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Independent-Particle Model of the Nucleus. II. Weak Surface Coupling*

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The role of weak surface coupling is discussed for multiparticle nuclear configurations. Matrix elements are given for the standard form of the coupling of nucleons to nuclear surface vibrations. Energy spacings, configuration interaction, magnetic moments, quadrupole moments, and $E2$ transition rates are discussed, with emphasis on features independent of special details of particle configurations. Magnetic moments, in particular, become orderly when account is taken of variations of surface coupling strength together with a directly induced configuration mixing. Experimental evidence favors the idea that a true weak coupling situation exists for many nuclei near closed shells. In these nuclei, collective quadrupole effects may be appreciable while all other collective effects are negligible.

I. INTRODUCTION

THE possibly important role in low-energy nuclear phenomena of coupling of nucleons to the nuclear surface has first been pointed out by Foldy and Milford[1] in connection with magnetic moments, and by Rainwater[2] in connection with quadrupole moments. The existence of such surface coupling is inescapable if the nucleons within the nucleus behave in any way like independent particles: the nucleus is certainly deformable, and the (linear) variation of independent particle energy with nuclear deformation leads to surface coupling. In this sense surface coupling and the collective model of the nucleus are automatic consequences of the shell model. In view of the strong evidence for the independent particle behavior of nucleons in the nucleus, the relative importance rather than the existence of surface coupling is in question. Elementary calculation indicates that surface coupling energies may be appreciable relative to direct particle coupling energies for deformations of only a few percent, and one is led to expect surface coupling to play a dominant role in heavy nuclei with more than a few nucleons outside closed shells. Very eloquent and complete discussions of the properties of strongly deformed nuclei have been given by Hill and Wheeler[3] and by Bohr and Mottelson,[4] the latter authors working from the very fruitful strong coupling approximation of Bohr.[5] Certain predictions of the strong coupling model, those regarding the nuclear rotational states,[4,6,7] have been startlingly well confirmed in two regions of the periodic table, comprising roughly neutron numbers 90 to 115 and 135 or more. The agreement with experiment has come in energy level ratios,[4,8] in transition rates,[4,8,9] and in correlations among energies, quadrupole moments,[7] transition rates,[4,10] and atomic isotope shifts.[11] Some important quantitative discrepancies remain,[10,12] but it seems to be well established that nuclei in these regions are very substantially deformed from the spherical shape, which is most easily explained in terms of strong coupling of nucleons to the nuclear surface.

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[2] J. Rainwater, Phys. Rev. **79**, 432 (1950).
[3] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953); see also J. Griffin, Bull. Am. Phys. Soc. **30**, No. 3, 48 (1955).
[4] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
[5] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, No. 14 (1952).
[6] A. Bohr, *Rotational States of Atomic Nuclei* (E. Munksgaard, Copenhagen, 1954).
[7] K. W. Ford, Phys. Rev. **90**, 29 (1953).
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[11] Wilets, Hill, and Ford, Phys. Rev. **90**, 1388 (1953).
[12] K. W. Ford, Phys. Rev. **95**, 1250 (1954).

In contrast, there is little experimental evidence for the existence in any nuclei of a weak (but appreciable) coupling of nucleons to nuclear surface. Some rather successful applications of the shell model have been made[13,14] near double closed shells without reference to surface coupling. One motivation in undertaking the present work was the desire to learn whether the existence of weak surface coupling near closed shells or in light nuclei could be detected. It is theoretically expected[3] that the nuclear surface tension, and therefore the strength of particle-to-surface coupling, should show a marked dependence on the shell structure, the coupling being a minimum near closed shells. An analysis of quadrupole moments in terms of single particle effect plus collective deformation[15] indicates that this is the case. More detailed studies[16] near the double closed shell nucleus Pb^{208} and, in a succeeding paper,[17] Ca^{40}, indicate that the effective surface tensions for these doubly-magic cores are at least several times the "hydrodynamic" values,[4] while deviations from pure rotational spectra indicate that the hydrodynamic values are approximately correct in the region of strong coupling.[18] Evidence so far, both theoretical and experimental, then leads to the interesting conclusion that the strength of surface coupling is self-reinforcing, being very strong when strong and almost negligible when weak.

Because of the variation of nuclear surface tension with shell structure, it seems reasonable to expect that there will exist nuclei for which a treatment of weak surface coupling with only one or two phonons of surface excitation included will be adequate. Previous work on weak surface coupling[4,19,20] is extended to several extra shell nucleons in this paper, a numerical example is given of the role of weak surface coupling for configurations $(7/2)^2$ and $(7/2)^3$, and nuclear moments and transition rates are discussed.

II. MATRIX ELEMENTS OF SURFACE INTERACTION

We follow the theory and notation of Bohr and Mottelson and Choudhury,[20] replacing the interaction constant times the radial integral of the surface interaction by an energy, k, whose order of magnitude is twice the kinetic energy of nucleons within the nucleus, and whose value should depend weakly on the angular quantum numbers of the extra nucleons. With this simplification, and the restriction to ellipsoidal surface vibrations, the particle-to-surface interaction becomes:

$$H_{\text{int}}=-k[\hbar\omega/2C]^{\frac{1}{2}}\sum_i\sum_\mu(b_\mu+(-1)^\mu b_{-\mu}^*)\times Y_{2\mu}(\theta_i\varphi_i), \quad (1)$$

where $\hbar\omega=\hbar(C/B)^{\frac{1}{2}}$ is the phonon excitation energy, B is the mass constant in the surface kinetic energy, C is the surface tension constant in the surface potential energy, b_μ and b_μ^* are destruction and creation operators for the phonons with spin two and Z-component of angular momentum μ, $Y_{2\mu}$ is a normalized spherical harmonic, and the index i labels the particles. As is well known, the average of H_{int} over a closed shell is zero, and in practice the sum extends only over those nucleons in unfilled shells. To simplify notation, we define,

$$\mathcal{Y}_{2\mu}=(\pi/5)^{\frac{1}{2}}Y_{2\mu}, \quad (2)$$

and

$$\gamma=(5\hbar\omega/2\pi C)^{\frac{1}{2}}=(5\hbar/2\pi B\omega)^{\frac{1}{2}}, \quad (3)$$

whence

$$H_{\text{int}}=-k\gamma\sum_i\sum_\mu(b_\mu+(-1)^\mu b_{-\mu}^*)\mathcal{Y}_{2\mu}(\theta_i\varphi_i). \quad (4)$$

