curve agrees well with the previous calculations—since the interaction is strongly repulsive, the curve obtained does not depend sensitively on the details of the kernel. Proposals for the explanation of the disagreement with experiment in terms of a meson-meson attraction have been made by Mitra and Dyson and by Ross.[15] We note that a coupling constant of 2.0 is required to give rough agreement with experiment. A typical wave function $f_3(p)$ is shown in Fig. 5. Its rapid fluctuations for low momenta are characteristic for pion energies $\epsilon > M+2\mu$ and represent the reactive effect of the competing meson production on the elastic scattering. In the "no-pair" theory, the S-wave interaction is due to recoil effects and the phase shift is small, with momentum dependence $\sim k^5$.

For both kernels (2.2) and (2.6), the P-wave phase shifts δ_{13} and δ_{31} are each quite small, roughly equal and differ little from Born approximation, as shown in Figs. 6 and 7. (It is interesting to compare this result with the requirement of the pseudovector theory that $\delta_{13}=\delta_{31}$ exactly.) Experimentally, these phase shifts are known only to be small and of uncertain size. The various D-wave phase shifts are also shown in Fig. 6 and are very little different from the Born approximation values, which had been calculated earlier by Nelkin.[16]

ACKNOWLEDGMENTS

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[15] A. Mitra and F. J. Dyson, Phys. Rev. **90**, 372 (1953); M. Ross, Phys. Rev. **95**, 1687 (1954); R. E. Marshak Phys. Rev. **88**, 1208 (1952). The evidence for a curvature in the plot of δ_3 *vs* k (which is what the meson-meson force must explain) is no longer very firm.

[16] Quoted by Bethe and de Hoffmann, reference 2, Sec. 34.

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Isobar Role in Two-Nucleon Processes: Deuteron Photoeffect*

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An elementary model of the $\frac{3}{2}$, $\frac{3}{2}$ nucleon isobar is applied literally as an intermediate state in two-nucleon processes. The mechanism of deuteron photodisintegration which is thus implied is found to contribute most of the observed cross section at high energy. The isobar does not seem to be as important in nucleon-nucleon scattering, or in reducing the photomeson production (and similar processes) below the estimates of impulse approximation.

An estimate of single-nucleon Compton effect also is given, in the context of the isobar model.

I. INTRODUCTION

THE $J=\frac{3}{2}$, $T=\frac{3}{2}$ nucleon isobar idea has been applied extensively, and with considerable success, in studies of high-energy reactions involving one nucleon.[1] So far, however, these have been almost the full extent of its application, probably because the degree of rigor which makes these applications interesting is only possible for two-body collisions, as well as because it frequently is possible in the one-nucleon situation to use the isobar only as a guide in calculations of more general validity.

For multinucleon systems, perhaps the most plausible use so far made of the isobar has been the demonstrations[1,2] that in the reaction $p+p\rightarrow\pi^++d$ it gives a dominating role to the $^1D_2$ state. The present paper will extend this type of discussion, making very free use of the isobar for all high-energy reactions involving two nucleons.[3] The isobar idea will be used quite crudely. Nevertheless, if the absence of rigor be tolerated, it will be seen that several interesting results follow, notably that the more striking features of the high-energy deuteron photodisintegration are reproduced by the calculation.

The approach used in this paper takes quite literally all of the more elementary ideas about the isobar. We employ these to make a model of the isobar state of a

* This work was carried through in most part early in 1952, when the author held a U. S. Atomic Energy Commission Postdoctoral Fellowship at the Cornell University Laboratory of Nuclear Studies. He is very grateful for the fellowship, and for the warm hospitality of Cornell University.

[1] M. Gell-Mann and K. M. Watson, Ann. Rev. Nuclear Sci. **4**, 219 (1954). These authors summarize much of the work that has been done with isobar models for single-nucleon processes, and in π^+ production in proton-proton collisions. They also summarize the data, and give an extensive collection of references.

[2] S. Matsuyama and H. Miyazawa, Progr. Theoret. Phys. (Japan) **9**, 492 (1953).

[3] The use of isobar ideas in this connection was first suggested to the author by Professor H. A. Bethe. An earlier publication, N. Austern, Phys. Rev. **87**, 208(A) (1952), correctly described the model. However, the formulation was complicated, and subsequent work seemed to show that the model gave wholly insignificant contributions in deuteron photodisintegration. The present reformulation is very much simpler and more straightforward, and so more reliable.

nucleon: assuming it to have a discrete energy, to be localized in space (i.e., normalizable), to have even parity, and to have spin $\frac{3}{2}$, and isotopic spin $\frac{3}{2}$. We assume further that all processes involving two nucleons proceed in two steps, via an intermediate state in which only one nucleon is raised to the isobar, the other one not being excited, somewhat of a spectator. Also the model is simplified, for the present, by assuming the two nucleons in the intermediate state to move with zero relative angular momentum and zero energy of relative motion.

Our intermediate state is seen to have three possible channels for decay, as illustrated in Fig. 1. Not only can the one nucleon which is excited decay simply by pion or γ-ray emission, as it can when in isolation, but now it also can make a collision of "second kind" with its neighbor (as molecules do in a gas discharge), the energy of isobar excitation going over to kinetic energy of relative motion of two nucleons in their ground states. For such a collision decay our simplifying assumptions about the intermediate state lead by inspection to the well-known result[1,2] that the state of relative motion of the two normal nucleons must be ${}^1D_2$, this being the only state with even parity, isotopic spin unity, and J equalling 1 or 2.

