Small-Angle Scattering of Neutrons by Intermediate and Heavy Nuclei*

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Differential cross sections for scattering of neutrons at forward angles have been measured at three energies for several elements. The results are compared with predictions of the complex square well potential of Feshbach, Porter, and Weisskopf.

A SENSITIVE test of the complex square well nuclear model of Feshbach, Porter, and Weisskopf[1] is provided by comparing measured cross sections for heavy elements at a fixed energy as a function of atomic weight with the predictions of the model.[2–4] For example, the variation with atomic weight of the differential elastic scattering cross section $\sigma(\theta)$ measured at a fixed angle and energy permits comparison with theory without the need of complete angular distribution measurements.

Previous determinations[2] of $\sigma(\theta)$ at 1 Mev show qualitative agreement with the predictions of the complex square well model. In an attempt to provide further comparison of theory with experiment, measurements of $\sigma(\theta)$ have been made at small angles. Since $\sigma(\theta)$ is usually largest at small angles, such measurements are somewhat easier to perform for small values of θ.

Differential cross sections for elastic scattering have been measured for several elements at neutron energies of 0.50, 1.00, and 1.55 Mev with a neutron energy spread of about 100 kev. Figure 1 shows the experimental arrangement. Neutrons produced by the $Li^7(p,n)$ reaction were scattered through the angle θ by annular rings of the element being investigated and were detected by a gas recoil counter. A polythene cone shielded the counter against neutrons from the target. The neutron flux incident on the scattering samples was measured by moving the counter to the position shown by the dashed line drawing in Fig. 1. The angle θ was varied by changing the distance D and using two sizes of rings of mean diameters 4.22 and 8.54 cm, respectively. The rings were of the order of one cm thick. With this arrangement the angular resolution was approximately $\frac{1}{3}\theta$.

In order to compute $\sigma(\theta)$, the effective distance from scattering sample to counter must be known. This was obtained by comparing the counting rate with the counter at a distance $D+12$ inches (see Fig. 1) from the source to the counting rate with the counter in the same position relative to the source as it occupied relative to a typical element of the scattering ring as shown in Fig. 1.

At 1 Mev, experiments were performed on ten elements[5] at scattering angles of 8°, 12°, 16°, and 22°, and on fifteen elements at 30°. At 0.50 Mev, only $\sigma(30°)$ was measured and at 1.55 Mev, $\sigma(14°)$ and $\sigma(30°)$. For a neutron energy of 1 Mev, data were taken at two biases set so that no neutrons with energies less than 660 kev and 790 kev, respectively, were detected. There were no significant differences in cross sections measured at the two biases except in the case of thorium, where a 7% difference was observed. At 0.50 Mev, the two biases were set to discriminate against neutrons having energies less than 200 and 300 kev, respectively. In this case, only tantalum and thorium showed any appreciable bias effect. At both 0.50 and 1.00 Mev, results for both biases were averaged, except for the cases just mentioned, where only the high-bias data were used. At 1.55 Mev, the data presented were taken with the discriminator set such that neutrons of energy less than 1.1 Mev were not detected.

Multiple scattering corrections were applied to the observed cross sections by using the method of Walt and Barschall.[2] This procedure requires some knowledge of the angular distribution over the entire angular range. For the 1.00-Mev data the results of reference 2 were used. At 0.50 Mev and 1.55 Mev, corrections were estimated on the basis of the measured anisotropy $\sigma(\theta)4\pi/\sigma_t$. The magnitude of the corrections for a given anisotropy were obtained from the 1-Mev experiments. There is considerable uncertainty in corrections estimated in this way, but since the average value of the correction was about 5.5% for the measurements made at 30° and about 9% for those made at smaller angles, the uncertainty introduced into the final results is probably not more than 3%.

Table I lists the elements investigated with the corresponding values of the nuclear radius R, where R is taken to be $1.45\,A^{\frac{1}{3}}\times 10^{-13}$ cm. Figure 2 shows the

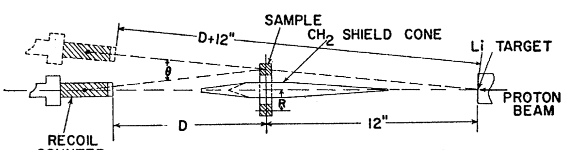

FIG. 1. Diagram of geometry used in measuring $\sigma(\theta)$.

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[1] Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954).
[2] M. Walt and H. H. Barschall, Phys. Rev. 93, 1062 (1954).
[3] R. K. Adair, Phys. Rev. 94, 737 (1954).
[4] W. S. Emmerich, Phys. Rev. 98, 1148 (1955).
[5] Darden, Haeberli, and Walton, Phys. Rev. 96, 836 (1954).

