_{\text{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A)$ is to be regarded as an operator inducing transitions of the nuclear system as well as deflections of the scattered particle. In particular, if we are interested in elastic scattering, we must employ the diagonal element of $\Gamma_{\text{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A)$ in the nuclear ground state.[8] For the reasons previously stated,[7] we may ignore the recoil momentum of the nucleus as a whole by letting its center of mass remain fixed at the origin. Then the elastic scattering amplitude is

$$F(\mathbf{k}',\mathbf{k})=\frac{ik}{2\pi}\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}d^{(2)}\mathbf{b}\times\int|\psi(\mathbf{r}_1\cdots\mathbf{r}_A)|^2\Gamma_{\text{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A)\prod_j d\mathbf{r}_j, \quad (8)$$

where the $\mathbf{r}$-integrations are over the configuration space of the $A-1$ independent nucleon positions.

To apply this approximation to scattering by deuterons, we introduce the relative coordinates

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2, \quad \mathbf{q}=\mathbf{q}_1-\mathbf{q}_2, \quad (9)$$

and let $\phi(\mathbf{r})$ be the wave function of the deuteron ground state. Then the elastic scattering amplitude according to (8) reduces to

$$F(\mathbf{k}',\mathbf{k})=\frac{k}{2\pi i}\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}\times\left\{\int|\varphi(\mathbf{r})|^2e^{-i[\chi_1(\mathbf{b}+\mathbf{q}/2)+\chi_2(\mathbf{b}-\mathbf{q}/2)]}d\mathbf{r}-1\right\}d^{(2)}\mathbf{b}, \quad (10)$$

where use has been made of the normalization

$$\int|\varphi(\mathbf{r})|^2d\mathbf{r}=1.$$

Now the expression $\exp\{-i[\chi_1+\chi_2]\}$ may be written in the form

$$\exp\{-i[\chi_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})+\chi_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})\}=[1-\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})][1-\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})], \quad (11)$$

so that the scattering amplitude may be resolved into the terms

$$F(\mathbf{k}',\mathbf{k})=\int|\varphi(\mathbf{r})|^2d\mathbf{r}\left(\frac{ik}{2\pi}\right)\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}+\int|\varphi(\mathbf{r})|^2d\mathbf{r}\left(\frac{ik}{2\pi}\right)\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}+\frac{k}{2\pi i}\int|\varphi(\mathbf{r})|^2d\mathbf{r}\times\int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}}\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}. \quad (12)$$

[8] We neglect the influence of nuclear spins in the present work.

This expansion separates the scattering into the amplitudes contributed by the nucleons individually plus a term representing the corrections due to the presence of two nucleons. The $\mathbf{b}$-integrations of the first two terms, after translation of their origins in the $\mathbf{b}$-plane, indeed reduce to the neutron and proton scattering amplitudes given by (1).

Since our interest centers, for the present, on the total cross section, and this may be found from the forward scattering amplitude, we specialize at once to the case $\mathbf{k}=\mathbf{k}'$, and introduce the abbrevaitions

$$F(0)=F(\mathbf{k},\mathbf{k}), \quad f(0)=f(\mathbf{k},\mathbf{k}). \quad (13)$$

Then, introducing the individual forward scattering amplitudes via (3) and (4), we have

$$F(0)=f_1(0)+f_2(0)+\frac{k}{2\pi i}\int|\varphi(\mathbf{r})|^2\times\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}d\mathbf{r}. \quad (14)$$

To find the total cross section of the deuteron, we make use of the fact that the attenuation of the incident beam may be described as a destructive interference with the coherent amplitude scattered in the forward direction. This relates the total cross section to the imaginary part of the forward amplitude in the familar way:

$$\sigma_d=(4\pi/k)\,\text{Im}F(0), \quad (15)$$

and for the individual nucleons

$$\sigma_j=(4\pi/k)\,\text{Im}f_j(0). \quad (16)$$

Then, taking the imaginary part of $F(0)$ as given by (14), we find the general expression for the deuteron cross section:

$$\sigma_d=\sigma_1+\sigma_2-2\,\text{Re}\int|\varphi(\mathbf{r})|^2\times\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}d\mathbf{r}. \quad (17)$$

The relation of the two-particle term to the "eclipse" corrections mentioned in the introduction may be seen in a general way at this point. The functions $\Gamma(\mathbf{b})$ defined by (4) go to zero for impact parameters lying outside the range of interaction. The integral over $\mathbf{b}$ in the two-particle term therefore vanishes except when the relative position of the nucleons is such that the region of interaction about one casts its "shadow" on the other.