The full nature of our model now is evident. It provides a sort of "extended" theory of detailed balance, based upon the dynamical assumption of a single, dominating intermediate state. Three matrix elements, which we label Π, Ω, T, as shown, inter-relate six distinct cross sections. These matrix elements are functions only of the total energy of the system, E. Various physical assumptions could be made about their energy variations, along the lines of the resonance theory studies of Gell-Mann and Watson. Fortunately, such detailed considerations about Π, Ω, T are not needed if we seek only the interrelation of cross sections. It is sufficient to recognize that the matrix elements are independent of the J_z and T_ζ quantum numbers, that their energy variations are given automatically from the three cross sections which are chosen to measure them, and that their phases do not enter in the calculations of this paper. Actually the only work involved in this theory consists of adding up correctly the combination coefficients which tell how the matrix elements enter into the different cross sections, and of keeping correct normalizations.

From a somewhat more fundamental standpoint the above model is rather difficult to justify. However it does represent certain terms which arise in field theory, it does contain a simple version of dispersion theory, and it does indicate to some degree whether the two-nucleon reactions can be understood in terms of those of one nucleon. The reliability of our model may be defended by noting that to a considerable degree the other terms of field theory do not interfere with those the model includes, and so these other terms either are negligible when cross sections are found, or may be

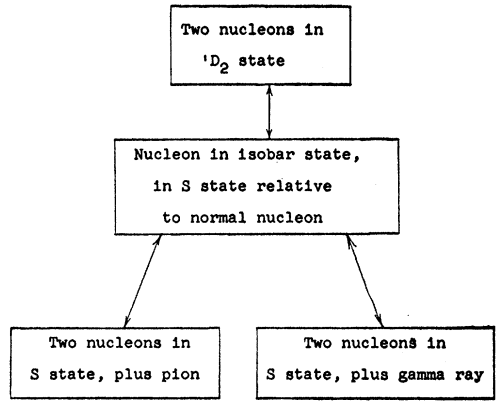

FIG. 1. The scheme of the isobar model.

discussed in terms of simple subtraction of cross sections. The "discrete" state assumption for the isobar is of little consequence, despite the actual short lifetime, and is related to one of the older derivations of nuclear dispersion theory. Here it should only be understood as a device to help keep correct normalizations.

The idea of dispersion theory provides an alternative context in terms of which to understand the results below, although the subsequent discussion is all in terms of perturbation theory. However, in dispersion theory Π, Ω, and T become reaction widths, to be evaluated from experiment. Our "spectator" assumption gives Π and Ω in terms of reactions involving one nucleon, so that the model is in this sense less adjustable than a dispersion theory. This assumption is a consequence of the idea that a meson can be in *very* strong interaction with only one nucleon at a time. Perhaps this, like the other assumptions, is best justified by seeing what are the results of calculation.

In the sections which follow we treat in turn each of the relevant cross sections, first giving quantitative definitions of Π, Ω, T, and evaluating them, and then using the results thus found. Section II treats π^+-proton scattering, so as to find Π. Section III treats (γ,π^0) production from a proton, so as to find Ω. Section IV treats π^+ disintegration of the deuteron, so as to find T. Then in Sec. V we compute deuteron photoeffect, and in Sec. VI proton-proton scattering in the ${}^1D_2$ state.

One interesting, though subsidiary, result comes from the fact that the existence of the collision channel permits the isobar to decay somewhat faster than for a nucleon in isolation. As a result the state is broader in the two-nucleon application, the energy denominator having a larger imaginary term, and the photoproduction, γ-ray scattering, and meson scattering cross sections are several percent smaller than in the predictions given by impulse approximation. Other than this one effect, the model gives no new results for those processes. This effect is discussed briefly in Sec. VII. Also in Sec. VII a discussion is given of Compton scattering from a *single* nucleon, as this process does not seem to have been treated elsewhere from the standpoint of the elementary form of the isobar model.

With regard to deuteron photoeffect the present model is somewhat of a formalization of ideas first given by Wilson.[4] Indeed, it was designed for that purpose. However it certainly does not give a complete theory of deuteron photoeffect, the isobar model only including magnetic dipole γ-ray interactions. These dominate in (γ,π^0) production, which we use to measure Ω, but account for only about half the (γ,π^+) production. Electric dipole radiative interaction, leading to the meson S wave, and also to both isotopic spins $\frac{1}{2}$ and $\frac{3}{2}$, is important in π^+ production. This interaction is not included in the present model. Accordingly, it is satisfying that the formula we will obtain for deuteron photodisintegration,

$$\sigma(\gamma+d\rightarrow n+p)=\frac{9}{4}\frac{\sigma(\gamma+p\rightarrow\pi^0+p)\,\sigma(\pi^++d\rightarrow p+p)}{\sigma(\pi^++p\rightarrow\pi^++p)},$$

with the angular distribution

$$\sigma(\gamma+d\rightarrow n+p)\sim 1+\tfrac{3}{2}\sin^2\theta,$$

gives values falling somewhat below those of experiment.

II. PION-PROTON SCATTERING

At this point we treat π^+ scattering from an isolated proton, so as to evaluate the matrix element Π.

The normalized isobar wave function will be labeled $Z^{\mu,\nu}$, where μ is the J_z quantum number, running through the set $\{\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\}$, and ν is the corresponding T_ζ quantum number. A nucleon in its ground state will be labeled $N^{\sigma,p}$ or $N^{\sigma,n}$, according as it has spin projection σ, and is a proton or a neutron.

The P part of a meson plane wave with momentum q has the form

$$\sum_{m=-1}^{1} Y_{1,m}^*(\mathbf{q}\cdot\mathbf{z})Y_{1,m}(\mathbf{r}), \qquad (1)$$

where $\mathbf{z}$ is the axis of directional quantization, and it will be understood that all boldface symbols which appear in this paper are unit vectors. We find the ${}^2P_{\frac{3}{2}}$ part of the product of (1) with $N^{\sigma,p}$, getting[5]

$$\sum_{m=-1}^{1} Y_{1,m}^*(\mathbf{q}\cdot\mathbf{z})\langle 1,\tfrac{1}{2};m,\sigma|\tfrac{3}{2},m+\sigma\rangle|\tfrac{3}{2},m+\sigma\rangle. \qquad (2)$$

Here the incident state has uniquely $T=\frac{3}{2}$, $T_\zeta=\frac{3}{2}$, so the isotopic functions are not indicated in (2).

