for $\omega > \omega_0$. In Eq. (35), ω_0 has the same meaning as before and $\tau = x_0 k_x^0 / 2\omega_0$ is the transit time. We now consider a number of limiting cases in order to appreciate the significance of Eq. (35). When $x_0 \rightarrow 0$, and, therefore, $\tau \rightarrow 0$, Eq. (35) reduces to the previous result of Eq. (33). In the classical limit $\hbar \rightarrow 0$ and $\omega_0 \rightarrow \infty$ in which case, Eq. (35) has the limit

$$\langle G_s(\nu)\rangle = 2e^2 \sin^2(\tfrac{1}{2}\omega\tau)/(\tfrac{1}{2}\omega\tau)^2. \qquad (36)$$

Equation (36) is the familiar classical result for this case.

In the case of $x_0 = 0$, the previous result of Eq. (33) for the spectral density was singular at $\omega = \omega_0$. The present result for finite x_0 also has this property. Also, for $x_0 = 0$, $\lim_{\nu\rightarrow\infty}\langle G_s(\nu)\rangle = \infty$. In the case of finite x_0, $\lim_{\nu\rightarrow\infty}\langle G_s(\nu)\rangle = 0$. In Fig. 2, some numerical calculations of the spectral density for various cases are shown.

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Transmission of Positrons and Electrons*†

H. H. SELIGER
National Bureau of Standards, Washington, D. C.

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The transmissions of monoenergetic beams of positrons and electrons with energies up to 960 kev have been measured in aluminum, brass, silver, tin, lead, and gold. The absorber forms the window of a 2π counter whose counting efficiency is better than 99% down to a few hundred electron volts. Particles from a radioactive source, focused into a beam by a 90-degree magnetic analyzer impinge perpendicularly on the absorber window of the 2π counter. The total transmission is therefore measured independently of forward angle of emergence or of partial energy loss. Positrons are found to be transmitted to a greater extent than electrons except at low energies in aluminum. These results are correlated with previous backscattering experiments and are in qualitative agreement with theoretical calculations of Rohrlich and Carlson. The shapes of the transmission curves are compared semiquantitatively with predictions of the Spencer theory of electron penetration.

I. INTRODUCTION

THE author has previously reported an excess of electron backscattering over positron backscattering.[1–3] The question naturally arises as to whether, in view of their excess backscattering, electrons are transmitted to a lesser degree than positrons. As will be shown in this paper, a lower transmission of electrons is usually, but not always, observed.

The problem of the penetration of positrons and electrons in thick foils has not been calculated at the present time. Bothe[4] has made estimates of the backscattering of electrons using nonrelativistic single-scattering cross sections. Miller[5] used Bothe's results, substituting the relativistic cross sections obtained by Bartlett and Watson[6] and Massey[7] for electrons and positrons, respectively, to show that an excess of electron backscattering over positron backscattering is to be expected. Rohrlich and Carlson[8] have published the results of theoretical calculations of range and stopping power of positrons and electrons, providing a qualitative interpretation of the experimental results to be reported here.

Recently, a theory of electron penetration in infinite media has been developed by Spencer.[9] This theory does not quite apply to the present experiments because

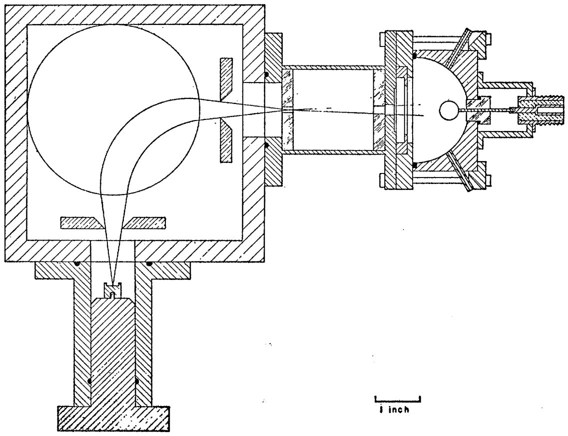

FIG. 1. Scale drawing of transmission geometry. The Lucite lining is not shown.

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* This work was reported at the Washington, D. C., meeting of the American Physical Society in May, 1954 [H. H. Seliger, Phys. Rev. **95**, 610(A) (1954)].

† This paper is a portion of a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Maryland.

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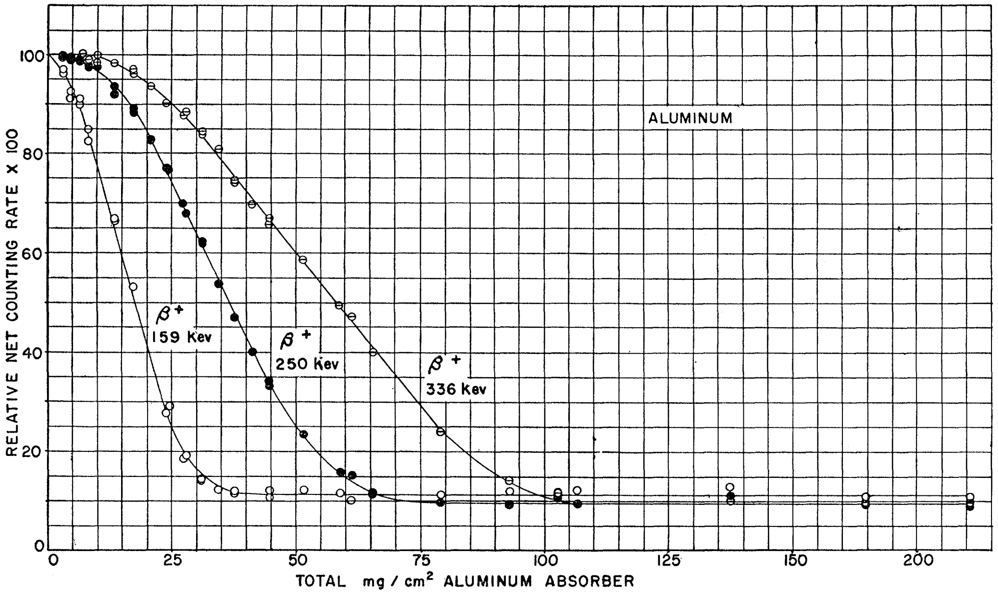