Since the wave function of the deuteron is spherically symmetrical to a good approximation, it is convenient to express the two-particle term in a way which takes advantage of this. As a first step we calculate the term for a spherically symmetric wave function in which the neutron and proton remain a fixed distance ρ apart;

that is, we take

$$|\varphi(\mathbf{r})|^2=(4\pi\rho^2)^{-1}\delta(|\mathbf{r}|-\rho). \quad (18)$$

The **r**-integration in (17) may then be separated into an integration over z, the component of **r** in the direction of incidence, and an integration of **q** over the perpendicular plane. The z-integration is easily carried out. Writing $S(\rho)$ for the two-nucleon correction in this case, we have

$$\sigma_d=\sigma_1+\sigma_2+S(\rho), \quad \rho \text{ fixed} \quad (19)$$

where

$$S(\rho)=-(2\pi\rho^2)^{-1}\,\mathrm{Re}\int d^{(2)}\mathbf{b}\,d^{(2)}\mathbf{q}\,\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})$$

$$\times\int_{-\infty}^{\infty}\delta\{(q^2+z^2)^{\frac{1}{2}}-\rho\}dz=-\frac{1}{\pi\rho}\int_{|q|<\rho}\frac{d^{(2)}\mathbf{q}}{(\rho^2-q^2)^{\frac{1}{2}}}$$

$$\times\mathrm{Re}\int\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}. \quad (20)$$

This result for a fixed distance between the nucleons may now be employed to find the correction to the cross section for any spherically symmetric representation of the deuteron wave function. The correction in the general case, which we represent as $\delta\sigma$, is obtained by averaging $S(\rho)$ over the radial distribution function.

$$\sigma_d=\sigma_1+\sigma_2+\delta\sigma, \quad (21)$$

$$\delta\sigma=\int_0^{\infty}|\varphi(\rho)|^2S(\rho)4\pi\rho^2d\rho. \quad (22)$$

For neutron-proton separations **r** much larger than the interaction ranges, $S(r)$ reduces to

$$S(r)\sim-(\pi r^2)^{-1}\,\mathrm{Re}\int\Gamma_1(\mathbf{b}+\tfrac{1}{2}\mathbf{q})\Gamma_2(\mathbf{b}-\tfrac{1}{2}\mathbf{q})d^{(2)}\mathbf{b}\,d^{(2)}\mathbf{q},$$

or, introducing $\mathbf{q}_1=\mathbf{b}+\mathbf{q}/2$, $\mathbf{q}_2=\mathbf{b}-\mathbf{q}/2$,

$$S(r)\sim-(\pi r^2)^{-1}\,\mathrm{Re}\left\{\int\Gamma_1(\mathbf{q}_1)d^{(2)}\mathbf{q}_1\int\Gamma_2(\mathbf{q}_2)d^{(2)}\mathbf{q}_2\right\}.$$

The latter integrals are proportional to the individual forward scattering amplitudes given by (3) and (4), so that $S(r)$ reduces to

$$S(r)\sim(4\pi/k^2r^2)\,\mathrm{Re}\{f_1(0)f_2(0)\}. \quad (23)$$

This is of course an asymptotic expression, valid at large distances, but its r-dependence is roughly correct down to the radii of the interaction regions themselves. Hence, if there is a sufficiently small probability that the interaction regions overlap, we have

$$\delta\sigma=(4\pi/k^2)\,\mathrm{Re}\{f_1(0)f_2(0)\}\langle r^{-2}\rangle_d. \quad (24)$$

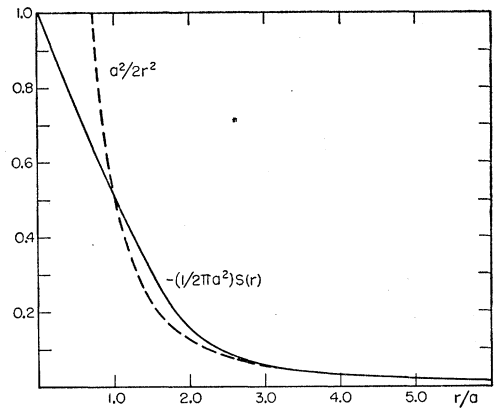

FIG. 1. The function $S(r)$, which represents the correction to the total cross section when the neutron-proton separation is held at the fixed value r, is plotted in units of $2\pi a^2$. The dashed curve represents the asymptotic form the function assumes at large distances.