Now we define Π by the assertion that the ${}^2P_{\frac{3}{2}}$ (and $T=\frac{3}{2}$) part of a meson plane wave of unit amplitude couples to the appropriate $Z^{\mu,\nu}$ with amplitude Π. Thus the amplitude with which the isobar is formed in a π^+-p collision is[6]

$$\Pi\sum_{m=-1}^{1} Y_{1,m}^*(\mathbf{q}\cdot\mathbf{z})\langle 1,\tfrac{1}{2};m,\sigma|\tfrac{3}{2},m+\sigma\rangle Z^{m+\sigma,\frac{3}{2}}. \qquad (3)$$

If $\mathbf{z}$ in (3) be identified with $\mathbf{q}$, this reduces to

$$\Pi Y_{1,0}^*(1)\langle 1,\tfrac{1}{2};0,\sigma|\tfrac{3}{2},\sigma\rangle Z^{\sigma,\frac{3}{2}}, \qquad (4)$$

$$=[\Pi/(2\pi)^{\frac{1}{2}}]Z^{\sigma,\frac{3}{2}}. \qquad (5)$$

The transition matrix of second-order perturbation theory is obtained by combining (5) with (3), using (5) as the matrix element for formation of the isobar, and (3) for its decay. For this application, $\mathbf{q}$ in (3) is replaced by $\mathbf{q}'$, the outgoing momentum, and σ by σ', the outgoing spin. However $\mathbf{z}$ remains $\mathbf{q}$, as in (5). Then the transition matrix becomes

$$\mathfrak{M}(\pi+p)=(2\pi)^{-\frac{1}{2}}D^{-1}\Pi^2Y_{1,\sigma-\sigma'}^*(\mathbf{q}\cdot\mathbf{q}')\times\langle 1,\tfrac{1}{2};\sigma-\sigma',\sigma'|\tfrac{3}{2},\sigma\rangle, \qquad (6)$$

where

$$D\equiv E-E_0+i\Gamma/2. \qquad (7)$$

The quantities in (7) are,[1] of course, the isobar energy, $E_0=159$ Mev, and the isobar width,

$$\Gamma=\left[\frac{2(qa/\hbar)^3}{1+(qa/\hbar)^2}\right]\times 58\text{ Mev}, \qquad (8)$$

with $a=0.88(\hbar/\mu c)$. The use of a width in (7) goes beyond the immediate results of second-order perturbation theory, except if one imagine the isobar to have a complex energy, by virtue of its decay. This we do.

From (6) the meson scattering cross section is

$$d\sigma(\pi^++p\rightarrow\pi^++p)$$
$$=(2\pi/\hbar v(\pi))\textstyle\sum_{\sigma'}|\mathfrak{M}(\pi^++p)|^2\rho_E(\pi)d\Omega/4\pi, \qquad (9)$$
$$=(8\pi\hbar)^{-1}[\rho_E(\pi)/v(\pi)]\Pi^4|D|^{-2}(1+3\cos^2\theta)d\Omega/4\pi, \qquad (10)$$

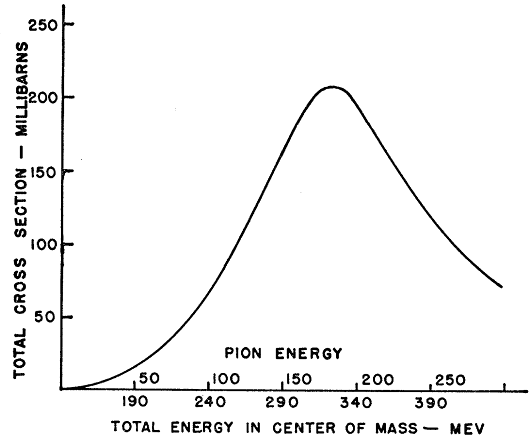

FIG. 2. Total cross section for π^+ scattering by protons. Taken from Fig. 5 of reference 1.

[4] R. R. Wilson, Phys. Rev. 86, 125 (1952).

[5] The notation used for the Clebsch-Gordan coefficients is

$$\langle j_1,j_2;m_1,m_2|J,m_1+m_2\rangle.$$

[6] Tables of relevant coefficients and functions may be found in the Appendix.

where θ is the scattering angle. Finally,

$$\Pi^4(E)=4\pi\hbar[v(\pi)/\rho_E(\pi)]|D|^2\sigma(\pi^+ + p\rightarrow\pi^+ + p). \quad (11)$$

A graph of $\sigma(\pi^+ + p\rightarrow\pi^+ + p)$ is provided as Fig. 2.

III. PHOTO π^0 PRODUCTION

Other studies[1] have made it clear that π^0 photoproduction goes overwhelmingly by way of the isobar. This makes it very proper to use the π^0 cross section as a measure of Ω. However, the simple isobar theory then gives a very bad fit for charged pion photoproduction. This point will be of importance later.

We define Ω by asserting that the isobar state $Z^{\sigma,\frac{1}{2}}$ is formed from $N^{\sigma,p}$ with amplitude

$$\Omega Z^{\sigma,\frac{1}{2}}, \quad (12)$$

if the γ ray is incident with a normalization of one photon per cm³. Here the γ-ray absorption is presumed to be of magnetic dipole type, as it largely is, so the relevant vector is the polarization vector of the γ ray, $\mathbf{u}$. This vector is used as the axis of quantization, with the result that σ is the total J_z quantum number.