FIG. 2. Curves showing $N(x)/N(0)$ for positrons in aluminum.

it disregards the effect of transitions between the foil material and the surrounding gas. Nevertheless, it provides a frame for a semiquantitative analysis of the experimental data.

II. EXPERIMENTAL PROCEDURE

The purpose of the present experiments was to compare the transmissions of monoenergetic collimated beams of positrons and electrons. The only suitable long-lived positron emitter available was Na^{22}; this fact limited the range of electron energies used in the initial measurements. At energies lower than 150 kev, the attenuation was rapid, especially for higher-Z absorbers, and accuracy was therefore difficult to obtain. Data were therefore taken initially between 159 kev and 360 kev for both positrons and electrons. Subsequently, transmission measurements were made at 960 kev using 9-hour Ga^{66} and long-lived $Ce^{144}-Pr^{144}$ as sources of positrons and electrons, respectively.

In view of the large backscattering differences previously observed, a momentum resolution of 5 percent was assumed to be adequate to demonstrate any transmission differences. A 90-degree magnetic analyzer was designed, based on improvements suggested by

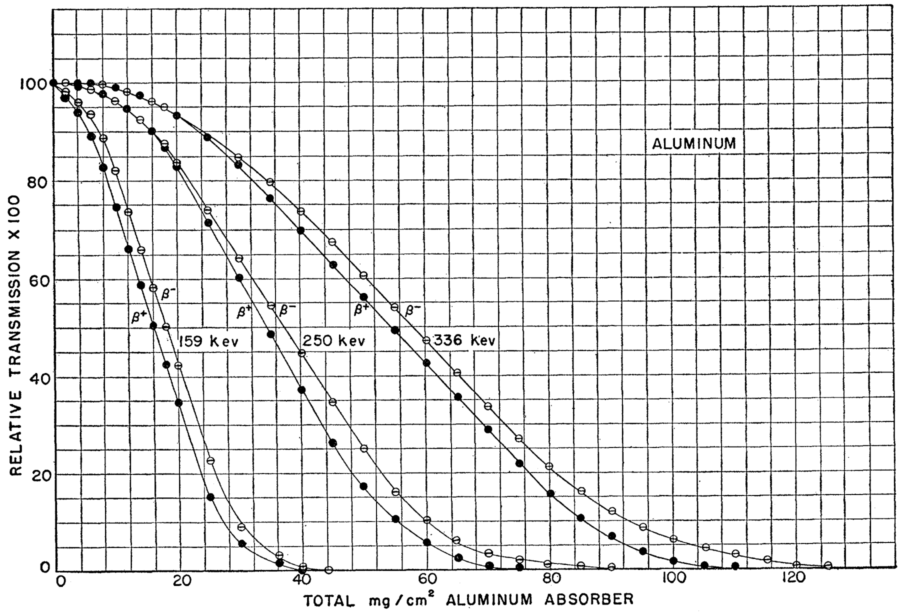

FIG. 3. Relative transmissions of positrons and electrons in aluminum.

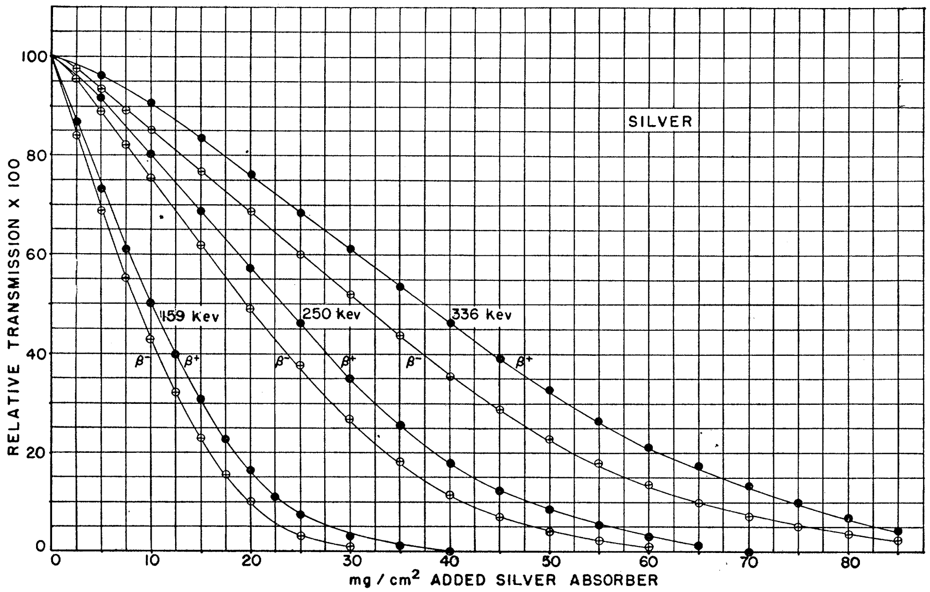

FIG. 4. Relative transmissions of positrons and electrons in silver.

