Magnetoresistance of Germanium-Silicon Alloys

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Magnetoresistance measurements of n-type germanium-silicon single crystals reveal a change in band structure occurring at about 14% silicon content. For silicon contents of less than 8%, the magnetoresistance is that of a crystal with ellipsoidal conduction minima along [111] axes in reciprocal lattice space. In the region 11–14% silicon, the magnetoresistance changes toward that expected for a crystal with ellipsoidal conduction minima along [100] axes. These results agree with Herman's speculations on the alloy band structure.

ALLOYS of germanium and silicon have been prepared[1–3] in homogeneous form, and a number of the physical and electrical properties measured. In particular, measurements of the optical band gap reported by Johnson and Christian[3] showed that the alloys have a forbidden band gap which may be represented by the equations:

$$E=0.72+1.5x \text{ ev} \tag{1}$$

in the region of x between 0 and 14%, and

$$E=0.89+0.31x \text{ ev} \tag{2}$$

in the region of x between 14 and 100%. Here x is the mole percent of silicon in the germanium.

To explain this behavior of the forbidden band gap, Herman suggested[4] that the alloys exhibit a band structure like germanium and silicon. Germanium has a conduction band with minima along the [111] axes[5] in the reduced zone; the conduction minima in silicon are along the [100] axes[5,6] in the reduced zone. Herman suggested that in germanium, beside the [111] states which are the lowest conduction minima, there are [100] minima about 0.17 ev above the [111] states. As we add silicon to form the alloys, both the [111] and the [100] minima move away from the valence band, at rates given respectively by Eqs. (1) and (2). Therefore, with less than 14% silicon we have conduction in [111] minima; for greater alloy compositions we would expect conduction in [100] minima.

In order to test this suggestion of Herman's, magnetoresistance measurements are being made on single-crystal samples of germanium-silicon alloys. This is a preliminary report on the results obtained to date.

For cubic crystals, the change of resistivity upon application of a magnetic field can be represented by the following expression[7]:

$$\frac{\Delta\rho}{\rho H^2}=b+c\frac{(\mathbf{I}\cdot\mathbf{H})^2}{I^2H^2}+d\frac{I_1^2H_1^2+I_2^2H_2^2+I_3^2H_3^2}{I^2H^2} \tag{3}$$

where ρ is the resistivity, **I** and **H** the electric current and magnetic field, respectively, with the components taken along the crystal axes. The constants b, c, and d can be determined from the measurement of the magnetoresistance for certain directions of **I** and **H**, and compared with theory. If we assume a relaxation time dependent on the energy only and conduction minima ellipsoidal in shape and oriented along the [100] axes of the [111] axes in the reduced zone, we obtain the following predictions[8]:

$$\begin{aligned}&[100]\text{ axes: } b+c+d=0,\quad d<0;\\ &[111]\text{ axes: } b+c=0,\quad d>0.\end{aligned} \tag{4}$$

To a first approximation, the data for silicon satisfy the first condition; those for germanium, the second. The constants b, c, and d are functions of the scattering law assumed, the ratio of the effective masses in the minima, and the Hall mobility R/ρ. However, the above symmetry relations [Eqs. (4)] do not depend on these quantities.

In the region where the two levels may be of the order of kT apart, both bands will contribute to the conduction. The magnetoresistance observed will be intermediate between the two cases. A calculation shows that there is no simple symmetry relation as in the cases where one set of valleys alone takes part. However, there is a condition among the constants b, c, and d which involves the effective masses of the valleys.

Oriented samples of germanium, silicon, and germanium-silicon alloys, n-type in conductivity, were cut in a bridge shape.[9] The resistivity, Hall coefficient, and magnetoresistance were measured at about 290°K. Details of the samples and results of the measurements

[1] H. Stöhr and W. Klemm, Z. anorg. u. allgem. Chem. **241**, 305 (1939).
[2] C. C. Wang and B. H. Alexander, American Institute of Mining and Metallurgical Engineers Symposium on Semiconductors, New York, February 15–18, 1954 (unpublished).
[3] E. R. Johnson and S. M. Christian, Phys. Rev. **95**, 560 (1954).
[4] F. Herman, Phys. Rev. **95**, 847 (1954).
[5] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955) and previous notes by both the California and the Lincoln Laboratory groups, referred to in above paper.
[6] G. L. Pearson and C. Herring, Physica **20**, 975 (1954).
[7] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).
[8] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954); M. Shibuya, Phys. Rev. **95**, 1385 (1954).
[9] Similar in shape to those used by Pearson and Suhl (reference 7), except that two additional probes were at the center of the sample bar and were used for Hall measurements.

TABLE I. Resistivity, Hall mobility, and magnetoresistance measurements on various samples of Ge, Si, and Ge-Si alloys.