For the isobar decay to a plane wave state with momentum $\mathbf{q}'$, and $N^{\sigma',p}$ the amplitude (3) applies again, multiplied with the one additional factor $\langle 1, \frac{1}{2}; 0, \frac{1}{2}|\frac{3}{2}, \frac{1}{2}\rangle=\sqrt{\frac{2}{3}}$, which picks out the $T=\frac{3}{2}$ part of (π^0+proton). Then

$$\mathfrak{M}(\gamma,\pi^0) = D^{-1}\Omega\Pi\sqrt{\tfrac{2}{3}}\, Y_{1,\sigma-\sigma'}{}^*(\mathbf{u}\cdot\mathbf{q}')\langle 1, \tfrac{1}{2}; \sigma-\sigma', \sigma'|\tfrac{3}{2}, \sigma\rangle; \quad (13)$$

$$d\sigma(\gamma,\pi^0) = \frac{2\pi}{\hbar c}\frac{1}{2}\sum_{\mathbf{u},\sigma'}|\mathfrak{M}(\gamma,\pi^0)|^2\rho_E(\pi)\frac{d\Omega}{4\pi}, \quad (14)$$

$$= (12\hbar c)^{-1}\rho_E(\pi)\Omega^2\Pi^2|D|^{-2}\sum_{\mathbf{u}}\{1+3(\mathbf{u}\cdot\mathbf{q}')^2\}\frac{d\Omega}{4\pi}, \quad (15)$$

$$= (6\hbar c)^{-1}\rho_E(\pi)\Omega^2\Pi^2|D|^{-2}(1+\tfrac{3}{2}\sin^2\theta)d\Omega/4\pi, \quad (16)$$

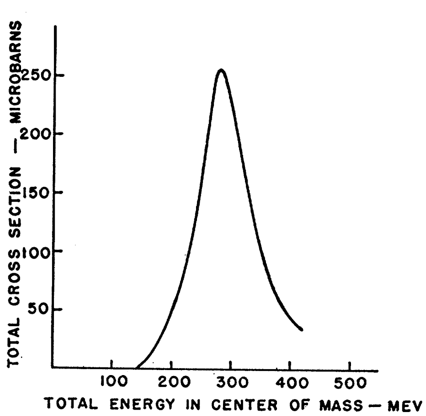

FIG. 3. Total cross section for photoproduction of π^0 mesons from the proton. This graph is the transcription to center-of-mass coordinates of Fig. 7 of reference 1. It should be noted that to obtain their Fig. 7 Gell-Mann and Watson used a theoretical angular distribution.

where θ is the angle the outcoming π^0 makes with respect to the γ-ray beam. Finally,

$$\Omega^2(E)\Pi^2(E)=[3\hbar c/\rho_E(\pi)]\sigma(\gamma,\pi^0)|D|^2. \quad (17)$$

Figure 3 is a graph of the values which will be used for $\sigma(\gamma,\pi^0)$.

The operation of averaging on polarizations was shown explicitly, in going from (15) to (16), in order to make it plain that the photoproduction gives the same angular distribution (in this model) as does the scattering, provided it be observed with respect to the polarization vector. Only the averaging over polarizations, with the change to the propagation vector as the reference direction, gives the $(1+\frac{3}{2}\sin^2\theta)$ expression. This same relationship will occur again in the two-nucleon situation.

IV. MESONIC DISINTEGRATION OF THE DEUTERON

The matrix element T will now be given a standard definition, and its value found. The calculation is a little more difficult than in II and III.

It is not necessary to consider the deuteron space wave function, and we choose to ignore the deuteron D state. Spin states $J_z=\pm 1$ will be treated separately from $J_z=0$.

For $J_z=1$ the spin and isotopic spin wave function is

$$2^{-\frac{1}{2}}(N_1{}^{\frac{1}{2},p}N_2{}^{\frac{1}{2},n}-N_1{}^{\frac{1}{2},n}N_2{}^{\frac{1}{2},p}). \quad (18)$$

According to the definition given for Π in Sec. II, the absorption of a π^+ wave of unit amplitude by nucleon 1 gives the isobar amplitude[7]:

$$\Pi(2\pi)^{-\frac{1}{2}}2^{-\frac{1}{2}}\{Z_1{}^{\frac{1}{2},\frac{3}{2}}N_2{}^{\frac{1}{2},n} - \langle 1, \tfrac{1}{2}; 1, -\tfrac{1}{2}|\tfrac{3}{2}, \tfrac{1}{2}\rangle_\tau Z_1{}^{\frac{1}{2},\frac{1}{2}}N_2{}^{\frac{1}{2},p}\}. \quad (19)$$

From (19) the $T=1$, $J=2$ part is selected, the only part which can decay into two nucleons, giving

$$\Pi(4\pi)^{-\frac{1}{2}}\langle\tfrac{3}{2}, \tfrac{1}{2}; \tfrac{1}{2}, \tfrac{1}{2}|2, 1\rangle_\sigma\{\langle\tfrac{3}{2}, \tfrac{1}{2}; \tfrac{3}{2}, -\tfrac{1}{2}|1, 1\rangle_\tau - \langle 1, \tfrac{1}{2}; 1, -\tfrac{1}{2}|\tfrac{3}{2}, \tfrac{1}{2}\rangle_\tau\langle\tfrac{3}{2}, \tfrac{1}{2}; \tfrac{1}{2}, \tfrac{1}{2}|1, 1\rangle_\tau\}|2, 1; 1, 1\rangle = \Pi(4\pi)^{-\frac{1}{2}}|2, 1; 1, 1\rangle, \quad (20)$$

where the ket, $|2, 1; 1, 1\rangle$ is defined as being normalized to unity.

The D part of a two-nucleon plane wave of momentum $\mathbf{k}$ has the form

$$\sum_{m=-2}^{2} Y_{2,m}{}^*(\mathbf{k}\cdot\mathbf{z})Y_{2,m}(\mathbf{r}), \quad (21)$$

with $\mathbf{z}$ once again the reference direction. We now define T by the assertion that a nucleon plane wave of unit amplitude forms the appropriate $|2, 1; J_z, T_\zeta\rangle$ with amplitude T. Thus the intermediate state amplitude when two protons collide is

$$T\sum_{m=-2}^{2} Y_{2,m}{}^*(\mathbf{k}\cdot\mathbf{z})|2, 1; m, 1\rangle. \quad (22)$$

[7] Here we begin the use of subscripts on the Clebsch-Gordan coefficients to indicate whether they originate in spin space or in τ space.