Kerwin[10] and Hintenberger.[11] Second-order focusing conditions in the median plane were obtained with circular pole faces of only 10 centimeters diameter. All inside surfaces of the analyzer chamber were lined with $\frac{1}{16}$-in. Lucite to minimize scattering. The analyzer beam was collimated into a cone of half-angle 2 degrees whose central axis impinged perpendicularly on the face of an absorber which formed the window of a 2π absolute beta counter. All particles emerging from the opposite face of the absorber were detected regardless of energy loss down to an energy of a few hundred electron volts. The techniques of absolute beta counting which made the experiments feasible have been described previously.[12,13]

Figure 1 is a top-view scale drawing of the experimental arrangement showing source, pole faces, particle trajectories, collimating holes, and the 2π counter in position.

The absorbers were introduced into the beam by means of a sliding window assembly. The 2π counter

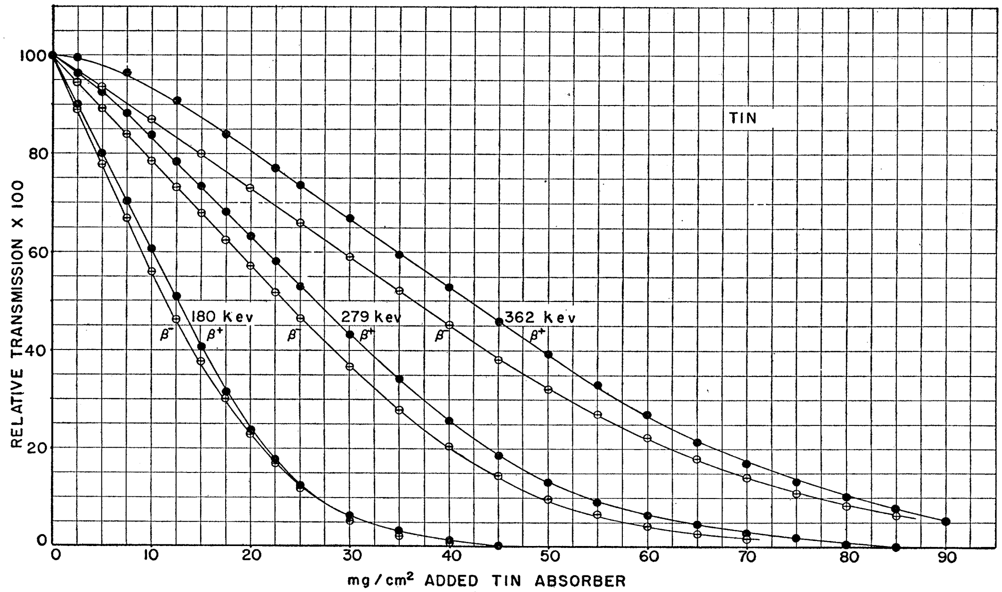

FIG. 5. Relative transmissions of positrons and electrons in tin.

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[10] L. Kerwin, Rev. Sci. Instr. **20**, 36, 381 (1949).

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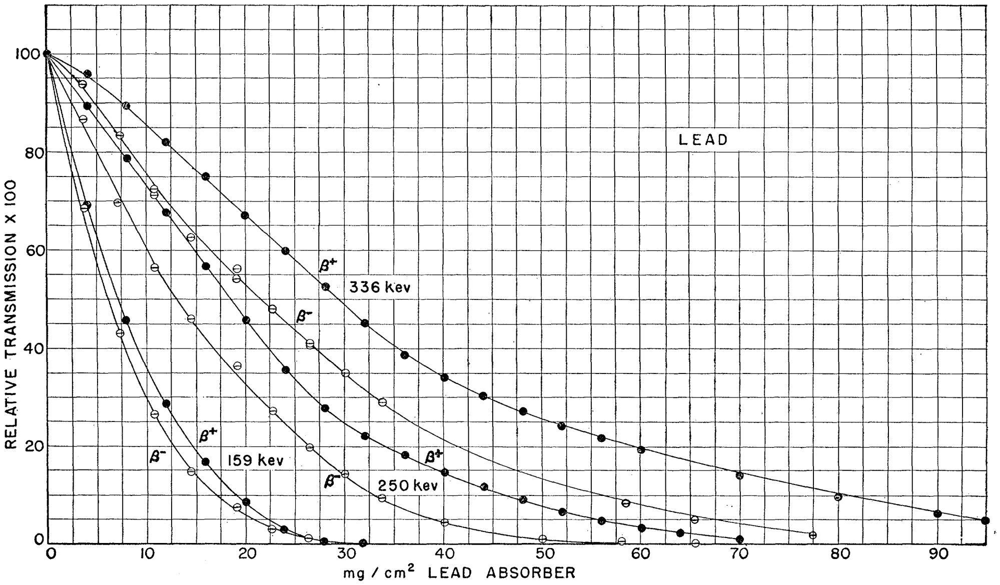

FIG. 6. Relative transmissions of positrons and electrons in lead.

was clamped on the outside of this assembly so that an absorber, placed in the path of the beam by sliding the window into position, formed the window of the 2π counter. The uniformity of the beam was verified by measuring the counting rate along the horizontal diameter of the exit hole, using a lead diaphragm with a 0.028-inch central hole.

The magnet constant-current control circuit was basically a voltage regulator. The current was read directly from a $\frac{1}{2}$ percent 0–300 milliampere meter.

