# Electrons from Muon Capture*

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We have searched for the process $\mu^- + p \to p + e^-$ or $\mu^- + n \to n + e^-$ for $\mu$ mesons stopped in a Cu target. Scintillation counters were employed to detect the electrons from the process. No counts attributable to the electrons were obtained and we place an upper limit of $\sim 5\times 10^{-4}$ for the relative rate of this process to that for the usual nuclear capture reaction.

## I. INTRODUCTION

NEGATIVE muons stopped in matter may disappear by means of two competing reactions, $\beta$ decay and nuclear capture.[1] $\beta$ decay is most probable in matter composed of light nuclei, the rates are approximately equal for the nuclei $Z=11$ and 12, and for the heavier nuclei, nuclear capture predominates.[2–4] In the case of copper, capture accounts for about 92%, and in the case of lead for approximately 95% of the muon annihilations.[3] The lifetimes are correspondingly reduced; the mean lives of $\mu^-$ mesons in copper and lead are of the order of $10^{-7}$ sec.

A series of experiments in emulsions,[5,6] cloud chambers,[7,8] and with counters,[9–12] has shown that $\mu^-$ capture normally results in the emission of a light neutral particle, which is not observed. This particle may be identical with the neutrino of $\beta$ decay and the normal capture reaction may then be written: $\mu^- + p \to n + \nu$ (1). This reaction involves four spin-$\frac{1}{2}$ particles and is in this respect analogous to nuclear $\beta$ decay: $n \to p + e^- + \nu$. One may then ask if the similar reactions $\mu^- + p \to p + e^-$ (2) and $\mu^- + n \to n + e^-$ (2′) occur to any appreciable extent.

The electrons of this reaction (2), when it occurs in a complex nucleus, will exhibit a continuous energy distribution which should be rather similar to that of the neutrinos of reaction (1). This latter spectrum has been discussed theoretically by several authors[4,13,14] and we will consider this in more detail in connection with the analysis of the experimental results. Here we point out only that the expected energy distribution is peaked sharply in the neighborhood of 90 Mev. Lagarrique and Peyrou[8] have made a search for these electrons by counting the number of minimum ionizing tracks originating from cosmic-ray muons stopped in the copper and tin plates of a cloud chamber. After subtracting for $\mu^+$ decay positrons and the free decays of $\mu^-$ mesons, it is concluded that nuclear capture with electron emission has a probability of $0.04\pm 0.05$ compared to capture without electron emission. The slight positive effect is obscured by the error.

We report here an experiment in which the sensitivity is increased one hundred-fold.

## II. EXPERIMENTAL METHOD

Experimentally, the chief problem in this search is that of distinguishing the capture electrons and the decay electrons. The technique used here is identical with that described in a search for the $\pi$-electron decay,[15] and we refer to the report on that experiment for details. In brief, the 85-Mev $\pi^-$ beam of the Columbia University Nevis Cyclotron is moderated by means of absorber in the monitoring counter telescope 1–2 (see Fig. 1). The beam contains approximately 7% $\mu^-$ mesons which have a larger range than the pions, and the moderator is adjusted in thickness to permit a maximum stopped $\mu$ flux in the target. As target for the bulk of the experiment we chose a sheet of copper $\frac{1}{8}$-inch thick. This is a compromise between the desirability of high $Z$ so that the $\mu$ mesons are largely captured, and low $Z$ so that the resulting electron spectrum will not be excessively modified by radiation in the target.

The resulting electrons are detected in a four-counter telescope, 3456, into which absorber can be inserted to reduce its efficiency for decay electrons. Without absorber the detector counts approximately 90% of the decay spectrum. With 9 inches of polyethylene inserted —1 inch each between 3–4, 4–5, and 5–6, and 6 inches between the target and counter No. 3—the efficiency for counting the decay electrons is 0.2%,[15] negligible in this experiment. On the other hand, the efficiency for counting the capture electrons is approximately 50%. We record the following events: $M$= counts in the monitor 1–2, $D$= counts in the detector 3456, $MD_f$= "fast"

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TABLE I. Experimental results.

| Run No. | Target | Absorber in detector | 1–2 (×4096) | 3456 $D$ | 12-3456 $MD_s$ (2-μsec gate) | 12-3456 $MD_f$ (0.2-μsec gate) |
|---|---|---|---|---|---|---|
| 1 | 2.14 g/cm² C | None | 250 | 739 | 231 | 21 |
| 1′ | Background | None | 220 | 318 | 12 | 2 |
| 2 | 2.85 g/cm² Cu | None | 513 | 581 | 123 | 75 |
| 2′ | Background | None | 320 | 384 | 16 | 3 |
| 3 | 2.85 g/cm² Cu | 31 g/cm² $CH_2$ | 4746 | 288 | 5 | 0 |

coincidences between monitor and detector in which the pulse $D$ comes not later than $2\times10^{-7}$ sec after pulse $M$, and $MD_s$ = "slow" coincidences in which $D$ comes not later than $2\times10^{-6}$ sec after $M$.

