Possibility of a Zener Effect

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THE motion of an electron in a crystal under the action of a static uniform field is a problem which has been treated since the early days of quantum mechanics. New insight into it can be gained, however, if account is taken of the fact that the field modifies the band structure. An elegant procedure consists in constructing an auxiliary equation from the Schrödinger equation of the problem; this equation is obtained by replacing $\mathbf{x}$ by $\mathbf{x}+i\partial/\partial\mathbf{k}$ in the (scalar or vector) potential defining the field. This auxiliary equation is periodic and has Bloch-type solutions $b_q(\mathbf{x};\mathbf{k})$ with stationary energies $W_q(\mathbf{k})$. It is then found that if the state of a particle was at one time describable by a single function $b_q(\mathbf{x},\mathbf{k})$ this property is preserved in time, the complete solution being

$$\psi_q(\mathbf{x},t;\mathbf{k})=b_q\left(\mathbf{x};\mathbf{k}+\frac{e\mathbf{A}}{\hbar c}\right)\exp\left[-\frac{i}{\hbar}\int W_q\left(\mathbf{k}+\frac{e\mathbf{A}}{\hbar c}\right)dt\right].$$

In interpreting this formula, the antisymmetric gauge must be taken for the magnetic case and the time gauge for the electric case. (This is a different one from the one used in defining the b's.)

Little has been done up to this time in interpreting the result for the magnetic case. In the electric case, the result means that interband transitions have disappeared and that the entire change of the wave function in time is summed up by the linear increase of $\mathbf{k}$ with t. This implies that whenever bands are separated by energy gaps transitions across them are rigorously excluded by a selection rule; this situation presumably prevails for the usual field strengths. For those situations, therefore, the result of Zener[1] is wrong and no tunneling through the forbidden band is possible. It is possible to show, however, that for very high fields the band gaps all have disappeared; the above equation must then be read to imply a smooth acceleration from band to band, as for a free particle. Thus the Zener effect is to be replaced by a picture whereby no current whatever takes place up to a threshold field, whereupon conduction becomes metallic. As a restriction to this, it should be added that processes of the Zener type are still possible in crystals whose periodicity has been disturbed, for instance by lattice defects or the inhomogeneity of the field itself. The process is then of a higher order, however, with a probability which is decreased correspondingly.

In conclusion we want to add that we have not yet succeeded in constructing an example which demonstrates explicitly that the electric field closes the bands as it becomes large. This particular feature is therefore inferred from the other pieces of information.

I want to thank Dr. C. Herring for calling my attention to the qualitative change taking place at high fields.

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[1] C. Zener, Proc. Roy. Soc. (London) **A145**, 523 (1934).

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ABSTRACTS OF PAPERS PRESENTED AT THE EIGHTH ANNUAL GASEOUS ELECTRONICS CONFERENCE HELD AT SCHENECTADY, NEW YORK, OCTOBER 20–22, 1955*

A. Breakdown

A1. First and Second Townsend Coefficients in Hydrogen and Nitrogen.* D. J. DEBITETTO AND L. H. FISHER, *New York University.*—Ionization currents were measured in uniform fields in hydrogen and nitrogen at pressures from 100 to 400 and 300 mm Hg, respectively. Log i *vs* δ curves yielded accurate values of α/p in the low E/p region reproducible to 2%. Current-voltage measurements taken in the same gas samples to within 0.05% of the sparking potential check the consistency of the α/p measurements and provided numerous evaluations of γ. The values of γ are about 10^{-3} in both gases (nickel cathode) and are independent of E/p in the range covered. The values of γ are reproducible to within 20% and are not affected by breakdown or by large changes in initial current. At all pressures studied, self-sustained currents of the order of 10^{-7} amp were obtained in both gases and in nitrogen

* *Note.*—The usual preamble to these abstracts will be published in the Bulletin early in 1956. Abstracts of meetings in 1956 and subsequent years will not be published in *The Physical Review.*