Cf^{250} and Cf^{252} each populate first excited states in their respective daughters of about 44 kev in an abundance corresponding roughly to the predictions of alpha-decay theory. More precisely these alpha groups are hindered by about a factor of 3, similar to Cf^{246}, somewhat lower than in 100^{254} decay and somewhat higher than in the decay of elements of lower atomic number. The spin and parity of the first excited state populated by Cf^{252} decay was deduced from the conversion coefficient to be 2+ in common with nearly all other even-even nuclei.

The 100-kev gamma ray in Cf^{252} decay is interpreted as the transition from the second even state to the first even state. The energies of the first and second states are such that they can be interpreted as a Bohr-Mottelson rotational band with a consequent spin of 4+ for the second even state. The alpha decay to the second even state as deduced from the abundance of the 100-kev gamma ray is lower by over two orders of magnitude from the predictions of spin-independent alpha-decay theory. This hindrance factor is very similar to that of Cf^{246} and appears to follow the general trends observed for corresponding transitions in other even-even nuclides.

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Double K Capture and Single K Capture with Positron Emission

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Transition probabilities are estimated for double K capture and for single capture with single positron emission. With real neutrino emission, the mean lives for both processes should be greater than 10^{24} years. Without real neutrino emission, as with Majorana neutrinos, the mean life for K capture in conjunction with positron emission is about 10^{15} years in the allowed approximation. Double K capture, however, is then at least a third-order process because an additional step is necessary to remove the energy, with the result that the mean life exceeds 10^{18} years.

I. INTRODUCTION

FOUR possible types of double beta processes[1] are double negatron emission, double positron emission, double negatron capture, and single capture with single positron emission. Double negatron emission probabilities have been calculated with various theories, while double positron probabilities are given, except for the Coulomb distortion of the wave functions of each of the emitted electrons, by similar calculations. Most experiments have been searches for these double emission processes.

One published experiment[2] and some private speculation, however, involved attempts to detect double capture or single capture with single positron emission. Double capture will be the only energetically possible transition if the mass difference between the parent atom and the isobaric atom with atomic number less by two lies between 0 and $2mc^2$. Either double capture or one capture and one positron emission can occur if the atomic mass difference is between $2mc^2$ and $4mc^2$. Both of these processes and double positron emission can occur if the difference is greater than $4mc^2$.

It is the purpose of this note to exhibit rough estimates of the transition probabilities for double capture and for single capture with positron emission.

[1] See references in Rolf G. Winter, Phys. Rev. **99**, 88 (1955).
[2] Berthelot, Chaminade, Levi, and Papineau, Compt. rend. **236**, 1769 (1953).

II. DIRAC NEUTRINOS

If the neutrino is a Dirac particle, double capture probabilities can be calculated from a second order perturbation. Both steps consist of the capture of a K negatron and the emission of a neutrino. The calculation is like that used by Goeppert-Mayer,[3] except that K negatrons are captured rather than free negatrons being emitted. If one intermediate nuclear state s, about mc^2 above the initial state i, contributes most of the result, the probability of transition to the final state f is given by

$$w_{if}=\frac{2\pi}{\hbar}\rho\left|\sum_s \frac{H_{is}H_{sf}}{mc^2+E_\nu}\right|^2, \tag{1}$$

where the energy of the first neutrino is E_ν.

If the total energy that must be removed by the neutrinos is E, one obtains, for unit volume normalization,

$$\rho\left(\sum_s \frac{1}{mc^2+E_\nu}\right)^2 =(2\pi^2\hbar^3c^3)^{-2}\int_0^E (E-E_\nu)^2E_\nu^2\frac{dE_\nu}{(mc^2+E_\nu)^2}. \tag{2}$$

If the lepton spin sums and nuclear wave function

[3] M. Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).

integrals are a maximum, the Fermi beta-decay matrix elements for K capture become

$$H_{is}\simeq H_{sf}\simeq g(Z^3/\pi a^3)^{\frac{1}{2}}, \quad (3)$$

where a is the Bohr radius, Z the atomic number, and $g=2\times 10^{-49}$ erg cm^3 is the Fermi coupling constant.

Integrating (2) and substituting the result and (3) into (1), one finds, with $E=mc^2\epsilon$, that

$$w_{if}\simeq\frac{m^3g^4Z^6}{2\pi^5\hbar^7a^6}\left[\frac{\epsilon^3}{3}+4\epsilon^2+4\epsilon -2(\epsilon+2)(\epsilon+1)\ln(1+\epsilon)\right]. \quad (4)$$

For $Z=30$, (Berthelot *et al.*[2] investigated Zn) and $\epsilon=2$, w_{if} becomes 4×10^{-25} per year. As might be expected, this result lies in the same neighborhood as low-energy double negatron emission probabilities in the Dirac neutrino theory.