The experiment consists essentially of three sets of data, one with a carbon target (chosen to have the same stopping power as the copper) and no absorber in the detector $D$, a second with copper target and no absorber in $D$, and a third with a copper target and 9 inches of polyethylene absorber.

## III. EXPERIMENTAL RESULTS

The experimental results are tabulated in Table I. The number of counts obtained with the C and Cu targets and their backgrounds (target out) are given. It should be noted that no counts attributable to high energy capture electrons from the Cu target were obtained in the fast coincidence with the absorber in position. This result represents a total of 20 hours of running time.

During this period the sensitivity of the apparatus was checked approximately every hour by removing the absorber in $D$ and observing counts due to decay electrons.

We thus find no evidence for high-energy capture electrons. The following discussion is an attempt to place an upper limit on the rate for the process.

## IV. ANALYSIS OF THE RESULTS

As can be seen in Table I, no counts were observed in the crucial category, that is in $MD_f$ with copper target and absorber in $D$. We have therefore no positive evidence for the production of electrons in $\mu$ capture, and proceed to a discussion of an upper limit for the branching ratio,

$$R=\frac{\text{Rate }\mu^-+\text{Cu}\rightarrow\text{Cu}+e^-}{\text{Rate }\mu^-+\text{Cu}\rightarrow\text{Ni}+\nu},$$

on the basis of our negative result.

To discuss $R$ we need the following quantities: $\epsilon$ = efficiency for counting the capture electron spectrum in $D$ with 9 inches polyethylene absorber; $\eta$ = the product of solid angle $\Omega$, number of $\mu$ mesons stopped in the copper target, and fraction captured within the time of resolution of $MD_f$, normalized to $10^6$ monitor counts $M$; $q$ = expected counts in $MD_f$ per $10^6$ monitor counts $M$, due to other causes, that is, the expected background rate.

The expected number of counts $\alpha$ in $MD_f$ with copper target and absorber in $D$ and given branching ratio $R$, is then

$$\alpha=(q+\epsilon\eta R)\times(M\times10^{-6}).$$

Since we observe a zero rate, we consider the probability $P_0(R)$, of observing no counts, as a function of $R$, where $P_0(R)=e^{-\alpha(R)}$, assuming a Poisson distribution.

We now discuss the separate quantities $\epsilon$, $\eta$, and $q$.

### A. Electron Detection Efficiency ε

To estimate $\epsilon$ we need the energy spectrum of the capture electrons. As has already been pointed out in the introduction, this is expected to be very similar to the neutrino spectrum of the normal capture process, with the only important difference that the Pauli principle is expected to play a much smaller role in the final nuclear state, since the nuclear charge is not changed. We assume that the meson is captured by a single nucleon. If this nucleon were unbound, the resulting electron would have an energy of 102 Mev. However, the nucleon is bound in the initial state and may be bound, and certainly interacts, in the final state. We have made a calculation of the electron energy distribution which may be expected due to the motion of the nucleon in the initial state. We have entirely neglected the binding in the final state, which favors emission of the nucleon with small momentum, and therefore higher energies for the electron. Our calculated electron energy distribu-

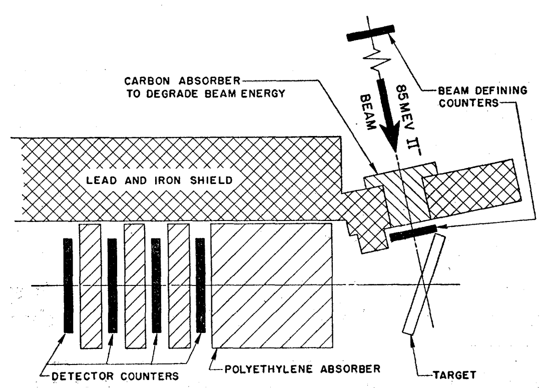


FIG. 1. Experimental arrangement.