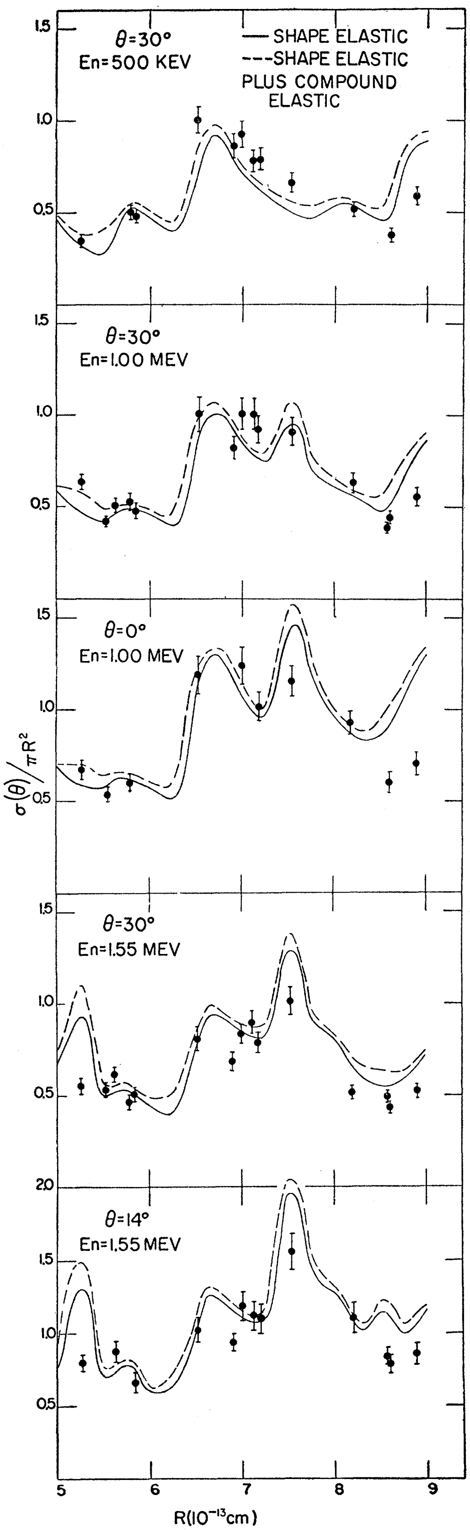

FIG. 2. Differential elastic scattering cross sections divided by πR^2 *versus* R, where $R=1.45\ A^{\frac{1}{3}}\times 10^{-13}$ cm. The black circles show the experimental results. Shape-elastic scattering calculated from the complex potential $-42(1+0.03i)$ Mev is indicated by the solid curves. The dashed curves include the contribution of compound-elastic scattering.

results. The differential cross sections divided by πR^2 are plotted *versus* R. The points shown in Fig. 2 for $\theta=0°$, $E_n=1.00$ Mev, were obtained by extrapolating the 1-Mev measurements to 0°. The principle uncertainties in the results are the statistical uncertainties (6%), uncertainties in the multiple scattering correction (3%), and uncertainty in determining the effective distance from sample to counter (5%). This last uncertainty will affect only the scale of the ordinates in Fig. 2, and is therefore not included in the error bars shown.

The curves shown in the figures were calculated[6] by using the complex potential[1]

$$\begin{aligned} V&=-42(1+\zeta i)\text{Mev}, & r&<R, \\ &=0, & r&>R. \end{aligned} \qquad \zeta=0.03$$

According to this model, the differential elastic scattering cross section consists of a shape-elastic or body-elastic cross section $\sigma_{se}(\theta)$, and a compound-elastic cross section $\sigma_{ce}(\theta)$. The latter cross section corresponds to scattering events in which a compound state is formed, which then decays by emitting a neutron having the same energy as the incident neutron. Only an upper limit on the total compound-elastic cross section is predicted by the model. The solid curves in Fig. 2 show the contribution of shape-elastic scattering alone. In the absence of detailed information concerning the angular dependence of $\sigma_{ce}(\theta)$, the maximum total compound-elastic cross section divided by 4π was added to the shape-elastic cross section to estimate the possible effect of compound-elastic scattering. This is shown by the dashed curves in Fig. 2. Since the compound-elastic cross section is not expected to be isotropic,[1] measured cross sections somewhat higher than those indicated by the dashed curves do not necessarily constitute a disagreement with the predictions based on the model.

The measured cross sections are fairly well reproduced by the theory, especially at 0.50 Mev. For silver, the cross sections are consistently lower than for the neighboring elements, cadmium and tin. This is probably a result of the large inelastic scattering cross section[7] in silver, rather than an indication of a shape effect in the elastic scattering cross section. Some disagreement between theory and experiment is indicated at the smaller angles in the atomic weight region above 200 at 1.00 and 1.55 Mev. Also, the predicted peak in the cross section for nuclear radii of about 5.3×10^{-13} cm which appears in the curves for $E_n=1.55$ Mev, is not shown by the measurements, although only titanium results are available for comparison.

Differential elastic scattering cross sections measured at 4 Mev neutron energy[8] indicate that values of the

[6] The authors are very grateful to Dr. R. G. Thomas of the Los Alamos Scientific Laboratory for supervising the calculations of $\sigma(\theta)$.