Crystal	Resistivity (ohm-cm)	Hall mobility (cm^2/v-sec)	Magnetoresistance constants (10^6 cm^4/v^2-sec^2) b	c	d	Ratio of effective masses (m_l/m_t)
Ge (average for 5 samples)	5–7	4250	8.57	−7.99	17.99	12.3±3
106*T* (6% Si)	1.5	2330	2.31	−2.27	5.42	12.8±3
87*J* (7.5% Si)	0.6	2100	1.72			12
87*H* (8% Si)	0.9	2110	1.61	−1.43	3.60	9 ±3
87*E* (8.5% Si)	1.0	2100	1.57			$8\ {}^{+3}_{-2}$
106*H* (12% Si)	6.2	840	0.55	−0.45	0.70	
106*G* (13% Si)	8.3	580	0.29	−0.18	0.24	
106*D* (13.5% Si)	9.2	540	0.21	−0.11	0.12	
106*F* (14% Si)	9.4	510	0.17	−0.09	−0.10	
Si (average for 6 samples)	8–10	1570	1.59	0.55	−2.05	5.5

are given in Table I. The composition of the alloy samples was estimated indirectly from measurements of the optical band gap[3] and x-ray determination of the lattice spacing.[1,3]

The mobility of these samples falls off very rapidly with the addition of silicon; indeed at 14% we are down to a mobility one-third that of pure silicon. This is in sharp contrast to measurements of R/ρ for p-type samples, where the decrease is less rapid in this region.[10,11] Mobility measurements on the p-type samples have been interpreted[11] in terms of a combination of alloy scattering and a lattice scattering which varies continuously from germanium to silicon. The rapid decrease in the n-type mobility may be due to additional scattering processes or an abrupt change in the lattice scattering or a combination of these. Measurements of the temperature dependence of the mobility are being made in the hope of clarifying this situation.

It is possible to estimate K, the ratio of longitudinal to transverse effective masses from the data, in the case where only one set of valleys need be considered. These values are given in the last column in Table I. The value 5.5 for silicon is in good agreement with previous measurements. However, the value 12.3±3 for germanium is to be compared with a ratio of 14.6 given by Benedek[12] and the value 19.3 determined from the cyclotron resonance measurements.[5] The errors in the magnetoresistance value of K for germanium are large because of a high sensitivity of this constant to the experimental data. However, an examination of the values for the crystals in the range 6–8.5% silicon shows a tendency for K to decrease, from 12.8±3 at 6%, to

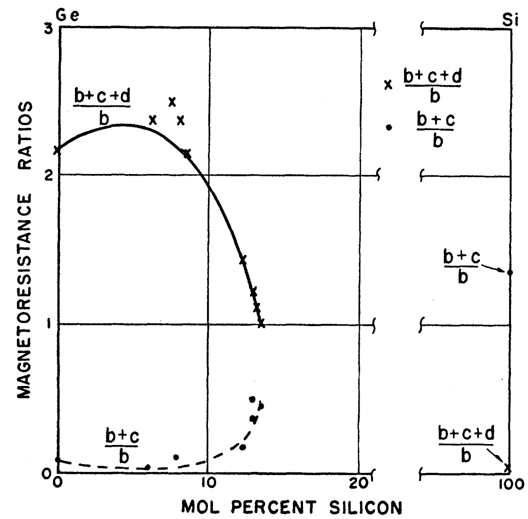

FIG. 1. The variation of the ratios $(b+c+d)/b$ and $(b+c)/b$ with composition in the germanium-silicon alloys. The curves drawn are for illustration purposes only.

8_{-2}^{+3} at 8.5%. This question will be examined in more detail when more data are available in the range 0–10%.

In the region 11.5–14% silicon, the magnetoresistance exhibits the changes in symmetry conditions expected for a competition of [111] and [100] minima. This behavior can be seen in Fig. 1, where are plotted $(b+c+d)/b$ and $(b+c)/b$ *versus* alloy composition. Since the constants b, c, and d are proportional to $(R/\rho)^2$, dividing by b eliminates the mobility variation from these functions. In the region 11–14%, $(b+c+d)/b$ drops off towards its value in silicon, whereas $(b+c)/b$ rises significantly from the low germanium value. This trend is well outside the estimated experimental error and tends to support Herman's speculations on the alloys' band structure.

If reasonable values are assumed for the effective masses of the valleys in this region, we can estimate the separation of the two sets of valleys. Some rough estimates indicate that samples 106*H*, 106*G*, 106*F*, and 106*D* have the [111] minima below the [100], with the two sets of the order of kT or less apart. The magnetoresistance data satisfy the new symmetry relation but are rather insensitive to changes in the effective masses of the [100] and [111] valleys. When more data become available, it may be possible to say something about the shape of the valleys in this region.

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[10] Similar measurements are being made on p-type alloy single crystals and will be reported later.
[11] A. Levitas, Phys. Rev. **99**, 1810 (1955).
[12] G. B. Benedek (private communication).