The zero order nuclear states are taken to be shell model states of configuration α and angular momentum J, vector-coupled to surface excitation states of N phonons and angular momentum R, to yield total angular momentum I, which states we indicate by

$$|\alpha J;NR;IM\rangle. \quad (5)$$

Matrix elements of the interaction between such states can be evaluated according to the methods of Racah,[21,22] since both b_μ and $\mathcal{Y}_{2\mu}$ are tensor operators of rank 2 according to the definitions of Racah. The result is:

$$\langle\alpha J;NR;IM|H_{\text{int}}|\alpha'J';N'R';IM\rangle = (-1)^{2J'+J+I+R+1}(k\gamma)W(RR'JJ';2I)(NR\|b\|N'R') \times(\alpha J\|\textstyle\sum_i\mathcal{Y}_2(\theta_i\varphi_i)\|\alpha'J'), \quad (6)$$

where the double bar reduced matrix elements are defined as in references 21 and 22. Selection rules are: $\Delta N=1$, $\Delta R\leq 2$, $\Delta J\leq 2$, configurations α and α' differ at most in the quantum numbers of one particle, and for that particle, $\Delta l=0$ or 2, $\Delta j\leq 2$. Values of $(NR\|b\|N'R')$ are given in reference 20 up to $N'=3$ (Choudhury's reduced matrix elements should be multiplied by $(2R'+1)^{\frac{1}{2}}$ to give ours). Values of the one-particle matrix elements $(lsj\|\mathcal{Y}_2\|l'sj')$ have been given by Kerman[19] and Choudhury,[20] but their expressions can be very substantially simplified; namely,

$$(lsj\|\mathcal{Y}_2\|l'sj')=\tfrac{1}{2}(2j+1)^{\frac{1}{2}}(j2-\tfrac{1}{2}0|j2j'-\tfrac{1}{2}), \quad \Delta l=0,\pm 2, \quad (7)$$

where $(j_1j_2m_1m_2|j_1j_2jm)$ is a Clebsch-Gordan co-

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[18] E. L. Church and M. Goldhaber, Phys. Rev. **95**, 626 (1954).
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[20] D. C. Choudhury, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 4 (1954).
[21] G. Racah II, Phys. Rev. **62**, 438 (1942); III, Phys. Rev. **63**, 367 (1943); IV, Phys. Rev. **76**, 1352 (1949).
[22] Simon, Van der Sluis, and Biedenharn, Oak Ridge National Laboratory ORNL-1679, 1954 (unpublished).

efficient.[23] This result is found as follows: the expressions of Choudhury and Kerman are proportional to the product of a Racah and a Clebsch-Gordan coefficient. By Eq. (2), *p. x*, in reference 22, this product may be set equal to a sum over three Clebsch-Gordan coefficients. But the sum has only two terms and two of the three coefficients can be written down explicitly since they involve spin $\frac{1}{2}$. The reduction to (7) then follows easily.

For two-particle configurations, Eq. (44) of Racah II[21] may be used to give the reduced particle matrix elements. Several cases are to be distinguished:

(a) off diagonal:

$$(j_1j_2J\|\sum_i\mathcal{Y}_2(i)\|j_1'j_2J')=(-1)^{j_2-j_1'-J}[(2J+1)(2J'+1)]^{\frac{1}{2}}\times W(j_1Jj_1'J';j_22)(j_1\|\mathcal{Y}_2\|j_1'),\quad(8)$$

where j is used as shorthand for all particle quantum number, e.g., nlj;

(b) diagonal, inequivalent particles:

$$(j_1j_2J\|\sum_i\mathcal{Y}_2(i)\|j_1j_2J')=(-1)^{j_2-j_1-J}[(2J+1)(2J'+1)]^{\frac{1}{2}}\times[(j_1\|\mathcal{Y}_2\|j_1)W(j_1Jj_1J';j_22)+(j_2\|\mathcal{Y}_2\|j_2)W(j_2Jj_2J';j_12)];\quad(9)$$

(c) diagonal, equivalent particles:

$$(j^2J\|\sum_i\mathcal{Y}_2(i)\|j^2J')=2(-1)^J[(2J+1)(2J'+1)]^{\frac{1}{2}}\times(j\|\mathcal{Y}_2\|j)W(jJjJ';j2).\quad(10)$$

For configurations of more than two particles, expansions of the particle wave functions in terms of fractional parentage coefficients become necessary. We state the result for the reduced particle matrix elements only for the special case of the configuration $(j)^n$ of equivalent particles. Then

$$(j^nJ\|\sum_i\mathcal{Y}_2(i)\|j^nJ')=n(-1)^{-j-J}[(2J+1)(2J'+1)]^{\frac{1}{2}}\times(j\|\mathcal{Y}_2\|j)\sum_{J_1}(-1)^{J_1}\times(J[\![J_1)W(jJjJ';J_12)(J_1]\!]J'),\quad(11)$$

where $(J_1]\!]J)$ is shorthand for the fractional parentage coefficient, i.e.,

$$(J_1]\!]J)=(j^{n-1}J_1jJ]\!]j^nJ).\quad(12)$$

Fractional parentage expansions of configurations of inequivalent particles have been discussed by Redlich[24] and by Meshkov[25] and are used in the succeeding paper.[17]

Fractional parentage methods may also be used to find the reduced surface matrix elements $(NR\|b\|N'R')$.

[23] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1951).
[24] M. Redlich, Ph.D. thesis, Princeton University, 1954 (unpublished).
[25] S. Meshkov, Phys. Rev. **91**, 871 (1953).

TABLE I. Two-phonon to three-phonon fractional parentage coefficients.