A further simplification may be achieved if it is assumed that the high-energy interactions are purely absorptive. The absorption, which represents incoherent processes such as meson production is, of course, always accompanied by elastic diffraction scattering. This assumption, that the incident wave suffers no real shift of phase, but only a decrease of amplitude (i.e., $\chi_j(\mathbf{b})$ is purely imaginary), implies that the scattering amplitudes (3) are purely imaginary. We then have $f_j(0)=(ik/4\pi)\sigma_j$, and substitution of these relations in (24) yields

$$\delta\sigma=-(1/4\pi)\sigma_1\sigma_2\langle r^{-2}\rangle_d. \quad (25)$$

This is just the expression found in the introductory considerations, which treated the interaction regions as black spheres. It is evident now, however, that the result is substantially more general. The interactions need not be perfectly opaque and indeed the opacity of a nucleon may be distributed arbitrarily within ranges small compared with the size of the two-body system. It must be mentioned, though, that the absorption of the system is no longer given in general by (1), since the absorption and total cross sections are no longer simply related by a factor of two.

It is instructive to calculate the absorption and scattering cross sections separately. Their sum, of course, furnishes a check on the total cross sections discussed thus far. The correction to the absorption cross section is still given by the argument of the introduction, but employing the individual absorption cross sections σ_{1a} and σ_{2a} instead of geometrical areas.

$$\delta\sigma_{\mathrm{abs}}=-(1/2\pi)\sigma_{1a}\sigma_{2a}\langle r^{-2}\rangle_d. \quad (26)$$

The considerations required to find the scattering are rather more lengthy. It is necessary to take account of all scattering processes involving breakup of the deuteron as well as the elastic collisions. This is accomplished by integrating the scattered intensity over angles and summing the squares of all matrix elements of the operator $\Gamma_{\text{tot}}(\mathbf{b},\mathbf{q}_1\cdots\mathbf{q}_A)$. The resulting expression is a sum of terms representing scattering by the individual nucleons, interference between them, and corrections due to double scattering and scattering by one followed by absorption by the other. When these are summed and added to (26), they indeed reduce to the much simpler expressions given above for the total cross section.

III. RESULTS AND DISCUSSION

In the foregoing work we have analyzed the total cross section of the deuteron in a way which remains valid for the wide range of high-energy interactions which may be treated by the methods of diffraction theory. While it is anticipated that the actual interactions of high-energy incident particles with neutrons and protons will be found to have this character, too few observations exist as yet to permit conclusive analysis of the data. In particular, lacking measurements of the differential cross sections for elastic scattering, very little can as yet be said concerning the phase change functions $\chi_j(\mathbf{b})$, or the "shadow" functions $\Gamma_j(\mathbf{b})=1-\exp[i\chi_j(\mathbf{b})]$, which figure in the general results (20) and (22). We shall therefore confine ourselves for the present to the use of simple models for the high-energy interactions. In this way rough estimates of the effect may be reached along with comparisons of the relative magnitudes involved.

The experimental total cross sections[1,2] for nucleon-nucleon encounters at 1.4 Bev are

$$\sigma(p,p)=48\text{ mb}\quad\text{and}\quad\sigma(n,p)=42\text{ mb}.$$

Two attenuation measurements have also been performed with deuterium. With a beam of incident protons, a subtractive experiment[2] involving deuterium- and hydrogen-bearing targets yields $\sigma(p,[d-p])=33$ mb which is 9 mb smaller than $\sigma(p,n)=\sigma(n,p)$. The subtractive measurement added to $\sigma(p,p)$ indicates a proton-deuteron cross section $\sigma(p,d)=81$ mb. With neutrons incident on deuterium,[1] the total cross section of the deuteron is found to be $\sigma(n,d)=84$ mb, a value whose closeness to the corresponding result for protons strongly supports the hypothesis of charge symmetry. If it is assumed then that the free neutron-neutron cross section is the same as $\sigma(p,p)$ given above, the sum of the free-particle cross sections $\sigma(n,n)+\sigma(n,p)$ is found to exceed $\sigma(n,d)$ by 6 mb. As a further consequence of charge symmetry, these differences of 9 mb and 6 mb must be considered as two measurements of the same number. Since they are differences of experimental cross sections, their errors may of course be relatively large, but it will be of interest to see whether or not the results predicted lie in the range they indicate.