As before, we write down the transition matrix in second order, now combining (22) with (20), obtaining

$$\mathfrak{M}(\pi^+ + \chi_1^1 \rightarrow 2p) = (D')^{-1}(\Pi(\pi)^{-\frac{1}{2}})(TY_{2,1}^*(\mathbf{k}\cdot\mathbf{q})). \quad (23)$$

To form (23) the matrix element (20) is multiplied by two, to account for the participation of both nucleons, and $\mathbf{z}$ in (22) is taken to be $\mathbf{q}$, the momentum of the incident meson. The energy denominator now carries a prime, where

$$D' = E - E_0 + i\Gamma'/2. \quad (24)$$

The width Γ' is greater than Γ, collision decay having become possible. Also, the χ_1^1 symbol in $\mathfrak{M}$ in (23) means that this is only the transition matrix from the $J_z=1$ deuteron state.

The $J_z=1$ part of the cross section is

$$\begin{aligned} d\sigma(\pi^+ + \chi_1^1 \rightarrow 2p) &= [2\pi/\hbar v(\pi)]\rho_E(2n)|\mathfrak{M}(\pi^+ + \chi_1^1 \rightarrow 2p)|^2 d\Omega/4\pi, \\ &= (2/\hbar)[\rho_E(2n)/v(\pi)]|D'|^{-2}\Pi^2 T^2 |Y_{2,1}(\mathbf{k}\cdot\mathbf{q})|^2 d\Omega/4\pi, \\ &= (15/4\pi\hbar)[\rho_E(2n)/v(\pi)]|D'|^{-2}\Pi^2 T^2 \\ &\quad \times \sin^2\theta \cos^2\theta d\Omega/4\pi. \quad (25) \end{aligned}$$

It remains to find $d\sigma(\pi^+ + \chi_1^0 \rightarrow 2P)$, for the $\chi_1^{\pm 1}$ contributions are equal. The χ_1^0 wave function is

$$\tfrac{1}{2}(N_1^{\frac{1}{2},p}N_2^{-\frac{1}{2},n} + N_1^{-\frac{1}{2},p}N_2^{\frac{1}{2},n} - N_1^{\frac{1}{2},n}N_2^{-\frac{1}{2},p} - N_1^{-\frac{1}{2},n}N_2^{\frac{1}{2},p}). \quad (26)$$

We follow the earlier procedure identically. First Π is introduced, so as to find the isobar amplitude formed when nucleon 1 absorbs a positive pion. From this the $T=1$, $J=2$ part is found. The result is

$$\Pi(3\pi)^{-\frac{1}{2}}|2, 1; 0, 1\rangle, \quad (27)$$

in analogy with (20). The χ_1^0 part of the cross section is

$$d\sigma(\pi^+ + \chi_1^0 \rightarrow 2p) = (5/6\pi\hbar)[\rho_E(2n)/v(\pi)]|D'|^{-2} \times \Pi^2 T^2 (3\cos^2\theta - 1)^2 d\Omega/4\pi. \quad (28)$$

At last, we can evaluate the average mesonic disintegration cross section, one third the sum of (28) with twice (25).

$$d\sigma(\pi^+ + d \rightarrow 2p) = (5/18\pi\hbar)[\rho_E(2n)/v(\pi)]|D'|^{-2} \times \Pi^2 T^2 (3\cos^2\theta + 1) d\Omega/4\pi, \quad (29)$$

and

$$\sigma(\pi^+ + d \rightarrow 2p) = (5/9\pi\hbar)[\rho_E(2n)/v(\pi)]|D'|^{-2}\Pi^2 T^2. \quad (30)$$

Equation (30) permits the evaluation of T^2 in terms of $\sigma(\pi^+ + d \rightarrow 2p)$, or $\sigma(2p \rightarrow \pi^+ + d)$ which of course will do equally well. The latter cross section has recently been evaluated for protons of energies up to 660 Mev in an experiment by Meshcheryakov and Neganov.[8] The cross section $\sigma(\pi^+ + d \rightarrow 2p)$, evaluated from their data by the use of detailed balance,[1] is given as Fig. 4.

[8] M. G. Meshcheryakov and B. S. Neganov, Doklady Akad. Nauk SSSR, **100**, No. 4, 677–9 (1955) [translated by S. Shewchuck, Information Division, University of California Radiation Laboratory, Livermore, April 20, 1955].

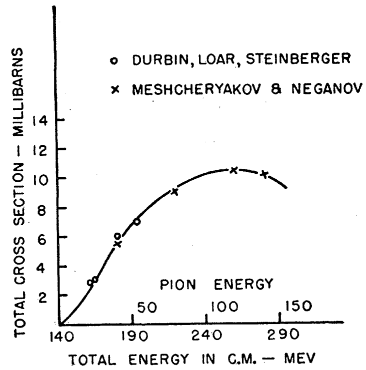

FIG. 4. Total cross section for disintegration of the deuteron by charged mesons. The crosses give the data of reference 8, the circles that of Durbin, Loar, and Steinberger, Phys. Rev. 84, 581 (1951).

V. DEUTERON PHOTODISINTEGRATION

The calculation proceeds in very close analogy to that of Sec. IV. The polarization vector, $\mathbf{u}$, is the axis of quantization, and we will average over polarizations after the cross section has been found for one polarization.