$R \backslash R'$	0	2	3	4	6
0		$(7/15)^{\frac{1}{2}}$			
2	1	$(4/21)^{\frac{1}{2}}$	$(5/7)^{\frac{1}{2}}$	$(11/21)^{\frac{1}{2}}$	
4		$(12/35)^{\frac{1}{2}}$	$-(2/7)^{\frac{1}{2}}$	$(10/21)^{\frac{1}{2}}$	1

Between 0 and 1 phonon,

$$(00\|b\|12)=5^{\frac{1}{2}}.\quad(13)$$

Between 1 and 2 phonons,

$$(12\|b\|2R')=2^{\frac{1}{2}}[5(2R'+1)]^{\frac{1}{2}}(00\|b\|12)W(022R';22)=[2(2R'+1)]^{\frac{1}{2}}.\quad(14)$$

Between 2 and 3 phonons,

$$(2R\|b\|3R')=3^{\frac{1}{2}}[(2R+1)(2R'+1)]^{\frac{1}{2}}(00\|b\|12)\times W(0R2R';R2)(R_0{}^2(R)R_0R']\!]R_0{}^3R')=[3(2R'+1)]^{\frac{1}{2}}(R_0{}^2(R)R_0R']\!]R_0{}^3R'),\quad(15)$$

where $R_0=2$, the angular momentum of each phonon. The phonon fractional parentage coefficients may be found, following Racah III,[21] by means of the relations (required to make the total wave function symmetric):

$$\sum_R(2R+1)^{\frac{1}{2}}W(22R'2;RR'')(2^2(R)2R']\!]2^3R')=0,\quad(16)$$

where R is summed over the allowed 2-phonon values 0, 2, 4; R' is the 3-phonon angular momentum; and R'' is 1 or 3, the unallowed 2-phonon angular momenta. Table I gives the two-phonon to three-phonon parentage coefficients.

III. DISCUSSION OF WEAK SURFACE COUPLING

A. Effective Scalar Interaction

The contribution of surface coupling to the nuclear energy takes a very simple form in the limit of strong coupling, giving the characteristic rotational states whose energies are independent of details of the particle structure. This is the limit of nuclear deformation large compared to amplitude of surface vibration, or of surface coupling energies large compared to interparticle interaction energies, or of surface rotation frequency slow compared to particle motion, so that the particle structure adjusts adiabatically to the surface shape. The opposite limit of weak coupling is also of special interest and simplicity, although of course it has no such marked consequences as the strong coupling limit. The weak coupling limit may be defined by mean deformation small compared to amplitude of surface vibration, by phonon energy large compared to spacing of particle levels, or by frequency of surface vibration large compared to particle frequencies, i.e., the adiabatic adjustment of the surface shape to the particle structure. By the nature of the surface interaction (its relation to the kinetic energy of the nucleons), a weak coupling situation with particle energy spacings larger than phonon energies is not reasonable, and therefore configuration interaction is an essential part of weak coupling.[26]

In a one-phonon approximation, the energy shift of the configuration $\alpha J(I=J)$ is, by second-order perturbation theory,

$$\Delta E(\alpha J) = -\sum_{\alpha' J'} \frac{|\langle \alpha J;00;IM|H_{\rm int}|\alpha' J';12;IM\rangle|^2}{\hbar\omega+E^{(0)}(\alpha' J')-E^{(0)}(\alpha J)}, \quad (16)$$

where $\hbar\omega$ is the phonon energy. In the weak coupling limit, $|E^{(0)}(\alpha'J')-E^{(0)}(\alpha J)|\ll\hbar\omega$, for many configurations. In the crudest approximation, the numerator in (16) is supposed to cut off before the denominator differs appreciably from $\hbar\omega$. Then

$$\Delta E(\alpha J) \cong -(\hbar\omega)^{-1}\sum_{\alpha' J'}|\langle \alpha J;00;IM|H_{\rm int}|\alpha' J';1,2;IM\rangle|^2$$
$$= -(\hbar\omega)^{-1}\langle \alpha J;00;IM|(H_{\rm int})^2|\alpha J;00;IM\rangle. \quad (17)$$

As has been previously pointed out,[4,19] the surface interaction then reduces in this approximation to an effective scalar two body interaction—where the surface phonon plays a role analogous to the meson in the ordinary nucleon-nucleon force. Integration over surface coordinates in (17) gives the effective scalar potential, due to surface interaction,

$$V_s = -[(k\gamma)^2/\hbar\omega]\sum_{ij}\sum_{\mu} \mathcal{Y}_{2\mu}{}^*(i)\mathcal{Y}_{2\mu}(j), \quad (18)$$

where it is to be noted that self-energy terms $(i=j)$ are included.

There are several flaws in these arguments leading to (18) coming from the assumed localization of the surface interaction at the nuclear surface, i.e., $H_{\rm int} \sim\sum_i\delta(r_i-R)$ in a square well model. This means that the matrix elements in (16) depend so weakly on the radial quantum numbers of the nucleons that the convergence of the sum (16) is questionable. If the surface interaction is spread out over a reasonable edge thickness, then the principal quantum number dependence of the terms in (16) will produce the desired cutoff, but only at very high excited states. Thus the energy $E^{(0)}(\alpha'J')-E^{(0)}(\alpha J)$ in the denominator of (16) may not be ignored relative to $\hbar\omega$. This difficulty may be avoided as follows. We note that for a given principal quantum number n, there are a finite number of configurations α' which can mix with α, limited by the selection rules stated after Eq. (6). These form a complete set for the angular dependence of $H_{\rm int}$, and in the weak coupling limit, this set will occur within an energy region less than $\hbar\omega$. If the weak dependence of the radial part of $H_{\rm int}$ on angular quantum numbers is ignored, (16) may be approximated as

$$\Delta E(\alpha J) = -\sum_n f(n) \times {\sum_{\alpha' J'}}' \frac{|\langle \alpha J;00;IM|H_{\rm int}{}'|\alpha' J';1\,2;IM\rangle|^2}{\hbar\omega+E^{(0)}(\alpha' J')-E^{(0)}(\alpha J)}, \quad (19)$$

where the primes on $H_{\rm int}$ and on the sum indicate that the radial dependence and radial quantum numbers respectively are not involved. Then for each n, the closure approximation may be applied to the angular sum in (19) with a suitable average energy denominator, $\hbar\omega+\Delta(n)$, to give

$$\Delta E(\alpha J) \cong -\{\sum_n[\hbar\omega+\Delta(n)]^{-1}f(n)\} \times\langle \alpha J;00;IM|(H_{\rm int}{}')^2|\alpha J;00;IM\rangle. \quad (20)$$

This expression differs from (17) in having $(\hbar\omega)^{-1}$ replaced by the curly bracket, and in having the radial dependence of $H_{\rm int}$ removed. The latter change is important, since in the delta function approximation of the radial dependence of $H_{\rm int}$, Eq. (17) leads to infinite self-energy terms in (18), while Eq. (20) leads properly to the relative magnitudes of mutual and self-energy given in (18).