Since little more is known about nucleon interactions at 1.4 Bev than is contained in the above total cross section measurements, we shall employ them to define the radii of opaque regions of interaction. The black-sphere model to which we thus return is at least roughly consistent with measurements[9] made at somewhat lower energies of the relative proportions of elastic and inelastic processes occurring in proton-proton scattering. Since the neutron and proton cross sections are not very different, we shall simplify the calculations by assuming their regions of interaction identical. The radius $a=0.85\times10^{-13}$ cm corresponds to a mean nucleon-nucleon cross section $2\pi a^2=45.4$ mb.

Whether this interaction range may be considered small or not depends in large measure on the form assumed for the deuteron wave function. As we shall see, the experimental data such a wave function must be required to fit, leave the radial distribution function quite undetermined at small neutron-proton separations. In particular for models which concentrate the probability densities at small distances, it will be necessary to use the general formulation of the result given by (20) and (22).

The functions $\Gamma_j(\mathbf{b})$ for the black-sphere interactions assume the particularly simple form

$$\Gamma_1(\mathbf{b})=\Gamma_2(\mathbf{b})=\begin{cases}1, & b<a\\ 0, & b>a.\end{cases}\tag{27}$$

With the aid of these, the function $S(r)$, which represents the correction to the total cross section when the neutron-proton separation is held at the fixed value r, is easily found. Introducing the variable

$$x=r/2a,\tag{28}$$

we have for $x\leqslant1$:

$$S(r)=-2\pi a^2\{1+(4/3\pi)(x^{-1}-x)K(x)-(4/3\pi)(x^{-1}+x)E(x)\}\tag{29}$$

and for $x\geqslant1$:

$$S(r)=-2\pi a^2\{1+(4/3\pi)(x^2-1)K(x^{-1})-(4/3\pi)(x^2+1)E(x^{-1})\},\tag{30}$$

where the functions K and E are the complete elliptic integrals of the first and second kind, respectively. A graph of $-(1/2\pi a^2)S(r)$ is given in Fig. 1, where it is compared with the asymptotic form $a^2/2r^2$, which is valid at large distances. It may be seen that the asymptotic form deviates widely from the correct function only for radii r smaller than the range of interaction a.

The deviation of the total cross section from the sum of the free-particle cross sections is found by averaging the function $S(r)$ over the radial distribution function of the deuteron. For this purpose we shall discuss three possible models of the deuteron, selected to illustrate different ways in which the probability density may behave at small distances. The constants which any representation of the deuteron wave function must be chosen to fit are: the triplet effective range $r_e=1.7$

[9] Smith, McReynolds, and Snow, Phys. Rev. 97, 1186 (1955).

$\times 10^{-13}$ cm, and the binding energy, which may also be stated in terms of the parameter α, the logarithmic derivative of the radial wave function at large distances. $\alpha=(1/4.31)\times 10^{13}$ cm^{-1}.

If the interaction which binds the deuteron is represented as a square-well potential, its radius is found to be 2.07×10^{-13} cm. The value of $S(r)$ averaged over the radial distribution function appropriate to this potential yields $\delta\sigma=5.7$ mb, a value quite consistent with the smaller of the two measurements. The value found by employing the asymptotic result (25) exceeds this by about 20%, an error which corresponds to a probability of only 4% that the neutron-proton separation is less than 0.85×10^{-13} cm. The use of the asymptotic approximation must therefore be restricted to distributions which are less concentrated at small distances.

A model of the deuteron somewhat more in keeping with the original proposals of the meson theory would be based on a potential which becomes singular as $1/r$ near the origin and decreases exponentially at large distances. The Hulthén potential, $V(r)\sim\lambda(e^{\lambda r}-1)^{-1}$ for which the wave function is proportional to $e^{-\alpha r}\times(1-e^{-\lambda r})/r$, represents a convenient choice of this type. The parameter λ is found to be 1.21×10^{13} cm^{-1}, a value which corresponds to an increased concentration of the wave function near the origin. The mean value of $S(r)$, in this case, yields $\delta\sigma=-7.2$ mb, which lies between the measured values of the cross-section defect.

A third type of deuteron model which has recently received considerable discussion is based on the assumption of a "hard core" in the low-energy neutron-proton interaction. The particles are assumed subject to an infinite repulsive potential which prevents their approach closer than a distance r_c. An example of this model is furnished by one of the computations of Blatt and Kalos based on a core of radius $r_c=0.531\times 10^{-13}$ cm surrounded by a potential of the type proposed by Lévy, with parameters providing an approximate fit to the known constants for the triplet state.[10] The average value of $S(r)$ in this case gives $\delta\sigma=-4.5$ mb. The asymptotic approximation (25), as may be expected, is more accurate here than in the square-well case. Since the radial distribution lies almost entirely in the region $r>a$, the asymptotic estimate is on the low side, by about 10%.