First consider the $J_z=1$ deuteron state, whose wave function is given by (18). According to Sec. III, the absorption of a γ ray of standard amplitude gives the isobar amplitude

$$\Omega 2^{-\frac{1}{2}}\{Z_1^{\frac{1}{2},\frac{1}{2}}N_2^{\frac{1}{2},n} - Z_1^{\frac{1}{2},-\frac{1}{2}}N_2^{\frac{1}{2},p}\}, \quad (31)$$

of which the $T=1$, $J=2$ part is

$$(3/4)^{\frac{1}{2}}\Omega|2, 1; 1, 0\rangle. \quad (32)$$

Then we proceed to find the $J_z=1$ part of the cross section, then in like manner the $J_z=0$ part, then combine the two to find the average photo cross section

$$d\sigma(\gamma_{\mathbf{u}} + d \rightarrow n + p) = (5/6\hbar c)\rho_E(2n)|D'|^{-2} \times \Omega^2 T^2 (3\cos^2\varphi + 1) d\Omega/4\pi. \quad (33)$$

Here $\gamma_{\mathbf{u}}$ means that the indicated cross section only refers to one polarization direction. Also φ is the angle the outgoing particles make with respect to $\mathbf{u}$. Averaging on $\mathbf{u}$ gives the final result,

$$d\sigma(\gamma + d \rightarrow n + p) = (5/6\hbar c)\rho_E(2n)|D'|^{-2}\Omega^2 T^2 \times (1 + \tfrac{3}{2}\sin^2\theta) d\Omega/4\pi, \quad (34)$$

and

$$\sigma(\gamma + d \rightarrow n + p) = (5/3\hbar c)\rho_E(2n)|D'|^{-2}\Omega^2 T^2. \quad (35)$$

In the expression (35) we may substitute for $\Omega^2 T^2$, using (11), (17), and (30), to obtain

$$\sigma(\gamma + d \rightarrow n + p) = \frac{9}{4}\frac{\sigma(\gamma + p \rightarrow \pi^0 + p)\,\sigma(\pi^+ + d \rightarrow 2p)}{\sigma(\pi^+ + p \rightarrow \pi^+ + p)}, \quad (36)$$

which was quoted in the introduction. Equation (36) is very convenient for calculation. Figure 5 shows the total cross section given by (36), along with the experimental points. Figure 6 shows the same comparison for several of the angular distributions.

At energies well below the meson threshold the photodisintegration process is dominated by the direct radiative interaction with the proton as a rigid charged particle.[9] This process gives a cross section which falls off rapidly with energy, a part of which is shown towards the lower energies in Fig. 5. This "low" energy cross section should be imagined[10] continued to higher energies, as *suggested* by the M-G line of Fig. 5, providing a background onto which to add the cross section of Eq. (36). (The two processes do not interfere, for the respective final states are orthogonal in spin.) Thus it is seen that the agreement between theory and experiment, at least for the total cross section, is very good, and most encouraging with regard to the model treated in this paper.

For the total cross section the outstanding discrepancy between experiment and theory occurs between about 100 and 300 Mev. In a sense this is hardly any discrepancy at all. As was emphasized earlier, the use of the photo-π^0 cross section to measure the radiative interaction strength is correct for the isobar model, but gives a bad fit to π^+ photoproduction. Especially over the first 200 Mev above threshold the π^+ production receives a large contribution from electric dipole γ-ray interaction, involving meson emission in $T=\frac{1}{2}$ states, and giving the very familiar meson S-wave term. Our model omits this entire process. Thus the gap between theory and experiment which we see in Fig. 5 should be regarded, for the present, as merely a convenient opening into which to fit the neglected processes.

The angular distributions, in Fig. 6, are more strongly affected by the shortcomings of the model, for interferences make small quantities important in the angular distribution. Nevertheless, it is seen that $(1+\frac{3}{2}\sin^2\theta)$ provides a good first approximation over much of the energy range. A first idea of what we have left out is the phase difference between the two nucleons. This would lead, if included, to a fore-aft asymmetry of the differential cross section, perhaps similar to that which is observed.

Another interpretation of the phase difference, in terms of the assumptions of our model, is that the nucleons in the intermediate state need not move, as we have assumed, with zero relative orbital angular momentum. Indeed a semiclassical estimate of the most likely maximum angular momentum gives it as about (γ-ray momentum)$\times\frac{1}{2}(\hbar/\mu c)$, thus $L\lesssim 1$ or 2. This failure of one of our assumptions need not much affect the total cross section for photodisintegration, for this has been measured in terms of the mesonic disintegration, which has similar angular momenta in the intermediate state. The S-state assumption would have to be abandoned if one were to consider phase differences between the two nucleons. However, the calculation then would become very much more complicated.

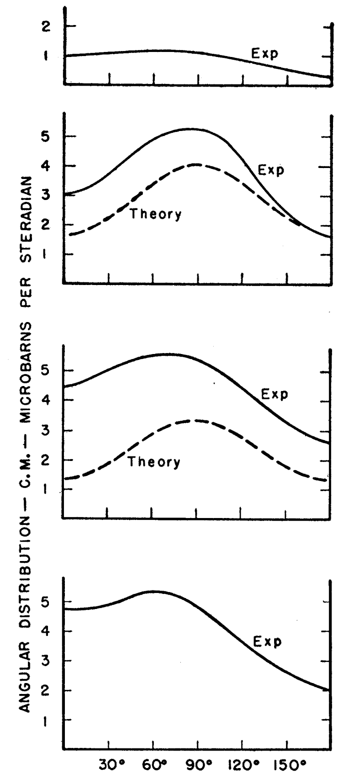

FIG. 6. Angular distributions for deuteron photodisintegration. The "experiment" curves are taken from a smoothed graph prepared by A. O. Hanson (private communication). Reading from the top down the gamma-ray energies are 455 Mev, 305 Mev, 220 Mev, and 140 Mev.