Neither of the possible uncertainties in the scalar interaction (18) is significant for energy levels. The over-all strength of interaction is uncertain, but should be regarded as an adjustable parameter of the theory in any case. The relative size of the mutual- and self-energy terms has no effect on level spacings of a given configuration, since the self-energy terms affect all levels of a given configuration equally. Thus for spacings within a given configuration, the self-energy terms may be ignored altogether. Then this limiting form of the surface interaction may be treated exactly as an ordinary two-body force, except that only diagonal elements are to be taken, since the surface induced configuration mixing is already included.

For configuration $(j)^n$ of equivalent particles, the interaction (18) leads to simple expressions for the surface contribution to nuclear energy levels. We rewrite (18) as,

$$V_s = \epsilon_0[\sum_{i>j} v_{ij} + \sum_i v_i], \quad (21)$$

where

$$v_{ij} = -\sum_\mu \mathcal{Y}_{2\mu}{}^*(i)\mathcal{Y}_{2\mu}(j) = -\sum_\mu(-1)^\mu\mathcal{Y}_{2,-\mu}(i)\mathcal{Y}_{2\mu}(j), \quad (22)$$

$$v_i = -\tfrac{1}{2}\sum_\mu(-1)^\mu\mathcal{Y}_{2,-\mu}(i)\mathcal{Y}_{2\mu}(i), \quad (23)$$

and ϵ_0 is the energy unit of the interaction, given roughly, as in (18) by $+2(k\gamma)^2/\hbar\omega$.

[26] See, however, a recent paper by G. Scharff-Goldhaber and J. Weneser [Phys. Rev. 98, 212 (1955)], who suppose that direct particle coupling energies may be larger than phonon energies.

The mutual interaction term v_{ij} has the form of the scalar product of two tensors of rank two, and can be treated by the standard Racah methods.[21] For the configuration $(j)^2$,

$$\langle j^2J|v_{12}|j^2J\rangle = |(j\|\mathcal{Y}_2\|j)|^2 W(jjjj;2J), \qquad (24)$$

a result already given by Kerman.[19] We note from (7)

$$(j\|\mathcal{Y}_2\|j) = -\frac{1}{8}\left[\frac{(2j-1)(2j+1)(2j+3)}{j(j+1)}\right]^{\frac{1}{2}}. \qquad (25)$$

The Racah coefficient in (24) is not in general a monotonic function of J (for J even), but it is monotonic if j is not too large. Two-particle configurations and level orders from (24) are: $(3/2)^2$, 0, 2; $(5/2)^2$, 0, 2, 4; $(7/2)^2$, 0, 2, 4 and 6 degenerate; $(9/2)^2$, 0, 2, 4, 8, 6.

The reduced one-particle matrix element of (23) is

$$(j\|v_1\|j) = \tfrac{1}{8}(2j+1)^{\frac{1}{2}}. \qquad (26)$$

Therefore the matrix elements of the self-energy term, $\sum_i v_i$, are, for the configuration $(j)^n$,

$$\langle j^nJ|\textstyle\sum_i v_i|j^nJ\rangle = -n/8, \qquad (27)$$

independent of J.

For more than two equivalent particles, the mutual interaction energy is,[21] in units of ϵ_0,

$$\langle j^n\alpha J|\sum_{i>j} v_{ij}|j^n\alpha' J\rangle = [n/(n-2)] \sum_{\alpha''\alpha'''J'} (\alpha J[\![\alpha''J')(\alpha'''J']\!]\alpha' J) \times \langle j^{n-1}\alpha''J'|\textstyle\sum v_{ij}|j^{n-1}\alpha'''J'\rangle, \qquad (28)$$

where an abbreviated notation is used for the fractional parentage coefficients:

$$(\alpha''J']\!]\alpha J) = (j^{n-1}(\alpha''J')j\alpha J]\!]j^n\alpha J). \qquad (29)$$

Surface contributions to nuclear energy in the weak coupling limit are given in Fig. 1 for all configurations $(j)^n$ of equivalent particles through $j=7/2$ and for $(9/2)^2$. Configurations $(j)^{-n}$ are equivalent to $(j)^n$. The required parentage coefficients were taken from Edmonds and Flowers.[27] For the configuration $(5/2)^3$, $I=j=5/2$ lies lowest, in contrast to the strong-coupling limit where $I=j-1$ lies lowest for all j. For $(7/2)^3$, $I=j-1$ also lies lowest in the weak coupling limit,[28] although by a relatively small amount. Thus the occurrence in Ca^{43}, with configuration $(f_{7/2})^3$, of a ground state spin 7/2 must be attributed to interactions other than surface coupling (this nucleus is discussed in detail in a succeeding paper).[17]

[27] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952). Among the coefficients of fractional parentage needed in our work, we noted several typographical errors in this paper. In first line of Table III, change $5/(14)^{\frac{1}{2}}$ to $(5/14)^{\frac{1}{2}}$ and $3/14$ to $3/(14)^{\frac{1}{2}}$. In last line of Table V, change $(5/22)^{\frac{1}{2}}$ to $-(5/22)^{\frac{1}{2}}$.

[28] A remark on p. 35 of reference 4 appears to be in error on this point.

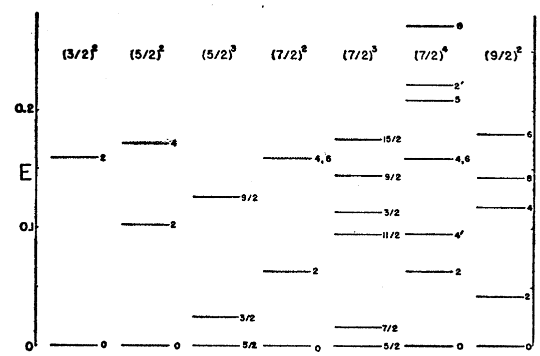

FIG. 1. Surface-induced energy spacings for simple configurations in weak coupling limit, in units of ϵ_0 [see Eqs. (18), (21), (24), and (28)]. For configuration $(7/2)^4$, primes on spins 2 and 4 indicate seniority 4 states.