The magnetic fields for both positrons and electrons were calibrated with a synchronous-motor rotating-coil assembly. The output of the rotating coil was rectified and then measured with a Type *K* potentiometer. In the course of the experiments, the magnet coil currents were varied from 52 milliamperes to 152 milliamperes. All currents were obtained from zero in a positive direction. The momentum calibration was obtained from the $H\rho = 3367$ gauss-cm conversion line of Ba^{137}.

The exit port of the spectrometer was sealed by a 1.8-mg/cm² aluminum foil. A pressure of the order of 0.1 micron was maintained for all measurements.

Thin films of aluminum and thicker films of brass and gold were available. For silver, tin, and lead, it was necessary to prepare thin films of the order of 1 mg/cm² by vacuum evaporation. The techniques used are thought to warrant a brief description.

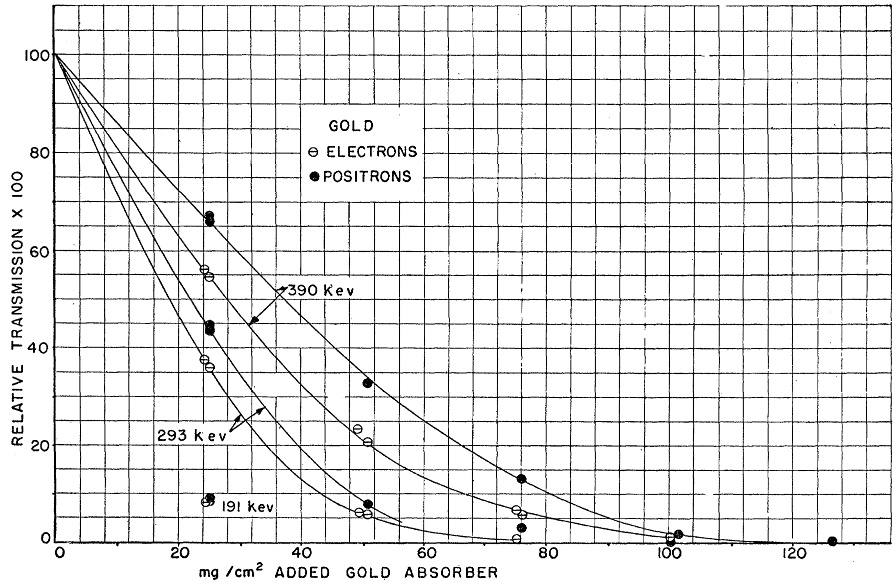

FIG. 7. Relative transmissions of positrons and electrons in gold.

The direct evaporation of the metal films onto thin (5–10 micrograms per square centimeter) plastic films was not too successful because of the formation of pin holes, owing to minute dust particles on the plastic film during drying causing weak spots in the film. Uniform lead films could be floated off onto water from the surfaces of clean microscope slides on which sodium chloride had previously been vacuum evaporated. These films could then be picked up and mounted on aluminum-disk absorber mounts. The above technique was possible because the thin lead films remained ductile.

The silver films were brittle and did not respond to the sodium chloride technique. The water, in lifting up part of the film, cracked and split the film. However, if silver is evaporated directly onto the grease-free surface of blued spring steel, films of the order of 1 in.×1 in. can be peeled off the surface of the steel by gentle flexing under water.

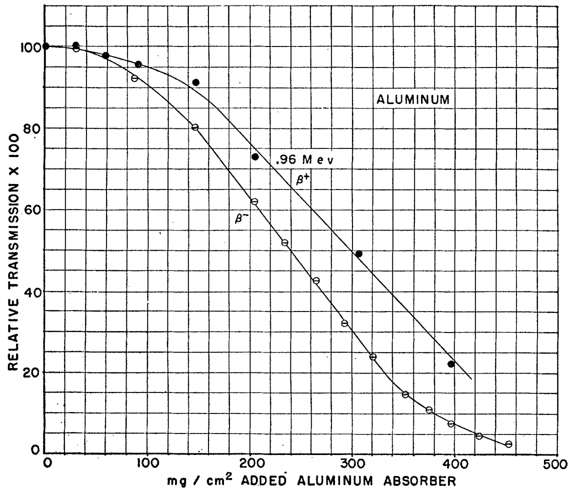

FIG. 8. Relative transmissions of 960-kev positrons and electrons in aluminum.

In the case of tin, it appeared that some lead was always present. Owing to the lower melting point of lead relative to tin, the thin tin films probably had a much higher percentage of lead than the original tin foil used as the evaporating source.

Cs^{137} and $Ce^{144}-Pr^{144}$ were used as sources of electrons and Na^{22} and Ga^{66} were used as sources of positrons. The sources were prepared by evaporation in air directly onto a cupped source mount. Activities of the order of 1–2 millicuries were necessary. All nuclides, with the exception of Ga^{66}, were obtained from the Oak Ridge National Laboratory. The Ga^{66} was prepared by a $Cu^{63}(\alpha,n)Ga^{66}$ reaction in the cyclotron of the Carnegie Institute of Washington. The chemical separation of gallium was made with ethyl ether.

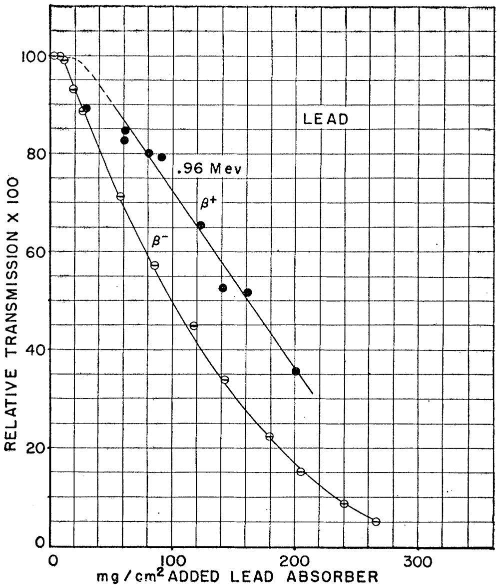

FIG. 9. Relative transmissions of 960-kev positrons and electrons in lead.