The value 4.5 mb for the cross-section defect in the hard-core example considered is smaller than either of the measured values, 9 mb and 6 mb, though perhaps not small enough to exclude this model in view of the various uncertainties involved. However, since the case is one in which the asymptotic approximation is serviceable, certain general remarks may be made about the dependence of the result on the form assumed for the high-energy interactions. In particular, the validity of the asymptotic expression (25) for arbitrary interactions of a purely absorptive character indicates that the result should not be very sensitive to variations in the opacity distributions which leave the total cross sections unchanged. If the interactions are assumed to be refractive as well as absorptive, the forward scattering amplitudes are no longer purely imaginary, and we must return to the more general relation (24) to find $\delta\sigma$. This may be rewritten as

$$\delta\sigma\sim-(1/4\pi)\{\sigma_1\sigma_2-(4\pi/k)^2 \times \mathrm{Re} f_1(0)\, \mathrm{Re} f_1(0)\}\langle r^{-2}\rangle_d, \quad (31)$$

from which it is evident that similar signs for the real parts of the forward scattering amplitudes would result in a decrease of the predicted effect, and hence a strong need for a more compact deuteron model. For models in which the probability density is substantially more concentrated, however, the asymptotic approximation becomes less accurate and the influence of other assumptions concerning the interactions can only be found by recalculating the general expression (22). While the black-sphere model we have employed in the absence of scattering data may well prove inaccurate, it seems clear that an effect of the magnitude observed may be explained with other strongly absorptive interactions by suitably adjusting the radial distribution of the deuteron.

A test of consistency may be made on the radial distribution required by analysis of the similar effect which is found to occur in collisions of π mesons with deuterons. The total cross sections at 800 Mev for collisions of $\pi^{\pm}$ mesons with protons are[3]

$$\sigma(\pi^-,p)=41 \text{ mb},\quad \sigma(\pi^-,p)=18 \text{ mb}, \quad (32)$$

while subtractive measurements performed with deuterium- and hydrogen-bearing targets yield

$$\sigma(\pi^+,[d-p])=35 \text{ mb},\quad \sigma(\pi^-,[d-p])=14 \text{ mb}. \quad (33)$$

These results, which involve probable errors of about 2 mb, may be added in pairs to yield the deuteron cross sections

$$\sigma(\pi^+,d)=53 \text{ mb},\quad \sigma(\pi^-,d)=55 \text{ mb}.$$

The agreement of these results offers further support for the hypothesis of charge symmetry, so that the cross sections (32) and (33) must be interpreted as implying that the deuteron cross section is less than the sum of the free-particle cross sections by about 5 mb. In the absence of other data on meson interactions at this energy, we may merely remark that the assumption of purely absorptive interactions, together with the use of the deuteron wave functions employed above, furnishes estimates of the effect in the range from 1 to 3 mb. A larger effect may be due to nonvanishing refractive effects or concentration of the density distribution of the deuteron at smaller radii.

[10] J. M. Blatt and M. H. Kalos, Phys. Rev. **92**, 1563 (1953). The wave function employed corresponds to the parameters listed on line 2 of Table I. The core radius is the smallest reported upon. We are indebted to Mr. J. Bernstein for supplying the tables.

In concluding we may mention still another factor whose possible influence on the effect may be of interest. We have assumed from the outset that the complex phase changes brought about by the passage of a wave through the two regions of interaction are additive. When the interaction regions overlap, any nonlinearity in the superposition of their fields may imply nonadditivity of the phase changes. This too would contribute to the observed cross-section defect, but its analysis must clearly follow a more complete investigation of the linear effects. The author is greatly indebted to Dr. Anatole Shapiro for calling the measurements of the effect to his attention, and to C. Sommerfield and J. Bernstein for aid with some of the calculations.

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A Theory of New Particles

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A tentative scheme is developed to formulate the behavior of the new unstable particles. This scheme is a straightforward generalization of the usual charge-independent meson theory. The selection rules for isotopic spin are identical with those suggested by Gell-Mann. Owing to the particular form of the interaction assumed in this scheme, we can derive a new selection rule which seems to be of some use in interpreting the metastability of the new particles.