Among the configurations shown in Fig. 1, only $(7/2)^4$ has more than one level of a given angular momentum. These levels are most conveniently classified according to seniority,[21,29] and, in fact, the inter-seniority matrix elements vanish for $(7/2)^4$. These matrix elements may be expanded in terms of the two particle matrix elements as,

$$(j^4v_1I|\sum_{i>j} v_{ij}|j^4v_2I) = 6\sum_{J_2}\{\sum_{J_3}(v_1I[\![J_3)\times(J_3]\!]v_2I)|(J_2]\!]J_3)|^2\}(j^2J_2|v_{12}|j^2J_2), \qquad (30)$$

where v is the seniority quantum number, and the parentage coefficients are abbreviated in an obvious way: J_2 is two-particle angular momentum, J_3 is three-particle angular momentum. For $j=7/2$, and $v_1\neq v_2$ ($I=2$ and 4), it has been verified[30] that the quantity in curly brackets vanishes. Therefore, for this configuration, seniority is a good quantum number for all scalar two-body operators. A more general result has not been found.

B. Configuration Interaction

The appropriate surface-induced potential (18), which springs from the assumption that many particle levels lie within an energy $\hbar\omega$, might appear inadequate in a particular nucleus, and one would be led to examine the perturbation expression (16) to see in greater detail the effect of specific configuration mixing induced by weak surface coupling. The interesting result is that for groups of equivalent particles, configuration mixing, like the self-energy contribution in (18), affects all levels of a given configuration equally, therefore the energy spacing[5] calculated from (18) are exactly the same as those calculated from only the diagonal elements in (16). That is, in the sum (16), all off diagonal (in *angular* quantum numbers) matrix elements give contributions independent of I. Moreover, this independence of I holds for the effect of each admixed

[29] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).

[30] C. Schwartz and A. de-Shalit, Phys. Rev. **94**, 1257 (1954).

configuration separately. This irrelevance of configuration mixing applies, of course, only to energies, not to magnetic moments or other nuclear properties.

Consider the mixing of a configuration, $\alpha=j^n$, with configuration $\alpha'=j^{n-1}j'$ (the only possibility in view of the selection rules). Then that part of the perturbation sum (16) due to this mixing will be,

$$\Delta E_{\alpha'}(\alpha\beta J)=-\frac{1}{\hbar\omega+\Delta E^{(0)}(\alpha,\alpha')} \times \sum_{\beta''J''J'}|\langle j^n\beta J;00;IM|H_{\text{int}}|j^{n-1}(\beta''J'')j'J';12;IM\rangle|^2, \quad (31)$$

where β represents other quantum numbers possibly required to specify the states, e.g., seniority. Note that no other quantum number β' is required with J'. From the general formula (6) for the matrix elements of H_{int}, one has

$$|\langle\alpha J;00;IM|H_{\text{int}}|\alpha'J';12;IM\rangle|^2 =5(2I+1)^{-1}(k\gamma)^2|(\alpha J\|\textstyle\sum_i\mathcal{Y}_2(i)\|\alpha'J')|^2. \quad (32)$$

Now one expands the antisymmetrized states,

$$\langle j^n\beta J|=\sum_{\beta_1J_1}(j^n\beta J[\![j^{n-1}(\beta_1J_1)j\beta J) \times\langle j^{n-1}(\beta_1J_1)j_n\beta J|, \quad (33)$$

and

$$|j^{n-1}(\beta''J'')j'J'\rangle=n^{-\frac{1}{2}}|j^{n-1}(\beta''J'')j_n'J'\rangle +\sum_{\beta'''J'''}a_{\beta'''J'''}|j^{n-2}j'(\beta'''J''')j_nJ'\rangle, \quad (34)$$

combinations of $(n-1)$ particles vector-coupled to the nth particle, and the subscripts n indicate the dependence only on the coordinates of the nth particle. Equation (34) is a straightforward generalization of a three-particle result of Redlich.[24] In this case the coefficients $a(\beta'''J''')$ do not concern us. The reduced matrix element in (32) becomes

$$(\alpha J\|\textstyle\sum_i\mathcal{Y}_2(i)\|\alpha'J')=n^{+\frac{1}{2}}(j^n\beta J[\![j^{n-1}(\beta''J'')j\beta J) \times(j^{n-1}(\beta''J'')j_n\beta J\|\mathcal{Y}_2(n)\|j^{n-1}(\beta''J'')j_n'J'). \quad (35)$$

The right side of (35) may be reduced by means of Eq. (44) in Racah II.[21] The shift due to configuration interaction, Eq. (31), then becomes

$$\Delta E_{\alpha'}(\alpha\beta J)=-\left[\frac{(k\gamma)^2}{\hbar\omega+\Delta E^{(0)}(\alpha,\alpha')}\right]|(j\|\mathcal{Y}_2\|j')|^2\cdot 5n \times\sum_{\beta''J''}|(j^{n-1}(\beta''J'')j\beta J]\!]j^n\beta J)|^2 \times\sum_{J'}(2J'+1)W^2(jj'JJ';2j). \quad (36)$$

The sum over J' in (36) is just $(2j+1)^{-1}$, by the orthogonality condition, while the normalization of the fractional parentage coefficients makes the sum over β'' and J'' equal to unity. Writing simply ϵ for the energy contained in the square brackets in (36), we obtain finally,

$$\Delta E_{\alpha'}(\alpha\beta J)=-5n\epsilon(2j+1)^{-1}|(j\|\mathcal{Y}_2\|j')|^2, \quad (37)$$

independent of J. It is to be noted that this result is not peculiar to the form of H_{int}, but holds for all scalar operators of the one-particle form, $\sum_i f_i$. The result depends essentially on the inequivalence of j and j'. If J' could not take on all values consistent with vector coupling of J'' and j', then the sum over J' in (36) would retain a dependence on J. Thus for more complicated initial configurations, α, the result that the configuration interaction energy shift is independent of J need not remain true.