[7] Beyster, Henkel, and Nobles, Phys. Rev. **97**, 563 (1955).

[8] M. Walt and J. R. Beyster, Phys. Rev. **98**, 677 (1955).

parameter ζ larger than 0.03 are required to fit the data at this energy. The effect of using values of ζ of 0.1 or 0.2 in calculating the curves of Fig. 2 is to smooth out the fluctuations and decrease the agreement between theory and experiment. A slightly better fit at 1.55 Mev might be obtained by using a value of ζ between 0.03 and 0.1. Such an energy dependence of the value of ζ necessary to fit the data has been suggested, and is indicated by several experiments.[9]

It has been suggested[4] that agreement between measured and predicted cross sections may be improved by using the formula $R^*=(1.26A^{\frac{1}{3}}+0.70)\times10^{-13}$ cm to determine nuclear radii. Values of R^* corresponding to the elements used in this experiment are listed in Table I. The effect of using R^* instead of R is to shift the points in Fig. 2 to smaller radius values as well as to raise them slightly. The shift is greatest for the heaviest elements. Although a slight change in this direction improves the agreement with theory for Pb, Bi, and Th, the effect of using R^* is to worsen the over-all agreement.

The present results for $\sigma(30°)$ at 1.00 Mev are about 12% larger on the average than the results of Walt and Barschall.[2] Since the uncertainty in the absolute cross sections is about 15% in both experiments, this difference is within the quoted errors.

The theoretical three-dimensional plot of $\sigma(\theta)$ at 1-Mev neutron energy shown in reference 1 indicates a sharp peak in $\sigma(0°)$ for atomic weights around 200, which corresponds to a value of R of about 8.5×10^{-13} cm. The failure of the curves in Fig. 2 for $E_n=1.00$ Mev to show this peak may be caused by the fact that values of orbital angular momentum only up to and including 5 units were considered in the present calculations.

TABLE I. Elements investigated and corresponding values of R and R^* in 10^{-13} cm. R is given by $1.45\,A^{\frac{1}{3}}\times10^{-13}$ cm and $R^*=(1.26\,A^{\frac{1}{3}}+0.70)\times10^{-13}$ cm.

Element	R	R^*	Element	R	R^*
Ti	5.26	5.28	Sn	7.12	6.90
Fe	5.53	5.52	Sb	7.18	6.94
Ni	5.63	5.60	Ce	7.53	7.24
Cu	5.79	5.73	Ta	8.20	7.83
Zn	5.84	5.78	Pb	8.57	8.15
Zr	6.52	6.37	Bi	8.60	8.18
Ag	6.90	6.70	Th	8.89	8.44
Cd	7.00	6.77			

[9] A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524 (1955).

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Energy Levels of S^{33}, S^{35}, Cl^{36}, Cl^{38}, and Ba^{139}†

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Barium-chloride targets have been bombarded with deuterons accelerated by an electrostatic generator to energies between 3.0 and 7.5 Mev. Charged reaction products have been observed with a high-resolution magnetic analyzer. The following ground-state Q-values have been measured: $Cl^{35}(d,\alpha)S^{33}$, 8.277 ± 0.010 Mev; $Cl^{37}(d,\alpha)S^{35}$, 7.783 ± 0.012 Mev; $Cl^{35}(d,p)Cl^{36}$, 6.354 ± 0.008 Mev; and $Cl^{37}(d,p)Cl^{38}$, 3.881 ± 0.008 Mev. Fifteen levels have been observed in S^{33}, four in S^{35}, twenty-three in Cl^{36}, and six in Cl^{38}. From the intensities of the observed proton groups and other considerations, a spin of $J=2^-$ can be assigned to the Cl^{38} ground state, and a spin of $J=5^-$ to the lowest (isomeric) level in Cl^{38} at 672 ± 5 kev.

The $Ba^{138}(d,p)Ba^{139}$ ground-state Q-value is 2.493 ± 0.010 Mev, and a level in Ba^{139} is observed at 623 ± 8 kev.

I. INTRODUCTION

THIS work has been undertaken primarily to obtain accurate ground-state Q-values for the chlorine (d,α) and (d,p) reactions. Together with other nuclear reaction data, chiefly beta-decay energies, they establish mass links between many sulfur, chlorine, and argon isotopes, and S^{33}, the heaviest nucleus included in Li's mass survey.[1] The masses to be computed from them will be published in a separate paper, awaiting the measurement of the $K^{39}(p,\alpha)A^{36}$ ground-state Q-value, which is now being undertaken. This Q-value will establish a link with the masses of several potassium, calcium, and scandium isotopes.

In the course of this investigation, alpha-particle and proton groups have been observed corresponding

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* On leave from the Stichting voor Fundamenteel Onderzoek der Materie, Physisch Laboratorium, Utrecht, The Netherlands.

[1] C. W. Li, Phys. Rev. 88, 1038 (1952).