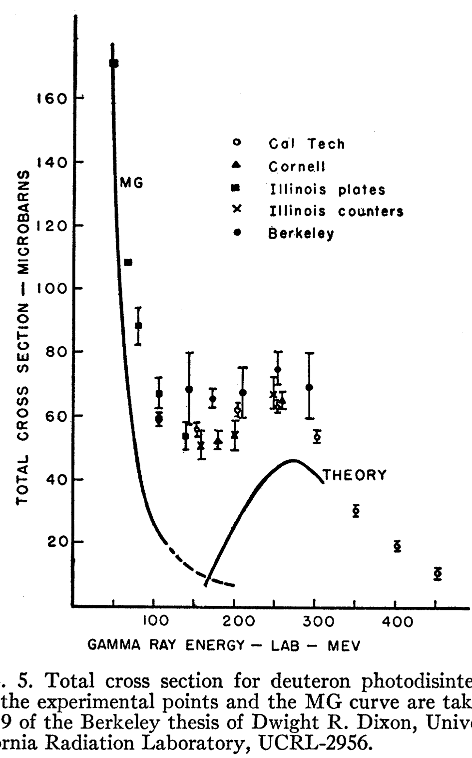

FIG. 5. Total cross section for deuteron photodisintegration. Both the experimental points and the MG curve are taken from Fig. 29 of the Berkeley thesis of Dwight R. Dixon, University of California Radiation Laboratory, UCRL-2956.

[9] L. I. Schiff, Phys. Rev. 78, 733 (1950). J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).

[10] Rather than simply graphing in Fig. 5 the result of subtracting the extended low-energy curve, the reader is asked to imagine the subtraction. This device is used to emphasize the qualitative nature of our discussion, for the high-energy end of the low-energy curve is very dependent on the details of the nuclear force, and the calculations of reference 9 and of subsequent workers certainly are not reliable in this region.

At this point it is relevant to remark again about the intimate relation between the angular distributions for mesonic disintegration and photodisintegration. This relation does not involve the question of the orbital angular momentum of the intermediate state, but relies only upon the broad features of the model, and upon the similarity of roles of the meson momentum and the γ-ray polarization. (The latter similarity breaks down at high energy.) An angular distribution peaked near 90° thus is to be expected as soon as one notes the strong fore and aft peaking of the mesonic absorption cross section.

One other failing of our result is the restricted energy range over which it is known, a consequence of the limited range of even the recent data for $\pi^+ + d \rightarrow 2p$. This range might be extended, both to higher energy and down to and below meson threshold, by going outside the model and introducing theoretical ideas about the energy variations of Π, Ω, T.

VI. PROTON-PROTON SCATTERING

Within the limitations of the model we can only calculate nucleon-nucleon scattering in the $^1D_2$ state. Utilizing the standard definition of T, as given in Sec. IV, the transition matrix is seen to be

$$M(p+p\rightarrow p+p)=(D')^{-1}T^2Y_{2,0}(1)Y_{2,0}(\mathbf{k}\cdot\mathbf{k}'). \quad (37)$$

The cross section follows as

$$d\sigma(p+p\rightarrow p+p)=[2\pi/4\hbar v(2n)]\rho_E(2n)|D'|^{-2}T^4 \times[Y_{2,0}(1)Y_{2,0}(\mathbf{k}\cdot\mathbf{k}')]^2 d\Omega/4\pi, \quad (38)$$

$$=[25/512\pi\hbar][\rho_E(2n)/v(2n)] \times|D'|^{-2}T^4(3\cos^2\theta-1)^2 d\Omega/4\pi. \quad (39)$$

The factor (1/4) appearing before (38) is the relative singlet statistical weight, among the incident spin states. Upon integrating (39),

$$\sigma(p+p\rightarrow p+p) =[5/128\pi\hbar][\rho_E(2n)/v(2n)]T^4|D'|^{-2}, \quad (40)$$

$$=\left[\frac{81}{2560}\right]\left[\frac{v(\pi)}{v(2n)}\right]\left[\frac{\rho_E(\pi)}{\rho_E(2n)}\right]\frac{|D'|^2}{|D|^2}\frac{\sigma^2(\pi^++D)}{\sigma(\pi^++P)}. \quad (41)$$

Even near resonance, where (41) is at its largest, it only gives a value of eight microbarns for the total cross section. Evidently, this is insignificant, in comparison with the experimental value of 50 mb.

The calculation given here for $\sigma(p+p\rightarrow p+p)$ is more an exercise than a practical application of the model in nucleon scattering. Even if the isobar dominates in nucleon scattering, the simple approach used in this paper must be inadequate. For this scattering process, even more than for the deuteron photoeffect, one must consider other than S states of relative motion of the two nucleons in the intermediate state. This has the consequences: (1) of invalidating the angular distribution of Eq. (39); (2) but even more importantly, of permitting nucleon isobars to be formed from other than $^1D_2$ incident states, so making possible considerably larger contributions to the scattering cross section. It is not obvious what is the best way to extend the simple model so as to include nucleon scattering, although related studies have been given by other authors.[11]

VII. CROSS SECTION REDUCTIONS, ALSO NUCLEON COMPTON EFFECT

The difference between D and D' [see Eqs. (7) and (24)] has not been important in the earlier sections. We now study this difference, so as to find a correction to the impulse approximation treatments of such reactions with the deuteron as γ-ray scattering, photopion production, and meson scattering.

The difference between D and D' lies solely in Γ' being greater than Γ, because of the existence of the collision channel for two nucleon systems. The ratio $(\Gamma'-\Gamma)/\Gamma$ is simply the ratio of the probability of collision decay to the probability for decay by meson emission (radiation being negligible). A convenient measure of $(\Gamma'-\Gamma)/\Gamma$ is

$$\frac{\Gamma'-\Gamma}{\Gamma}\approx\frac{\sigma(\gamma+d\rightarrow n+p)}{2\sigma(\gamma+p\rightarrow\pi^0+p)}, \quad (42)$$

the two cross sections in (42) providing similar initial conditions for isobar formation, but a choice of modes of decay. The factor $\frac{1}{2}$ in (42) corrects the π^0 photoproduction to a two nucleon situation. Upon eliminating deuteron photoeffect in favor of our primary cross sections, (42) becomes

$$\frac{\Gamma'-\Gamma}{\Gamma}\approx\frac{9}{8}\frac{\sigma(\pi^++d\rightarrow 2p)}{\sigma(\pi^++p\rightarrow\pi^++p)}. \quad (43)$$

Equation (43) predicts that Γ' is about ten percent bigger than Γ.