IV. NUMERICAL EXAMPLE: $f_{7/2}$ SHELL

As an example of the influence of weak surface coupling on the energies of multiparticle configuration, the surface interaction energy matrices have been numerically diagonalized for the configurations $(f_{7/2})^2$ and $(f_{7/2})^3$ [or equivalently, $(g_{7/2})^2$ and $(g_{7/2})^3$] including 1 or 2 phonons. Direct interparticle forces have been ignored, and configuration mixing has been ignored, in the example. As shown above, other configurations do not contribute to level splitting in the weak coupling limit, although they may become important when two phonons are important.

The calculation may conveniently be carried out in dimensionless form. The energy unit is $k\gamma$ [Eqs. (3) and (4)], the natural unit for expressing the interaction energy, and we define a dimensionless measure of the phonon energy, $\xi^{-1}=(\hbar\omega/k\gamma)$; ξ plays the role of coupling constant, and is related to the coupling constant x used in references 4 and 20 by

$$\xi=(8j)^{\frac{1}{2}}x. \quad (38)$$

The eigenvalues obtained by diagonalizing the energy matrices through one or two phonons are $\lambda=E/k\gamma$. These dimensionless results, λ *vs* ξ are given, for configurations $(7/2)^2$ and $(7/2)^3$ in Figs. 2 and 3 and Tables II and III. The translation to energy units de-

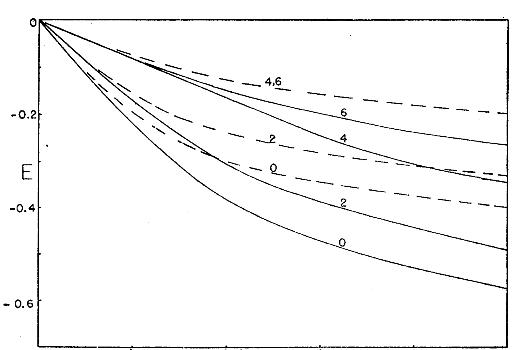

FIG. 2. Surface-induced energy shifts of levels of $(7/2)^2$ configuration. Dashed lines: one phonon included. Solid lines: two phonons included. Vertical scale, energy in units $k\gamma$ [see above Eq. (38)]. Horizontal scale, coupling strength ξ [see above Eq. (38)].

pends somewhat on one's assumptions about the parameters of the collective theory. For example, one may assume that the interaction constant, k, and the mass parameter, B, are known, and that all of the variation of phonon energy comes from the variation of effective surface tension, C. Thus, if we choose $k=40$ Mev, and $B=(3/8\pi)MR^2$, with $R=1.40\times10^{-13}A^{\frac{1}{3}}$ cm, or $\hbar^2/B=179A^{-5/3}$ Mev, and if C is eliminated in favor of $\hbar\omega$, we obtain the energy unit

$$k\gamma=478A^{-5/6}(\hbar\omega\text{ Mev})^{-\frac{1}{2}}, \tag{39}$$

and the coupling constant,

$$\xi=478A^{-5/6}(\hbar\omega\text{ Mev})^{-\frac{3}{2}}. \tag{40}$$

Table II gives the energies in the weak coupling limit, good in the linear regions of Figs. 2 and 3, roughly for $\xi<0.7$. The results of exact matrix diagonalization are given in Table III, for one, two, and, for the special case $I=0$, three phonons, and are plotted in Figs. 2 and 3. Because of the difficulties of exact matrix diagonalization, the results are incomplete. The lowest levels of spin 0, 2, 4, and 6 for $(7/2)^2$ are shown for one and two phonons included in Fig. 2. In order better to illustrate the range of validity, we show in Fig. 4 the energy shifts of the spin-zero state only, for one, two, and three phonons. It is evident from the figure that the one-phonon result is accurate for $\xi\lesssim0.8$, and the two-phonon result for $\xi\lesssim2$. The weak coupling two particle result differs from the strong coupling result principally in having a small spacing of the 4+ and 6+ states. Both weak- and strong-coupling results differ qualitatively from the results for short-range particle forces in having a smaller 0–2:2–4 ratio of spacings. The four-particle configuration, $(7/2)^4$ gives rise, in the weak-coupling limit, to the same surface-induced energy spacings as $(7/2)^2$.

The energy level order of the $(7/2)^3$ configuration due to weak surface coupling is 5/2, 7/2, 11/2, 3/2, 9/2, 15/2, while the lowest two states in strong coupling also are 5/2, 7/2. In Fig. 3, only the states of lowest spin, 3/2, 5/2, 7/2, have been calculated with two phonons. The surface-induced level spacings, for any

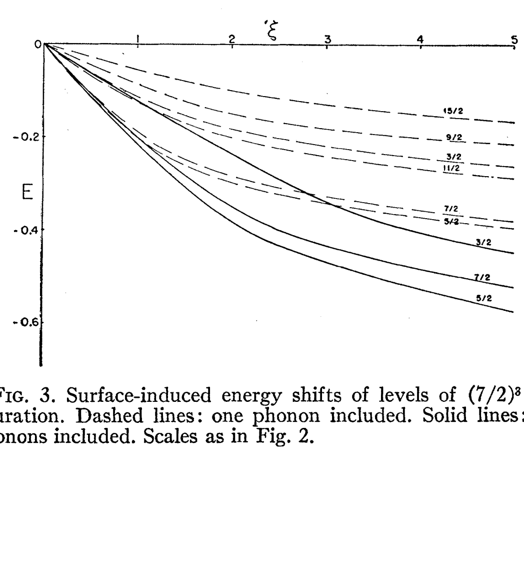

FIG. 3. Surface-induced energy shifts of levels of $(7/2)^3$ configuration. Dashed lines: one phonon included. Solid lines: two phonons included. Scales as in Fig. 2.

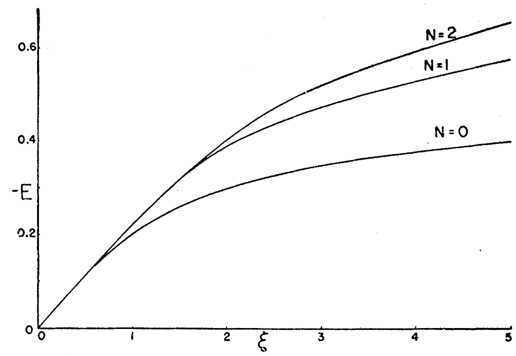

FIG. 4. Surface-induced energy of spin-zero state of $(7/2)^2$ configuration resulting from exact matrix diagonalization, including one, two, and three phonons. (*Note:* From top to bottom, the curves should read $N=3$, $N=2$, and $N=1$, respectively.)