III. MEASUREMENTS AND RESULTS

The transmissions of positrons and electrons were measured for discrete energies ranging from 159 kev to 960 kev. The sequence of measurement was as follows:

1. The absorber was placed in position in front of the 2π counter and a number of readings of counting rate were taken for zero magnetic field ($I=0$). Owing to the low transmission of the analyzer and the use of Lucite to minimize scattering, the net counting rate in the 2π counter for $I=0$ was the same as for fields sufficiently high to deflect all particles past the exit hole ($I=\infty$). These readings at $I=0$ constituted the background for the particular absorber used. This point is important, for an absorber in front of the 2π counter in the presence of gamma rays acts as a radiator. The background, as expected, increased slightly with thickness of absorber and with Z.

2. The magnetic field was increased from zero to the

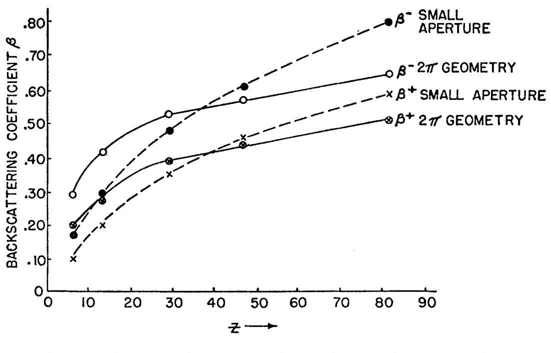

FIG. 10. Backscattering coefficients for positrons and electrons as functions of Z, measured with 2π geometry (solid curves) and with low geometry (dashed curves).

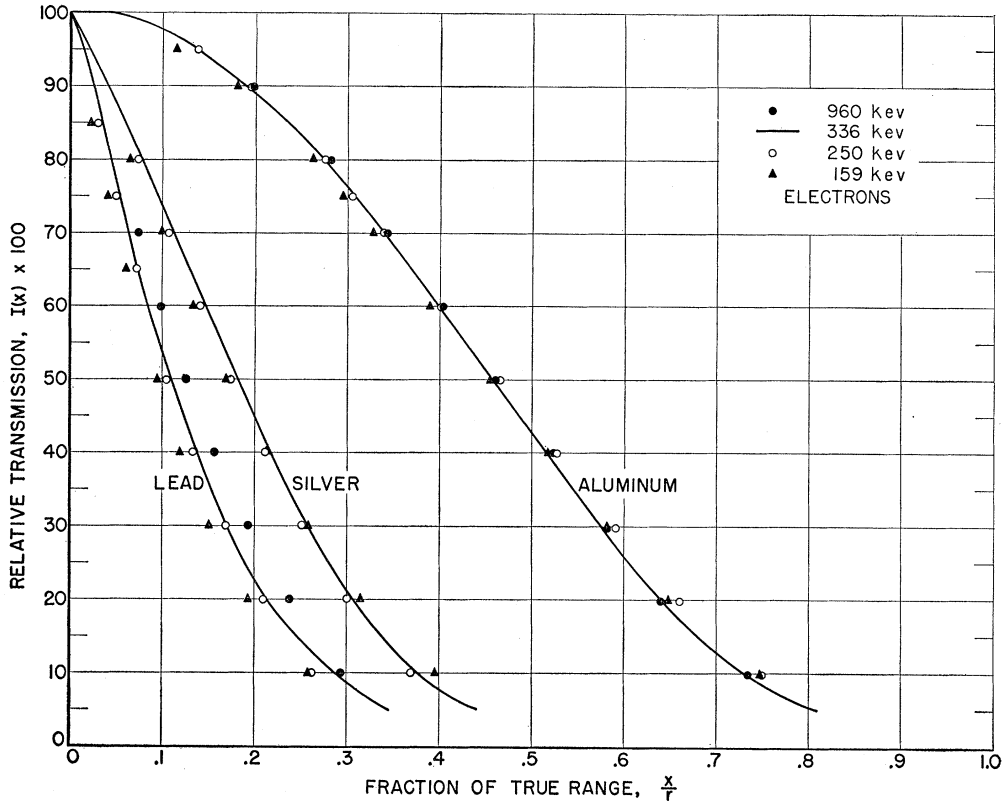

FIG. 11. Transmissions of electrons in aluminum, silver, and lead as functions of the fractional true range.

appropriate value and readings of counting rate were obtained.

The magnetic field was then increased for the next energy under consideration and a like set of readings was obtained.

3. Steps 1 and 2 were repeated for absorbers of different Z varying in thickness from 1 mg/cm^2 to 500 milligrams per square centimeter, depending on the beam energy.

The relative transmission of electrons is defined by

$$T_{\beta^-}=N(x)/N(0), \tag{1}$$

where $N(x)$ is the net observed counting rate for given absorber of thickness x and $N(0)$ is the net observed counting rate at "zero added absorber."

