and at the line center, taking a shape of the form

$$h(\omega,\omega_0)=(T_2^*/\pi)\exp[-T_2^{*2}(\omega-\omega_0)^2/\pi],$$

we have

$$\chi'(\omega)=\tfrac{1}{2}(0.62)\chi_0\omega T_2^*(\gamma H_m T_2^*)\cos(\omega_m t)+\tfrac{1}{4}\chi_0\omega T_2^*\epsilon\sin(\omega_m t)+O(\epsilon^3). \quad (2)$$

The lag angle, $\phi=\tan^{-1}(0.81/\gamma H_m T_2^*)\sim 45°$ for $\Delta H=1/\gamma T_2^*=19$ oersteds. In Fig. 3 we show the set of dispersion curves for KCl. The phase shift as determined from these curves is consistent with the above estimate. This confirms that rapid passage conditions are responsible for the observed effects.

It is a pleasure to thank Dr. F. Keffer for his interest in this work and Mr. D. Shaltiel for assisting with the preliminary measurements.

* This research was supported by the United States Air Force through the Office of Scientific Research of the Air Research and Development Command.

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Phenomenological Equations for Superconductors

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(Received May 9, 1955)

IN every physical system at finite temperature the momenta of distant particles are uncorrelated—that is, there exists a "correlation length" Λ such that particles which are farther apart than Λ have rapidly decreasing momentum correlation.[1] Λ is very large in a lattice of particles; for nearly freely moving particles Λ is usually of the order of several mean free paths against collisions, although there are exceptional systems in which it can be much longer. No matter what the size of the correlation length, its very existence has important effects on the behavior of a system in temperature equilibrium when this system is rotated uniformly, or when it is put into a magnetic field.

It has been shown[2] that London's equation,[3]

$$-\lambda c\,\mathrm{curl}\,\mathbf{j}_s=\mathbf{B}, \quad (1)$$

is incompatible with the existence of a finite correlation length; i.e., (1) implies an infinite correlation length (more precisely a correlation length proportional to the size of the container). This is illustrated by the fact that the only model known to obey Eq. (1), the ideal Bose-Einstein gas of charged particles below its condensation point,[4] does indeed have an infinite correlation length: the momenta of bosons occupying the ground state are correlated over the whole volume of the container.

It is therefore necessary to modify the London equation so as to be consistent with a finite correlation length Λ. This can be done by rewriting (1) in terms of the magnetization vector **M** rather than the supercurrent density vector $\mathbf{j}_s$. The two vectors are related through

$$\mathbf{j}_s=c\,\mathrm{curl}\mathbf{M}, \quad (2)$$

so that Eq. (1) becomes

$$-\lambda c^2\,\mathrm{curl}\,\mathrm{curl}\mathbf{M}=\mathbf{B}. \quad (3)$$

We now expand in an orthonormal set of vector eigenfunctions $\mathbf{u}_q(\mathbf{r})$ defined by

$$\mathrm{curl}\,\mathrm{curl}\mathbf{u}_q=q^2\mathbf{u}_q, \quad (4a)$$

$\mathbf{u}_q$ is normal to the surface of the superconductor, (4b)
and

$$\int(\mathbf{u}_q)^2d^3r=1, \quad (4c)$$

where the last integral extends over the volume of the superconductor. These functions form a complete set. We write

$$\mathbf{M}(\mathbf{r})=\textstyle\sum_q M_q\mathbf{u}_q(\mathbf{r}),\quad \mathbf{B}(\mathbf{r})=\sum_q B_q\mathbf{u}_q(\mathbf{r}). \quad (5)$$

Equation (3) becomes

$$M_q=-(\lambda c^2q^2)^{-1}B_q. \quad (6)$$

In this form the equation can be modified. We introduce a "kernel" $K(q)$ which is a (so far unknown) continuous function of q, and write

$$M_q=-(\lambda c^2)^{-1}K(q)B_q. \quad (7)$$

The values of q which appear in (7) are the discrete eigenvalues of Eq. (4) for the particular shape and volume of the superconductor under study. The construction of Eq. (7) in terms of the eigenfunctions (4) ensures automatically that the supercurrent is parallel to the superconductor at its surface, provided only that $K(q)$ decreases sufficiently rapidly for large q.

It can be shown that, in terms of the kernel $K(q)$, the existence of a finite correlation length implies that the quantity $qK(q)$ must not become infinite as q approaches 0. Thus the London equation (6) gives an infinite correlation length. An additional clue to the form of $K(q)$ can be derived from the observed agreement between the London equation and the Meissner-Ochsenfeld effect. This indicates that $K(q)$ must behave like q^{-2} in the relevant region of q, i.e., for $q\sim d^{-1}$, where $d=c(\lambda/4\pi)^{\frac{1}{2}}$ is the London penetration depth. A simple kernel which satisfies both requirements is

$$K(q)=1/[q(q+\mu)], \quad (8)$$

where μ is an inverse length, such that μ^{-1} is comparable to the correlation length Λ. In order that (8) be consistent with (1) in the region $q \sim d^{-1}$, the correlation length Λ must be much larger than the penetration depth d.