TABLE II. Energies in weak coupling limit.

Configuration $(7/2)^2$		Configuration $(7/2)^3$	
I	λ[a]	I	λ[a]
0	−15/63	5/2	− 59/252
2	−11/63	7/2	− 55/252
4	− 5/63	11/2	− 5/36
6	− 5/63	3/2	− 71/588
		9/2	−157/1764
		15/2	− 5/84

[a] λ is the energy shift due to the surface-induced interaction in the weak coupling limit, in units $\xi(k\gamma)=(k\gamma)^2/\hbar\omega$.

strength of coupling, differ from the direct particle force induced spacing in having the $I=5/2$ state lower than the $I=7/2$.

Energy matrices involving both surface coupling and direct particle coupling have been diagonalized for the same two and three particle configurations, but since special assumptions about the particle matrix elements were made, the results are primarily of qualitative significance. The approximate method of adding together the separate particle and surface induced shifts differs from the result of simultaneous diagonalization principally in overestimating the shifts of the spin 0 (for two particles) or spin 7/2 (for three) states. These results have been used in an attempt to set an upper limit on the strength of surface coupling in the calcium isotopes.[17]

V. MOMENTS AND TRANSITION RATES

A. Magnetic Moments

The existence of surface coupling contributes to nuclear magnetic moments by (1) mixing particle configurations, and (2) contributing directly due to the sharing of the particle angular momentum with the core.[4] For weak coupling, the first effect is likely to be of greater significance, but *neither* effect is important compared to the configuration mixing induced by direct particle forces. This is because weak coupling affects magnetic moments only in second order. Let the wave function be a superposition of the states given in Eq.

TABLE III. Energies resulting from matrix diagonalization for pure particle configurations, $(7/2)^2$ and $(7/2)^3$, through one or two phonons (or three phonons for special case $I=0$). ξ, defined above Eq. (38) and approximated by (40), measures the strength of coupling. The lowest eigenvalues for each angular momentum, λ, are given in units of $k\gamma$, which is defined by Eq. (3) and approximated by Eq. (39). The number of phonons included is indicated in parentheses at the right of each row.

A. $(7/2)^2$ configuration

$I \backslash \xi$	0.1667	0.25	0.75	1.00	2.50	2.86	5.0	N
	−0.0396	−0.0590	−0.1594		−0.3274		−0.3981	(1)
0				−0.2233		−0.4643	−0.5737	(2)
				−0.2249		−0.5001	−0.6517	(3)
	−0.0291	−0.0432	−0.1201		−0.2632		−0.3297	(1)
2				−0.1679		−0.3792	−0.4896	(2)
	−0.0132	−0.0198	−0.0570		−0.1455		−0.1989	(1)
4				−0.0812		−0.2320	−0.3414	(2)
	−0.0132	−0.0198	−0.0570		−0.1455		−0.1989	(1)
6				−0.0789		−0.1888	−0.2622	(2)

B. $(7/2)^3$ configuration

$I \backslash \xi$	0.1667	0.25	0.75	1.00	2.50	2.86	5.0	N
	−0.0390	−0.0580	−0.1570		−0.3235		−0.3941	(1)
5/2						−0.4604	−0.5739	(2)
7/2	−0.0364	−0.0538	−0.1473		−0.3081		−0.3777	(1)
							−0.5215	(2)
	−0.0232	−0.0344	−0.0970		−0.2230		−0.2859	(1)
11/2								
	−0.0201	−0.0300	−0.0850		−0.2009		−0.2615	(1)
3/2				−0.1232		−0.3253	−0.4448	(2)
	−0.0148	−0.0220	−0.0636		−0.1692		−0.2146	(1)
9/2								
	−0.0099	−0.0148	−0.0432		−0.1154		−0.1636	(1)
15/2								

(5). Then for weak surface coupling but no direct coupling, there will be only one component with amplitude of order unity, which will have $N=0$. Amplitudes of $N=1$ states will be of first order in the coupling strength, while $N=2$ states and other $N=0$ states will be of second order. The magnetic moment operator,

$$\mu_z=\sum_i g_i j_{iz}+g_R R_z, \tag{41}$$

is diagonal with respect to N, the number of phonons. Therefore the off diagonal contributions and the $N=1$ contributions, both diagonal and off-diagonal, are of second order. With direct particle coupling, however, other $N=0$ states and correspondingly the off-diagonal contributions to μ, are introduced in first order in the strength of direct coupling.

TABLE IV. Magnetic and quadrupole moments of $(f_{7/2})_N{}^3$, $J=7/2$, for various strengths of surface coupling. The coupling parameter, ξ, is defined in Eq. (38) and approximated in Eq. (40). Surface-induced mixing is ignored.

ξ	No. of phonons included	$\Delta\mu_c$	μ	Q (barns)
0	0	0	−1.91	0
0.75	1	+0.03	−1.88	0.0280
2.5	1	+0.10	−1.81	0.0695
5	2	+0.17	−1.74	
$\rightarrow\infty$	strong coupling limit	+2.21	+0.30[a]	$(1/15)Q_0$[a]
$\rightarrow\infty$	strong coupling limit	+0.90	−1.01[a]	$(5/14)Q_0$[a]

[a] The first value given in the strong coupling limit is for $\Omega=K=5/2$ $I=7/2$, but this is not the lowest state. The second value is for $\Omega=K=I=5/2$, the lowest state. $I=7/2$ is also not lowest in weak coupling unless direct particle coupling is included.

We conclude that for weak surface coupling, deviations of nuclear magnetic moments from the pure state values (in some cases the Schmidt lines) are due principally to configuration mixing induced by direct particle forces. In strong coupling, surface induced deviations from the $(l+\frac{1}{2})$ Schmidt line may be of the same order as the shifts due to configuration mixing, but even in strong coupling, shifts away from the $(l-\frac{1}{2})$ Schmidt lines are expected to be due primarily to configuration mixing. The smallness of the collective corrections to magnetic moments has been confirmed by calculations in weak coupling with various assumptions about the strength of coupling and the degree of mixing.[31] Predicted deviations from this source amount generally to less than a tenth of a nuclear magneton. Table IV, for example, shows magnetic moments predicted for Ca^{43} on the basis of weak and intermediate strengths of surface coupling. (In this case, surface induced mixing was neglected, so that predicted deviations are only correct as to order of magnitude.)