Equation (1) is not entirely correct for positrons because of annihilation radiation from positrons stopped in the absorber window. If $N'(x)$ is the number of positrons transmitted per unit time through the thickness x and $\epsilon_{\gamma a}$ the efficiency of the 2π counter for the detection of annihilation radiation originating at the absorber and defined by

$$\epsilon_{\gamma a}=N(\infty)/N(0), \tag{2}$$

we have

$$N(x)=N'(x)+[N(0)-N'(x)]\epsilon_{\gamma a}. \tag{3}$$

Thus, the relative transmission T_{β^+} is given by

$$T_{\beta^+}=\frac{N'(x)}{N(0)}=\frac{N(x)-N(\infty)}{N(0)-N(\infty)}=\frac{[N(x)/N(0)]-\epsilon_{\gamma a}}{1-\epsilon_{\gamma}}. \tag{4}$$

The magnitude of $\epsilon_{\gamma a}$ in the case of positron transmission in aluminum is shown in Fig. 2 where the ratios $N(x)/N(0)$ are plotted as functions of absorber thickness. The flat portions at ordinate 10 percent are $N(\infty)/N(0)=\epsilon_{\gamma a}$. The same data corrected for $\epsilon_{\gamma a}$ are shown in Fig. 3, together with T_{β^-} for electrons having the same initial energy. Similar transmission curves for silver, tin, lead, and gold are shown in Figs. 4, 5, 6, and 7, respectively.

Brass was used to observe the transmission trend in the intermediate region of atomic number. For thicknesses of 22.0 mg/cm^2 and 43.8 mg/cm^2, positron transmission exceeded electron transmission at all energies.

Transmission curves were also measured in aluminum and in lead at 960 kev and are shown in Figs. 8 and 9, respectively. In this case Ga^{66} was the positron source and Ce^{144}—Pr^{144} was the electron source.

IV. DISCUSSION

In the absence of knowledge of any differences in positron-electron stopping-power, one assumes that electrons, being backscattered to a greater degree than positrons, are transmitted to a lesser degree. Any reversal of this trend should be explained on the basis of stopping-power differences. Thus, the results of Fig. 3 for aluminum imply a larger stopping power for positrons than for electrons.

The excess of electron backscattering over positron backscattering observed in the previous experiments

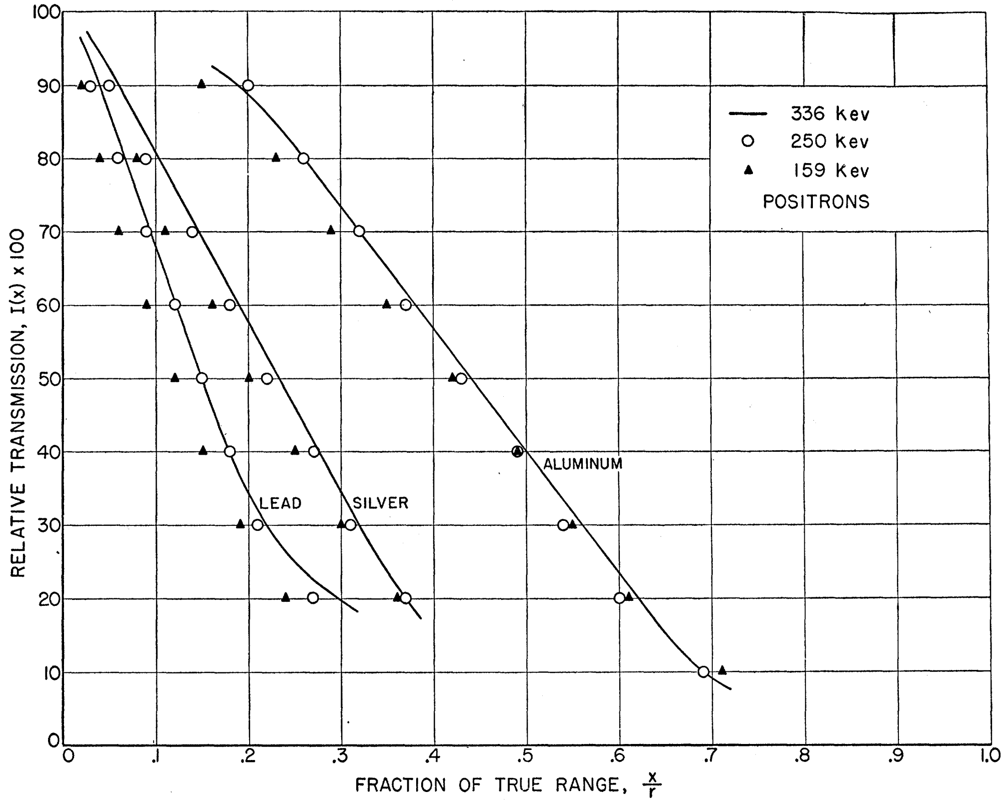

FIG. 12. Transmissions of positrons in aluminum, silver, and lead as functions of the fractional true range.

was explained on the basis of the cumulative effect in multiple scattering of the larger electron single-scattering cross sections. The experimental ratio of electron backscattering to positron backscattering, β^-/β^+, of 1.3 is in qualitative agreement with Miller's calculations. However, on the basis of scattering alone, β^-/β^+ should approach unity at low Z. That such is not the case is evident from the solid curves of Fig. 10, where β^-/β^+ for aluminum is even greater than for lead. This would attribute the large excess of electron backscattering at low Z to a greater stopping of positrons rather than to an excess scattering of electrons.

From the transmission data in aluminum given in Figs. 3 and 8, it follows that a cross over in transmission between positrons and electrons occurs between 336 kev and 960 kev. In all other cases studied the transmission of electrons was lower than that of positrons.