It is well known that London's second equation,

$$\lambda \partial \mathbf{j}_s / \partial t = \mathbf{E}, \tag{9}$$

cannot be derived from (1) in general. However, a partial derivation can be made as follows: We assume that all relaxation times are small compared to the time intervals of interest, so that we are allowed to differentiate the equilibrium equation (1) with respect to time. We then use Maxwell's equations to get the curl of (9). The divergence of both sides of (9) is automatically zero. The terms omitted in this quasi-derivation are precisely the ones responsible for supercurrent charge transport (constant vector fields in a simply connected region, "streamline flow" fields in a multiply connected superconductor). However, the method outlined here is presumably sufficient for the discussion of phenomena in which there is no actual charge transport. Straightforward application of this method to our modified equations (7) and (8) is therefore sufficient to discuss microwave measurements on superconductors.

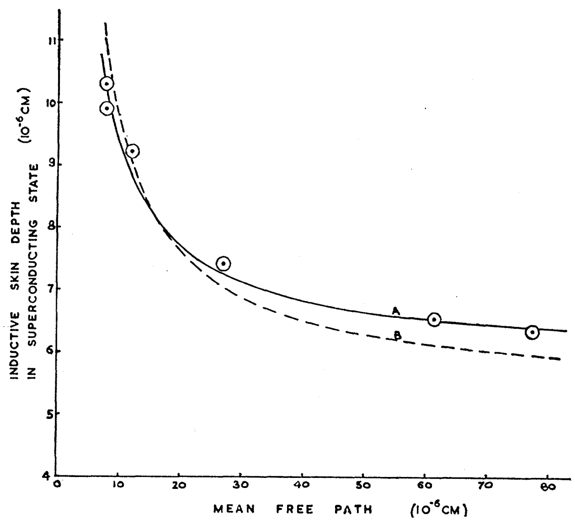

FIG. 1. Inductive skin depth d_{eff} of a superconductor *vs* mean free path ξ of the conduction electrons. The points are Pippard's measured values. Curve A is the best theoretical curve, with a correlation length $\Lambda \approx 3\xi$, and $\lim_{\xi\to\infty} d_{\text{eff}} \equiv d = 5.9 \times 10^{-6}$ cm. Curve B is the best curve under the assumption $d = 5.3 \times 10^{-6}$ cm, which is the value for very pure tin determined by Pippard in another measurement; the correlation length for curve B is $\Lambda \approx 2\xi$.

Equations (7) and (8) yield a static field penetration different from (1), but the penetration depth measured with small spheres in colloidal suspension remains equal to $d = c(\lambda/4\pi)^{\frac{1}{2}}$. At microwave frequencies, however, there are significant differences. The inductive skin depth determined from the inductive part of the surface impedance of superconductors is no longer equal to the statically measured penetration depth. The ratio between the inductive skin depth d_{eff} and the static penetration depth d can be used to determine the quantity μ in Eq. (8), i.e., to estimate the size of the correlation length Λ.

This provides a natural explanation for an effect found experimentally by Pippard.[5] Pippard observed a strong dependence of the measured penetration depth on the mean free path ξ of the electrons in the metal in its normal state. The measurements of Pippard are compared with our theory in Fig. 1. Curve A is the best fit and corresponds to a static penetration depth $d = 5.9 \times 10^{-6}$ cm, and $\mu\xi = 0.291$ (i.e., a correlation length $\Lambda \cong \mu^{-1}$ slightly more than 3 times the mean free path ξ). Not only is the ratio of correlation length to mean free path very reasonable, but the extrapolated static penetration depth d agrees quite well with Pippard's microwave value $d = 5.3 \times 10^{-6}$ cm in very pure tin (for which Λ is very large, and hence $d_{\text{eff}} = d$). It is difficult to determine whether the difference between these two values of d is significant, since the experiments are subject to considerable uncertainty. However, in curve B we show the best fit assuming $d = 5.3 \times 10^{-6}$ cm; the corresponding value of the product $\mu\xi = 0.493$, i.e., the correlation length Λ is equal to twice the mean free path ξ.

In view of the fact that the experimental measurements are subject to unknown errors, and that our theoretical kernel (8) is merely one possibility out of many others, this agreement between theory and experiment is entirely satisfactory.

It should be mentioned that the modified kernel (8) is basically different from the one proposed by Pippard himself in order to account for his measurements. Our modification cuts down the contributions from the *long*-wavelength parts ($q^{-1} \gg \Lambda$) of the magnetic field; Pippard's[5] equation (7) gives a smearing over small regions of space, i.e., a cutoff for the *short*-wavelength parts of the field. Pippard's equation, therefore, although in agreement with his experiments, contradicts the existence of a finite correlation length just as much as London's equation (1).

Similarly, Bardeen's[6] equation is inconsistent with a finite correlation length and can therefore not be accepted. Bardeen obtains his equation by assuming a finite energy gap between the lowest state of the electron gas and all excited states. Such an energy gap cannot occur in a system of finite correlation length.

We would like to thank Dr. S. T. Butler for valuable discussions.

* Also supported by the Nuclear Research Foundation within the University of Sydney.

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