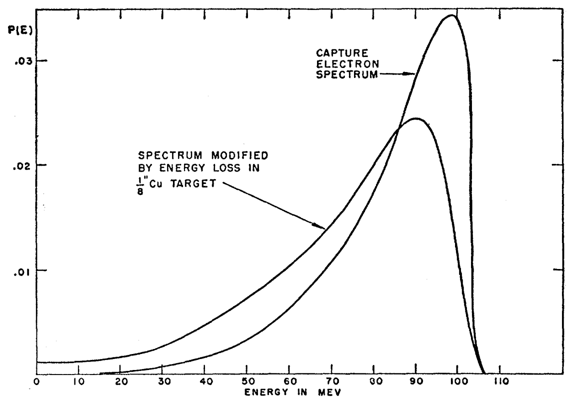


FIG. 2. The spectra of capture electrons calculated on the basis of the Chew-Goldberger model, with and without energy losses in the Cu target.

tion, being too low, will yield therefore a smaller detection probability $\epsilon$ than one which would result from a calculation in which the final nuclear state is properly treated. This tends to make our upper limit somewhat conservative.

We use the nucleon momentum distribution of Chew and Goldberger.[16] Following their assumptions, one then obtains for the electron energy spectrum (see Fig. 2):

$$P(E)=\frac{pE^2}{(p^2+E^2+\beta^2)-4p^2E^2},$$

where $p^2=2M(\mu-E)$, $\beta=4\sqrt{(ME_D)}$, $E_D$= deuteron binding energy, $M,\mu$= nucleon and muon masses respectively, and $\hbar=c=1$.

Continuing the discussion of the electron detection efficiency, we need to correct this spectrum for the radiation and ionization losses in the copper.

For the radiation losses we have averaged over a formula given by Heitler[17] to obtain:

$$\langle w(y)\rangle_{\text{Av}}dy=\frac{dy}{\Gamma(1+\frac{1}{2}bl)}\frac{e^{-y}}{y\ \ln y}e^{x}\left[e^{x}-\frac{\sinh x}{x}\right].$$

Here $\langle w(y)\rangle_{\text{Av}}dy$ is the probability of radiation loss into the region $dy$, for electrons originating uniformly from the volume of a plate of thickness $l$; $b$ is 1.35 times the radiation length of the material; $y=\ln E/E'$ where $E$ and $E'$ are initial and final energy respectively; $x=\frac{1}{2}bl\ \ln y$.

The average ionization loss is 2 Mev and we neglect straggling due to differences in ionization loss. We have included in Fig. 2 the calculated modification of $P(E)$ in the copper. The resultant spectrum is incident on the detector $D$.

For the probability that the electrons of this spectrum penetrate the detector $D$ with 9 inches of polyethylene absorber, we use the Monte Carlo range calculations of reference 15. The ionization loss for minimum-ionizing particles in the detector assembly is 53 Mev. In Fig. 3 we have plotted the probability $W(E)$ with which an electron of energy $E$ penetrates this absorber.

Finally the detection probability $\epsilon$ is the fold of $P(E)$ and $W(E)$:

$$\epsilon=\int P(E)W(E)dE\Big/\int P(E)dE=0.5\pm0.05.$$

### B. $\eta$, the Product of the Number of Stopped Muons Times the Solid Angle Times the Fraction Captured in the Copper Target within the Resolution Time of $MD_f$

When the target is carbon and no absorber is inserted in $D$, the mesons which come to rest mostly undergo free decay, and the probability of detecting the decay electron in $D$ is large. This then permits a calibration of the product of the stopping flux $F$ and solid angle $\Omega$ of the detector $D$. This will be directly applicable to the case of the copper target, since its stopping is the same as that of the carbon, and the solid angle of $D$ remains unchanged. To find $F\Omega$, we note that the detection probability for the $\mu$-electron spectrum in $D$ without absorber is 0.9, the fraction of muons decaying to electrons in carbon is 0.93,[2] and the fraction of decay electrons falling within the 2-$\mu$sec gate of $MD_s$ is 0.622. The counting rate $MD_s$ is 213±15 per $10^6$ $M$. Therefore

$$F\Omega=(213\pm15)\times(1/0.9)\times(1/0.93)\times(1/0.622)=410\pm24.$$

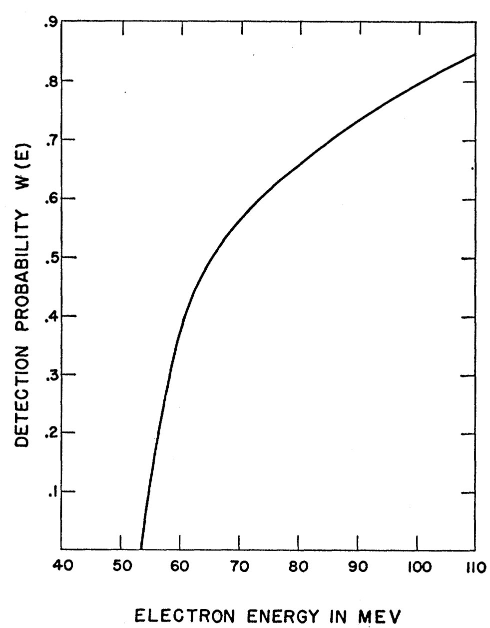


FIG. 3. Detection probability for electron detector with 21 g/cm² of polyethylene absorber added.