The calculation for one K capture followed by one positron emission is similar. The first neutrino with energy $E_{\nu 1}$, the second neutrino with energy $E_\nu - E_{\nu 1}$, and the positron with energy $E_\beta = E - E_\nu$ remove the total energy E. Therefore, instead of (2), one must calculate

$$(2\pi^2c^3\hbar^3)^{-3}\int_0^{E-mc^2}(E-E_\nu)[(E-E_\nu)^2-m^2c^4]^{\frac{1}{2}}dE_\nu \times\int_0^{E_\nu}\frac{(E_\nu-E_{\nu 1})^2E_{\nu 1}^2dE_{\nu 1}}{(mc^2+E_{\nu 1})^2}. \quad (5)$$

The K capture matrix element H_{is} is still approximated by (3), but the positron emission matrix element is given by

$$H_{sf}\simeq gf^{\frac{1}{2}}(E_\beta,Z). \quad (6)$$

Here $f^{\frac{1}{2}}(E_\beta,Z)$ is the Coulomb correction in the positron wave function.

The second integration in Eq. (5) can only be carried through numerically. The resulting transition probability is strongly energy dependent, but lies in the same range as the above double K capture result.

III. MAJORANA NEUTRINOS

If the neutrino is a Majorana particle, the neutrino emitted in the first step of a double beta process can be reabsorbed in the second step.[4] Its existence is then only virtual, it has all of phase space available, and therefore it usually produces a large increase in the transition probability.

Since, however, the neutrino is not actually emitted, it cannot carry away any energy. In general, then, double capture without real neutrino emission cannot bring the system to an available state; it is necessary to go to a higher order process that includes a mechanism for disposing of the right amount of energy.

Of course, there may exist an excited state of the product nucleus to which the transition can go. The initial state, however, has a completely negligible width, and the excited states of the product nucleus will generally have widths of less than 10^{-2} ev and spacings of about 10^5 ev. Therefore the existence of a state to which the transition can go with only double capture without real neutrino emission would be a monumental coincidence. Barring such a coincidence, the necessity of a third step to balance the energy leads to a large decrease in transition probability.

Double K capture with one internal bremsstrahlung appears to be the dominant process. The transition probability is

$$w_{if}=\frac{2\pi}{\hbar}\frac{\omega^2}{2\pi^2c^3\hbar}\left|\sum_t\sum_s\frac{H_{ft}H_{ts}H_{si}}{(E_t-E_i)(E_s-E_i)}\right|^2, \quad (7)$$

where ω is the angular frequency of the emitted gamma ray. The expression

$$\textstyle\sum_s H_{ts}H_{si}/(E_s-E_i) \quad (8)$$

corresponds to the capture of one K negatron with the emission of a bremsstrahlung photon and a virtual Majorana neutrino and brings the system to a state of energy E_t. The final step, given by H_{ft}, consists of the capture of the second K negatron and the absorption of the virtual neutrino.

Since the virtual neutrino can have any energy, one cannot approximate its wave function by a constant over the region of the nucleus. For optimum overlap, the nuclear matrix elements are, if the nuclear radius is R,

$$\frac{3}{4\pi R^3}\int_{r<R}e^{ik\cdot r}d\tau. \quad (9)$$

Otherwise, there is no difference between the calculation of (8) and the calculation of single bremsstrahlung matrix elements[5,6]; the result is

$$\frac{ge}{mc}\left(\frac{\pi\hbar}{\omega}\right)^{\frac{1}{2}}\Psi_{k1}(0)\frac{3}{4\pi R^3}\int_{r<R}e^{ik\cdot r}d\tau, \quad (10)$$

where $\Psi_{K1}(0)$ is the wave function, evaluated at the nucleus, of the electron that is captured. The remainder of the calculation is the same as that used by Furry.[4] The term H_{ft} is the usual matrix element for allowed K capture except for the use of (9) for the integration over the nuclear functions and is given by

$$g\Psi_{K2}(0)\frac{3}{4\pi R^3}\int_{r<R}e^{ik\cdot r}d\tau. \quad (11)$$

[4] W. H. Furry, Phys. Rev. 56, 1184 (1939).

[5] P. Morrison and L. I. Schiff, Phys. Rev. 58, 24 (1940).

[6] Richard E. Cutkosky, dissertation, Carnegie Institute of Technology, 1953 (unpublished).