1. INTRODUCTION

Several attempts[1] to interpret the contradictions between the copious productions of the new particles and their metastabilities have been published. Among these attempts the "two-coupling-constant theory," due to Pais, seems most successful. In this theory the production processes are due to an interaction with a large coupling-constant while the other small coupling-constant is responsible for decay processes. Pais' recent theory[1] based on the four-dimensional ω space seems especially attractive. However there may remain, of course, other kinds of formalisms within the framework of the "two-coupling-constant theory."

In the present paper, the details of a theory of the baryons and mesons will be presented. In this theory particles will be distinguished by (a) isotopic spin I, (b) curious particle constant A, and (c) intrinsic spatial parity ϵ in addition to the usual mass, spin, and charge. The theory is constructed so that the selection rules involving I, I_z, and A are identical with those proposed by Gell-Mann.[2] In addition we will show that it is natural to introduce an additional selection rule involving the intrinsic parity ϵ. This last aspect was briefly discussed in a paper by Tobocman and the author.[2]

In the usual formalism, the neutron and the proton are described by a spinor $\psi^\alpha(\alpha=1,2)$ in a three-dimensional τ space. Similarly the charged and neutral mesons are described by a vector $\boldsymbol{\phi}$ (or a symmetric spinor of the second rank $\chi^{\alpha\beta}=\chi^{\beta\alpha}$ $(\alpha,\beta=1,2)$) in τ space. The interaction Lagrangian between the two fields in the usual charge-independent theory is

$$g\boldsymbol{\phi}\bar{\psi}\gamma_5\boldsymbol{\tau}\psi=\sum_{\alpha,\beta=1,2} g\chi^{\alpha\beta}\bar{\psi}\gamma_5\tau_{\alpha\beta}\psi. \tag{1.1}$$

Here $\tau_{\alpha\beta}$ and $\chi^{\alpha\beta}$ are defined as follows:

$$\tau_{11}=\tau_{22}^{\dagger}=i(\tau_1+i\tau_2),\quad \tau_{12}=\tau_{21}=-i\tau_3,$$

and

$$\chi^{11}=\chi^{22*}=-(i/2)(\phi_1-i\phi_2),\quad \chi^{12}=\chi^{21}=(i/2)\phi_3.$$

Now we consider a straightforward generalization of (1.1), namely

$$g\sum_n \sum_{\alpha_1\alpha_2\cdots\alpha_n=1,2} \chi^{(\alpha_1\cdots\alpha_n)}\bar{\psi}\gamma_5\cdot T_{(\alpha_1\cdots\alpha_n)}\psi. \tag{1.2}$$

Here $\chi^{(\alpha_1\cdots\alpha_n)}$ is a symmetric spinor of the nth rank in τ space and is assumed to describe a meson field with the ordinary spin 0 and τ spin $n/2$, and ψ is a wave function corresponding to some assembly of baryons with ordinary spin $\frac{1}{2}$ and various values of τ spin. $T_{(\alpha_1\cdots\alpha_n)}$ is some square matrix which is considered to be a generalization of the usual τ matrix.

In this treatment, we shall include only three kinds of meson fields: the ordinary π-meson field ($\chi_{(\pi)}{}^{\alpha\beta}$ or $\boldsymbol{\phi}$),[3] the θ-meson field with τ spin $\frac{1}{2}(\chi_{(\theta)}{}^{\alpha})$, and the τ-meson field with τ spin $\frac{1}{2}$ $(\chi_{(\tau)}{}^{\alpha})$. As to the baryons we shall include the nucleon field $(\psi_{(N)}{}^{\alpha})$, the Λ particle with τ spin 0 $(\psi_{(\Lambda)})$, the Σ particles with τ spin 1 $(\psi_{(\Sigma)}{}^{\alpha\beta})$, and the cascade particle Y with τ spin $\frac{1}{2}$ $(\psi_{(Y)}{}^{\alpha})$. Con-

[1] A. Pais, Physica **19**, 869 (1953); A. Pais, Proc. Nat. Acad. Sci. **40**, 484 (1954); M. Gell-Mann and A. Pais, Proceedings of the International Physics Conference, Glasgow, July, 1954 (Pergamon Press, London, 1955); T. Nakano and R. Utiyama, Progr. Theoret. Phys. (Japan) **11**, 411 (1954); T. Nakano and K. Nishijima, Progr. Theoret. Phys. (Japan) **10**, 581 (1953).

[2] R. Utiyama and W. Tobocman, Phys. Rev. **98**, 780 (1955). In the present paper this will be cited as U.T.

[3] Indices α, β, $\cdots$ are spinor indices in τ space. The ordinary spinor indices of baryons are omitted.