Rohrlich and Carlson[8] have calculated positron-electron differences in energy loss and in multiple scattering in aluminum, tin, and lead, using the correct elastic and inelastic scattering cross sections. They find that, below 350-kev, positrons lose energy at a faster rate than electrons. This, coupled with the fact that at low Z the scattering of electrons is only very slightly greater than that of positrons, implies that at low energy and for low Z it is possible for a greater stopping power for positrons to overshadow a small excess scattering of electrons. At higher Z, the excess scattering (proportional to Z^2) overcomes any energy-loss differences. For energies above 345 kev, the stopping power for positrons becomes *smaller* than for electrons and works *with* the excess scattering of electrons to make positron transmission greater than electron transmission.

The multiple-scattering effect can be evaluated in terms of $\langle\cos\theta\rangle_{\text{Av}}$, the average obliquity of the electron trajectories at a path length s after entrance into the medium. This is given by

$$\langle\cos\theta\rangle_{\text{Av}}=\exp\left[-2\pi N\int_0^s ds'\int_{-1}^1 \sigma(\theta,s')\times(1-\cos\theta)d(\cos\theta)\right]. \quad (5)$$

A distance of penetration z_d at which the particles have lost their initial orientation can be defined by the condition $\langle\cos\theta\rangle_{\text{Av}}=1/e$, that is,

$$2\pi N\int_0^{z_d} ds'\int_{-1}^1 \sigma(\theta,s')(1-\cos\theta)d(\cos\theta)=1. \quad (6)$$

Rohrlich and Carlson use z_d as an index of the ability to penetrate. The larger the value of z_d, the farther the particles penetrate before being turned around and diffused. For an initial energy of 358 kev, $z_d^+/z_d^-=1.04$ for aluminum and $z_d^+/z_d^-=1.35$ for lead. The present experimental results also show the increase of the ratio of positron transmission to electron transmission with

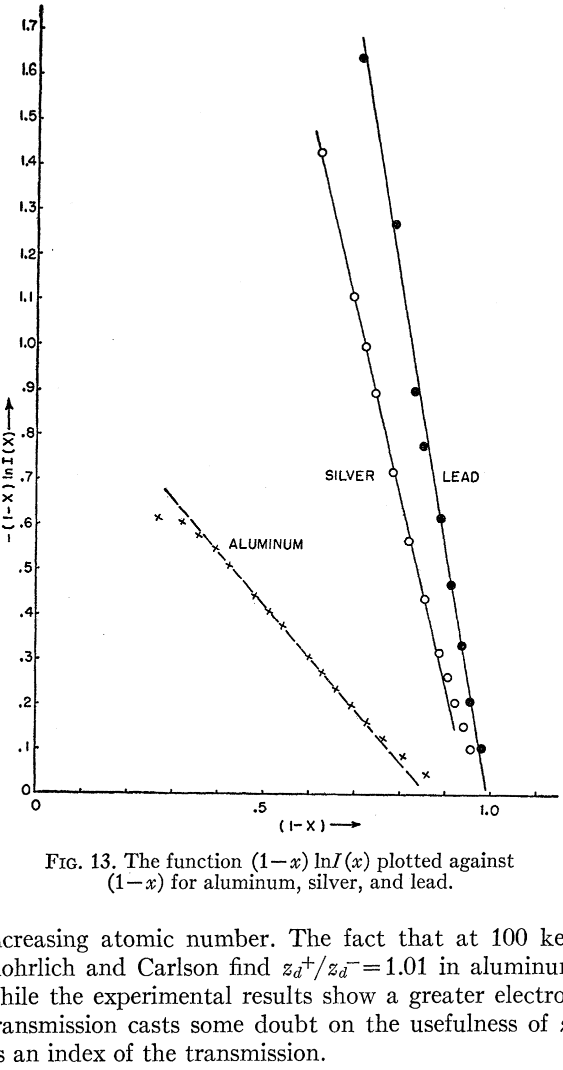

FIG. 13. The function $(1-x)\ \ln I(x)$ plotted against $(1-x)$ for aluminum, silver, and lead.

increasing atomic number. The fact that at 100 kev Rohrlich and Carlson find $z_d^+/z_d^-=1.01$ in aluminum while the experimental results show a greater electron transmission casts some doubt on the usefulness of z_d as an index of the transmission.

There are some further points that can be made regarding the shapes of the transmission curves. These concern (a) the effect of single scattering on the initial slope of the transmission curve, (b) the constancy of the shape of the transmission curves as functions of x/r_0 = absorber thickness/true range, and (c) the comparison of the transmission curves at deep penetrations with the predictions of the Spencer theory for infinite media.

The Rutherford single-scattering cross section modified for screening, inelastic collisions, and relativistic effects is given as

$$2\pi\sigma[r(T),\theta]=\tfrac{3}{4}(Z+1)\left(N_A\phi_0\frac{Z}{A}\right)\times\left[\frac{T+1}{T(T+2)}\right]^2(1+2\eta-\cos\theta)^{-2}, \quad (6)$$

where N_A is Avogadro's number, A is the atomic weight and η is given by the formula of Molière.[14] Integration of the single-scattering cross section from 90° to 180°, i.e., single scattering out of the forward direction, gives an estimate of the contribution of single scattering to the initial slope of the transmission curve. From this single-scattering initial slope, a thickness for 0.99 transmission can be derived. Table I is a summary of the experimentally observed transmissions at these calculated thicknesses. The departures from 0.99 transmission show the extent to which the multiple scattering is taking hold, even at small penetration thicknesses.

