however, are not completely independent, since the rotational data were used to supply the second-order correction.¹⁰ These values are also in good agreement with the earlier value of -2150 Mc/sec obtained by Sheridan and Gordy¹⁷ from observations of higher rotational transitions in the *K*-band.

¹⁷ J. Sheridan and W. Gordy, J. Chem. Phys. 20, 591 (1952).

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Threshold Law for Multiple Ionization

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On the basis of evidence from single ionization, an approximate threshold law for multiple ionization is derived.

TWO years ago, in a paper on the threshold law for single ionization¹ a serious effort was made to take into account the three-body character of the ionization process. It is a result of that calculation that the interaction of the two electrons is a relatively minor effect as far as the power law is concerned, the reason being that the two slow electrons, upon emerging from the ion, keep to opposite sides of it. A simplified type of reasoning which neglects the interaction of the emerging electrons outside the reaction zone has thus semiquantitative value. If it is applied to double ionization



FIG. 1. Region in which ionization occurs in the case of single ionization. T_1 and T_2 are the kinetic energies of the two emerging electrons, and ΔE is the energy excess.

a quadratic dependence of the yield on the energy excess results. This was communicated to Fox, Hickam, and Kjeldaas and is mentioned in their paper.² It appears that these ideas are of some interest to the experimental workers in the field, and the derivation of this approximate threshold law will therefore be presented here.

Draw around the ion a spherical surface (whose radius b is, as is usual in this type of argument, unspecified) and let the electrons emerge from the sphere with "switched off" mutual interaction. Each electron must then have a kinetic energy T greater than e^2/b in order to escape. In Fig. 1, the case of single ionization is illustrated. The kinetic energies T_1 and T_2 of the two emerging electrons are plotted on abscissa and ordinate. The region in which ionization occurs is limited by two straight lines parallel to the axes. Now a line of constant energy excess ΔE in this diagram is straight and of slope -1; the segment of it leading to ionization is marked in the figure. Clearly, the length of this segment is proportional to the energy excess and so is the yield unless the probability distribution has a strong anomaly in the corner. This possibility is highly improbable. If more than two electrons emerge from the reaction zone, then one can plot their respective kinetic energies in a space of higher dimensions, and reason as above. For the case of double ionization, we work in three dimensions: the straight line $\Delta E = \text{constant}$ becomes a plane and the segment a triangle. Its area, and thus the yield, varies as the square of the energy excess. For n-fold ionization, the threshold law thus becomes the nth power. One can reason by analogy to single ionization that the true exponent is probably slightly larger than n.

¹G. H. Wannier, Phys. Rev. 90, 817 (1953).

² Hickam, Fox, and Kjeldaas, Phys. Rev. 96, 65 (1954).