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[16] G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 471 (1950).

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To obtain the fraction which is captured in the copper within the gate time $MD_f$, we use lines 2 and 2′ of Table I. The counting rate with copper target with our effectively infinite gate duration ($MD_s$) is 46.4±5. This rate represents the decay electron rate. The total number of decay electrons with copper target is therefore $(46.4\pm5)/0.9=51\pm5.5$. The fraction captured in copper is therefore $[(410\pm29)-(51+5.5)]/(410\pm29) = (88\pm1.3)$ percent. The fraction of these which fall within the gate time $MD_f$ is also obtained from Table I, lines 2 and 2′, and is $MD_f/MD_s=0.72\pm0.12$. We have therefore

$$\eta=(410\pm29)\times(0.88\pm0.013)\times(0.72\pm0.12)$$
$$=260\pm45 \text{ per } 10^6\, M.$$
$$=5050 \text{ for the run of line 3, Table I.}$$

### C. $q$, the Expected Number of Background Counts

*1. Accidental coincidences.*—This follows from the known rates $M$ and $D$, the resolving time ($2.5\times10^{-7}$ sec), the duration of the run (20 hr), and the duty cycle of the cyclotron (1/30):

$$q_{\text{acc}}=4746\times4096\times288\times2.5\times10^{-7}\times30/(3600\times20)$$
$$=0.58.$$

*2. Decay electrons.*—In the previous experiment with this detector it was found that with 9 inches of polyethylene absorber, there is still a finite probability, 1/500, for the detection of the decay electrons. 12% of the stopping muons produce decay electrons in copper. The expected number of counts is therefore $q_{\mu\to e}=0.12\times(1/500)\times4746\times(4096/10^6)\times0.72\times410=1.38$, and

$$q=q_{\text{acc}}+q_{\mu\to e}=1.86.$$

The expected number of counts $\alpha$ is then

$$\alpha(R)=q+\epsilon\eta R$$
$$=1.86+2620R.$$

In Fig. 4 we show $P_0(R)=e^{-\alpha(R)}$. From this figure we see that the probability of obtaining this negative

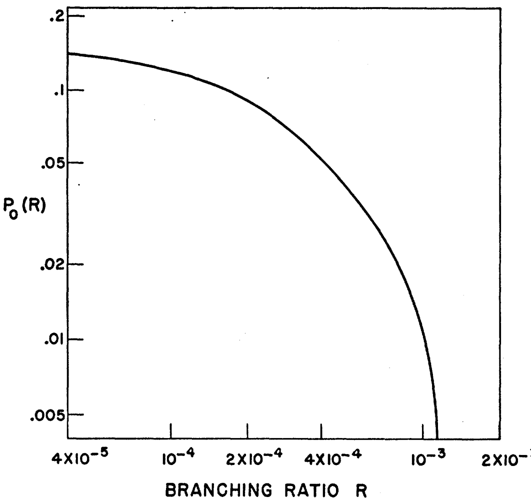


FIG. 4. Probability of obtaining no counts as a function of branching ratio $R$.

result is ~5% for the branching ratio $R=4\times10^{-4}$ and ~1% for $R=10^{-3}$. The experiment is not sensitive to values of $R\lesssim3\times10^{-4}$. We feel that the result of the experiment is best contained in the plot of $P_0(R)$, and that perhaps $R=5\times10^{-4}$ corresponding to $P_0(R)\simeq0.04$ may be reasonably safe upper limit for the branching ratio $(\mu^-+\text{Cu}\to\text{Cu}^*+e^-/(\mu^-+\text{Cu}\to\text{Ni}^*+\nu)$.

### V. DISCUSSION OF THE RESULT

The inhibition of the reaction here under study is even greater than indicated by the smallness of $R<5\times10^{-4}$. There are two factors which should favor this reaction: (1) Both protons and neutrons of the target nucleus could interact to give the electron, but only the protons can contribute to neutrino formation. (2) The Pauli principle inhibits the reaction in which the nuclear charge changes; this may be a factor also of the order 2. To compare the relative coupling strengths, we include these factors:

$$\frac{g^2(\mu^-+p\to n+\nu)}{g^2(\mu^-+p\to p+e^-)}>4\times2\times10^3=8\times10^3.$$