In order to study the shape of the transmission curves as a function of the initial energy of the electrons, the transmission data were replotted with absorber thickness in units of fractional true range x/r_0, the true range r_0 being interpolated from data in reference 9. The electron curves for 159 kev, 250 kev, 336 kev, and 960 kev are shown in Fig. 11 for aluminum, silver, and lead. The shape appears to be nearly a constant for the 150–350 kev range. The constancy of shape with energy implies a balancing of energy loss and scattering or "scaling" in this energy range. It was just this "scaling" property that was *assumed* in order to simplify the discussion of the previous backscattering results for heteroenergetic sources of electrons and positrons.[15] The corresponding curves for positrons, using positron true ranges calculated by Nelms,[16] are shown in Fig. 12.

In the Spencer theory, which neglects range straggling, the flux of electrons at deep penetrations has the form

$$I(x)=C(1-x)^{-\xi}e^{-A/(1-x)}, \quad (7)$$

where A is a function of source energy and scattering material, ξ is a constant, [equal to $\frac{3}{2}$] and x is the fraction of the true range traversed. In Fig. 13, the quantity $(1-x)\ \ln I(x)$ is plotted as a function of $(1-x)$ for electrons in aluminum, silver, and lead. Because of the "scaling" property, only one energy needed to be considered for each absorbing material. The curves can be approximated by straight lines over a large portion of their range, in qualitative agreement with the Spencer theory with $\xi=0$. The deviation at small absorber thicknesses in aluminum is due to the fact that the theory is an asymptotic one and the tail at large ab-

TABLE I. Experimentally observed transmissions at thicknesses calculated for 0.99 transmission based on single scattering.

	Aluminum		Silver		Lead	
Electron energy (kev)	Calc thickness for 0.99 trans. mg/cm²	Observed transmission	Calc thickness for 0.99 trans. mg/cm²	Observed transmission	Calc thickness for 0.99 trans. mg/cm²	Observed transmission
159	3.2	0.98	0.98	0.94	0.62	0.93
250	7.1	0.98	2.2	0.96	1.4	0.94
336	11.8	0.98	3.6	0.96	2.3	0.96

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sorber thicknesses is probably due to range straggling, which has been neglected in the theory.

The present data are in qualitative agreement with the results of Bascova and Dzhelepov,[17] who measured a higher positron to electron transmission in lead and with the measurements of Chang, Cook, and Primakoff[18] on positron-electron transmission through a 10.16-mg/cm^2 platinum end-window G-M counter. The latter workers made measurements in aluminum only at 10.83 mg/cm^2 and no significant transmission difference between positrons and electrons was observed. Thicker aluminum absorbers might have shown up the difference.

V. ACKNOWLEDGMENTS

The author wishes to thank Dr. U. Fano and Dr. L. V. Spencer for their helpful suggestions and advice on the analysis of these results, Dr. R. W. Hayward for his suggestions on the experimental arrangement, Dr. M. A. Tuve and Dr. G. M. Temmer of the Department of Terrestrial Magnetism, Carnegie Institute of Washington, for the use of the D.T.M. cyclotron for the production of Ga^{66}, and Dr. W. B. Mann for his continued interest and encouragement both during and after these experiments.

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PHYSICAL REVIEW VOLUME 100, NUMBER 4 NOVEMBER 15, 1955

Interaction of Electromagnetic Waves of Radio-Frequency in Isothermal Plasmas: Collision Cross Section of Helium Atoms and Ions for Electrons*

J. M. Anderson and L. Goldstein

Department of Electrical Engineering, University of Illinois, Urbana, Illinois

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The elements of the theory of interaction of radio-frequency electromagnetic waves in gaseous discharge plasmas are reviewed. The applicability of this phenomenon of wave interaction to the measurement of quantities involved in fundamental processes in such plasmas is considered.

A measurement of the "effective" electron-atom collision frequency in helium, found to be 3.12×10^8/sec/mmHg for room temperature electrons ($\sim$300°K), gives a value for the probability of collision for momentum transfer, P_m, of the helium atom equal to 24.0 cm^2/cm^3 at 0°C and 1 mm Hg.

An experimental determination of the "effective" electron-ion collision frequency (for singly charged ions) has also been determined which closely agrees with earlier theoretical predictions. The ratio of the effective cross sections, associated with electron collisions, of ions and helium atoms in the room temperature isothermal plasma is $\sim3.0\times10^5$ at an ion density of 10^{11}/cm^3.

INTRODUCTION

It has been known for some time[1] that when two low- or medium-frequency radio waves traverse a common region of the ionosphere, and one of the waves is of sufficient strength, an interaction between the two electromagnetic waves is observed. When the stronger wave is amplitude modulated, there is a measurable transfer of modulation from the stronger wave to the carrier of the weaker. Bailey and Martyn[2] proposed a theory for this phenomenon of "ionospheric cross-modulation."

According to this theory, absorption of the interfering waves increases the mean energy of the electrons in the appropriate region of the ionosphere. This increased mean energy of the electrons leads to a change in the collision frequency of the electrons. As the absorption of an electromagnetic wave in a region of the ionosphere is determined, in part, by the collision frequency of the electrons in that region, the absorption of the "wanted" wave is modified by the presence of the absorbed "disturbing" wave. Bailey[3] later extended this theory by considering the effect of the earth's magnetic field. According to this more complete theory, an enhancement of transfered modulation at radio-frequencies corresponding to the gyrofrequency of the electrons was predicted which has since been observed experimentally.[4,5]

We have recently[6] described experiments of interaction between microwaves, which are simultaneously propagated in gaseous discharge plasmas. These controllable laboratory experiments offer a method of more general exploration of the fundamental processes involved in these phenomena. The purpose of this paper is then twofold: First to describe the experimental techniques relative to the method, and second, to

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* This work was supported by Air Force Cambridge Research Center.

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