Thus at resonance we find the two-nucleon cross sections to be reduced by factors somewhat like those which are observed.[12] However, the whole effect is limited to the resonance region, for outside this region $(E-E_0)$ dominates both in D and in D'. It should be remembered that the applicability of the above result is limited to the domain of applicability of the isobar model.

The present section is a suitable one in which to give passing mention to Compton effect. Corrections to this effect for the deuteron are described above. However, the isobar prediction for Compton scattering from a

[11] For example, see J. Iwadare, Progr. Theoret. Phys. (Japan) **9**, 94 (1953); F. T. Solmitz, Phys. Rev. **92**, 164 (1953); Thaler, Bengston, and Breit, Phys. Rev. **93**, 644 (1954).

[12] Reductions of up to twenty percent were described by G. Bernardini in a lecture given at Columbia University. Also see Bethe and de Hoffman *Mesons* (Row, Peterson and Company, Evanston, Illinois, 1955), footnote on p. 165.

single nucleon does not seem to have been published elsewhere. The calculation yields

$$d\sigma(\gamma+p\to\gamma+p)=(2\pi/\hbar c)\rho_E(\gamma)|D|^{-2}\Omega^4 \times[\tfrac{7}{8}+\tfrac{3}{8}\cos^2\theta]d\Omega/4\pi, \quad (44)$$

and

$$\sigma(\gamma+p\to\gamma+p)=\frac{9}{2}\left(\frac{E}{qc}\right)^2\frac{\sigma^2(\gamma+p\to\pi^0+p)}{\sigma(\pi^++p\to\pi^++p)}. \quad (45)$$

Too much trust should not be put in (45). The direct γ ray scattering by the proton charge certainly also occurs, and *does* interfere with the scattering via the isobar. The angular distribution in (44) is just the average on polarizations of $[1+3(\mathbf{u}\cdot\mathbf{u}')^2]$.

The cross section of Eq. (45) is graphed as Fig. 7. The limited range in energy of the $\pi^++d\to 2p$ input cross section prevents carrying this curve past its peak, which seems to occur near 350 Mev. However, at the peak Fig. 7 shows the quite large cross section of 3 μb. It might be noted that the total cross section of Fig. 7 is not in too bad disagreement with some tentative predictions which were guided by causality ideas,[13] although the angular distributions are rather different.

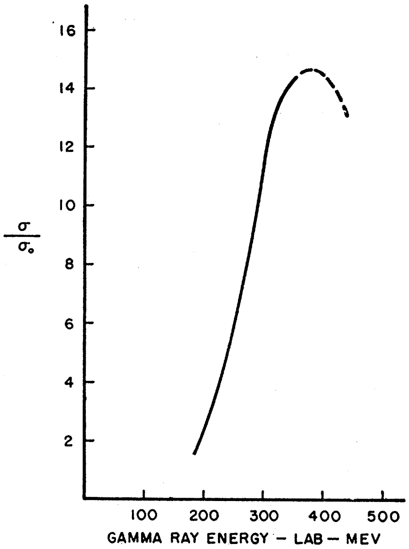

FIG. 7. Total cross section for the scattering of gamma rays by protons. Note, $\sigma_0=(8\pi/3)(e^2/Mc^2)^2$ $=0.198$ microbarn.

VIII. CONCLUSIONS

Among the two-nucleon processes treated in this paper the only one in which the isobar model gives important effects, beyond impulse approximation, is deuteron photodisintegration. Here the model gives such good agreement with experiment as to lend support to the further elementary use of the isobar, as in this paper, as a universal intermediate state in high-energy nucleon interactions.

Note added in proof.—The same result for deuteron photoeffect has also been presented to B. T. Feld [Nuovo Cimento **2**, 145 (1955)] in a paper which appeared after the completion of the foregoing work. A detailed comparison of (36) with the corresponding formula of Feld is facilitated by noting that $\sigma(\pi^0+p\to\pi^0+p)=4/9\sigma(\pi^++p\to\pi^++p)$ and $\sigma(\pi^0+d\to n+p)=\sigma(\pi^++d\to p+p)$. Professor Feld has informed me that he has also since carried his investigation beyond the simple model, considering many of the deficiencies of this model which are indicated in my paper. I am grateful to him for calling his work to my attention.

ACKNOWLEDGMENTS

The author is grateful to the many people, at Cornell and elsewhere, who have discussed this work with him. He is especially grateful, in this regard, to Professor H. A. Bethe, Professor A. O. Hanson, Professor A. Silverman, and Professor R. R. Wilson, and to Dr. M. Ross, who also was kind enough to check part of the calculations.

[13] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954).

APPENDIX

The coefficients $\langle\frac{3}{2},\frac{1}{2};m,m'|2,m+m'\rangle$:

↓ m' \ m →	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
$\frac{1}{2}$	1	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{4}}$
$-\frac{1}{2}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{4}}$	1

The coefficients $\langle\frac{3}{2},\frac{1}{2};m,m'|1,m+m'\rangle$:

↓ m' \ m →	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
$\frac{1}{2}$	0	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{3}{4}}$
$-\frac{1}{2}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{4}}$	0

The coefficients $\langle 1,\frac{1}{2};m,m'|\frac{3}{2},m+m'\rangle$:

↓ m' \ m →	1	0	-1
$\frac{1}{2}$	1	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$-\frac{1}{2}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$	1

Also,

$$|Y_{2,0}|^2=(5/16\pi)(3\cos^2\theta-1)^2,$$
$$|Y_{2,\pm1}|^2=(15/8\pi)\sin^2\theta\cos^2\theta,$$
$$|Y_{1,0}|^2=(3/4\pi)\cos^2\theta,$$
$$|Y_{1,\pm1}|^2=(3/8\pi)\sin^2\theta.$$