If one intermediate nuclear state contributes most of the result, the sum over t is simply an integration over the phase space of the virtual neutrino which does not cut off until the neutrino wavelength is of the order of the nuclear radius. Since most of the contribution to this integral comes from the high energy (about $100mc^2$) end, it is reasonable to use for E_t-E_i merely the neutrino energy, or $c\hbar k$. The term $\Psi_{K1}(0)\Psi_{K2}(0)$ can range from a maximum of

$$2\Psi_K^2(0)\simeq 2Z^3/\pi a^3, \tag{12}$$

down to zero in case of complete cancellation in the spin sum, depending on the particular form of the interaction; the maximum value will be used here. The sum over t is now a count of the number of cells in phase space available to the virtual neutrino and contributes

$$\frac{1}{2\pi^2}\int_0^\infty k^2dk, \tag{13}$$

where, of course, all of the above k dependent terms must be included before the integration is performed.

Collecting the above estimates, one finds that the transition probability becomes

$$w_{if}\simeq\frac{81}{16\pi^6}\frac{g^4}{R^4m^2c^6\hbar^2}\frac{e^2}{\hbar c}\frac{Z^6}{a^6}\omega. \tag{14}$$

If $\hbar\omega=2mc^2$, $Z=30$, and $R=6\times10^{-13}$ cm, $w_{if}\simeq10^{-18}$ per year. This result, which is smaller by 10^{-4} than allowed emission probabilities, might be expected; in ordinary K capture, inner bremsstrahlung occurs once in about 10^4 captures.

Of course, double capture without real neutrino emission can proceed with any third-order process that includes a mechanism for removing energy. Competition with inner bremsstrahlung photon emission can come from electron conversion, depending on the angular momentum that must be removed. Another class of processes involves double capture to a virtual state of the final nucleus followed by, for instance, gamma emission. Such processes, however, are considerably less probable. For double K capture followed by electric quadrupole gamma emission, a calculation much like the above, with similar approximations, gives for the transition probability

$$\frac{243}{4\pi^6}\frac{g^4}{m^2c^{10}\hbar^2}\frac{e^2}{\hbar c}\frac{Z^6}{a^6}\omega^5, \tag{15}$$

or 10^{-23} per year for $\hbar\omega=2mc^2$ and $Z=30$. This result can also be understood by means of a qualitative argument. Allowed double electron emission with Majorana neutrinos should have transition probabilities of about 10^{-14} per year.[4,7] Here, since no real state is available after only double capture, one might imagine virtual transitions with a probability of 10^{-14} per year. If the energy difference between the initial state and the virtual state is roughly mc^2, the virtual state can exist for about $\hbar/mc^2\simeq10^{-21}$ sec. Electric quadrupole transition probabilities are, for $2mc^2$ energy, of the order of 10^{12} per second. Therefore, the probability that there is gamma emission from the virtual state rather than return to the initial state is 10^{-21} sec$\times10^{12}$ per sec or 10^{-9}. Therefore the probability of a third order transition consisting of a double beta process followed by gamma emission is $10^{-9}\times10^{-14}$ per year or 10^{-23} per year.

It can be seen from these arguments, or from more rigorous calculation, that in other double beta theories that do not give neutrinos in the final state,[8,9] just as in the Majorana-Furry theory, double capture lifetimes should exceed double emission lifetimes by similar factors.

If single capture in conjunction with single positron emission is energetically possible, one can again consider a second order process, for now the positron can remove the energy. The transition probability has the form (1), the lepton spin sum can be at most, instead of (12),

$$2f^{\frac{1}{2}}(E_\beta,Z)(Z^3/\pi a^3)^{\frac{1}{2}}, \tag{16}$$

and the sum over the states of the virtual neutrino and the integration of its wave function over the nucleus appear as above. The density of final states involves only the states of the positron of definite energy, and is

$$(2\pi^2c^3\hbar^3)^{-1}m^2c^4\epsilon(\epsilon^2-1)^{\frac{1}{2}}, \tag{17}$$

where ϵ is the total energy of the positron in units of mc^2. The result is

$$w_{if}\simeq\frac{81}{16\pi^6}\frac{m^2g^4}{c\hbar^6R^4}\frac{Z^3}{a^3}f(E_\beta,Z)\epsilon(\epsilon^2-1)^{\frac{1}{2}}. \tag{18}$$

In the Zn^{64} transition for which Berthelot *et al.*[2] showed experimentally that $w_{if}\lesssim10^{-16}$ per year, the kinetic energy available to the single positron should, according to the decay of Cu^{64}, be only 0.08 Mev. The result for w_{if} is then 10^{-15} per year.

All of the above estimates of transition probabilities are upper limits. The nuclear matrix elements, which occur in each calculation to the fourth power, are certainly less than unity, and there will be some cancellation in the lepton spin sums.

[7] H. Primakoff, Phys. Rev. **85**, 888 (1952).
[8] Bruno Touschek, Z. Physik **125**, 108 (1948–1949).
[9] Rolf G. Winter, Phys. Rev. **83**, 1070 (1951).