There is experimental evidence for the general ideas expressed above. On the basis of other evidence (quadrupole moments, $E2$ transition rates, rotational energies, etc.), one can estimate the strength of surface coupling. In particular, one can distinguish two groups of nuclei for each of which the magnetic moment is insensitive to

[31] K. W. Ford, Phys. Rev. **92**, 1094 (1953).

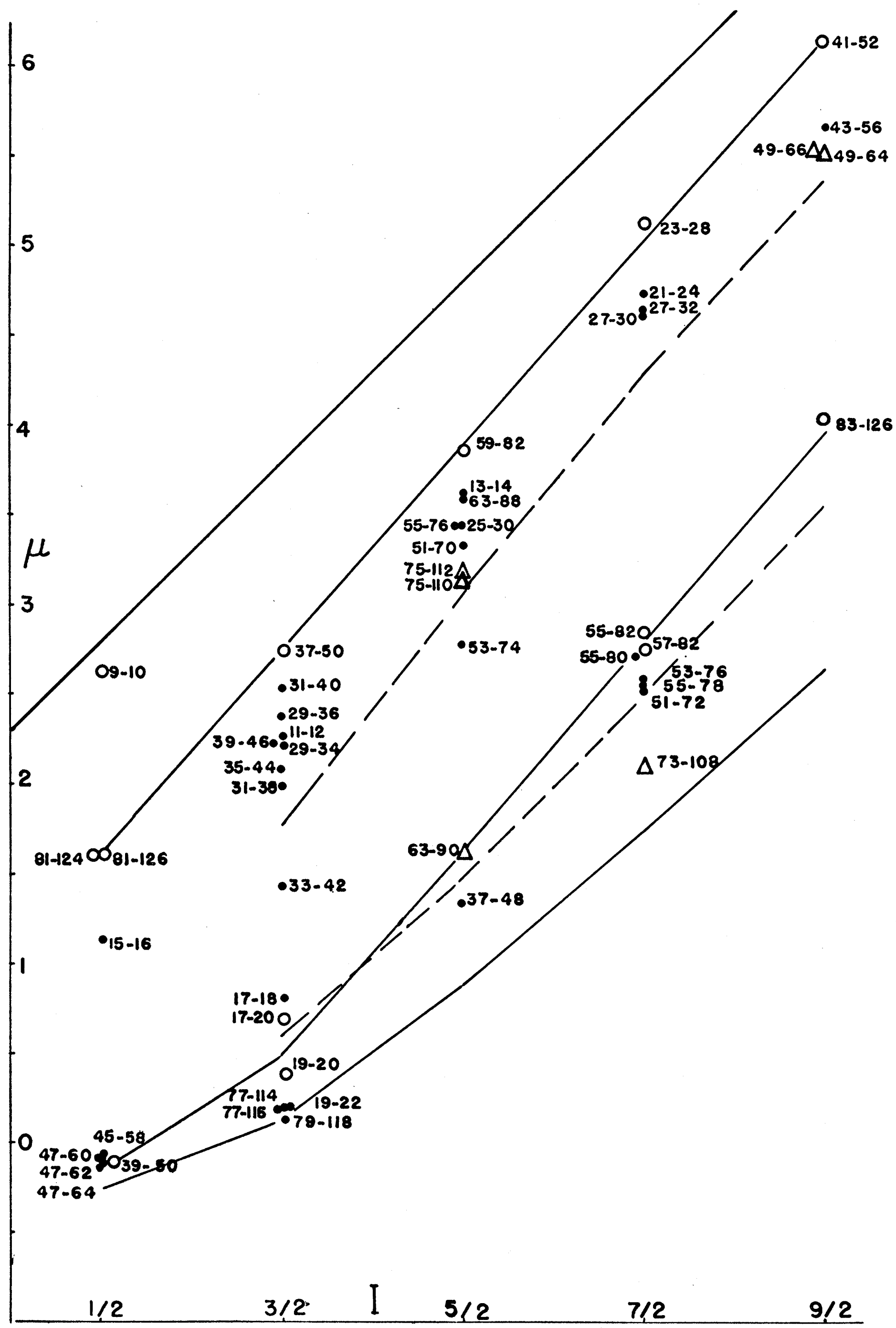

FIG. 5. Magnetic moments of odd-proton nuclei with $A>16$. Each point labeled by $Z-N$ of the nucleus. Open circles denote "spherical" nuclei, triangles denote "highly deformed" nuclei, and solid circles are nuclei which do not fit either extreme (see text for definitions). Outer solid lines are the Schmidt lines, giving the magnetic moments for pure configurations of equivalent protons. Inner solid lines are the "Blin-Stoyle Perks lines," drawn arbitrarily through the moments of spherical nuclei. Dashed lines are the "Bohr lines" giving the predicted position of the moments of highly deformed nuclei if the extra-particle g-factors are the same as for the spherical nuclei. Moments are taken from H. E. Walchli [Oak Ridge National Laboratory Report ORNL-1469, April 1, 1953 (unpublished)] and from the subsequent card file of the nuclear data group of the National Bureau of Standards.

the exact strength of surface coupling: I, the "highly deformed" nuclei, for which the collective contribution is given by the strong coupling limiting formula,[32]

$$\Delta\mu_{\text{collective}}=[I/(I+1)](g_R-g_J), \qquad (42)$$

where g_R and g_J are, respectively, the collective and particle g-factors; and II, the "spherical" nuclei, for which the surface coupling is sufficiently weak that

$$\Delta\mu_{\text{collective}}\ll\mu. \qquad (43)$$

The moments of the latter group should be determined almost entirely by the particle structure and the directly induced configuration mixing. The first group should show in addition the shift given by (42). If the shift (42) is larger than the fluctuations in the Blin-

[32] A. Bohr, Phys. Rev. 81, 134 (1951).

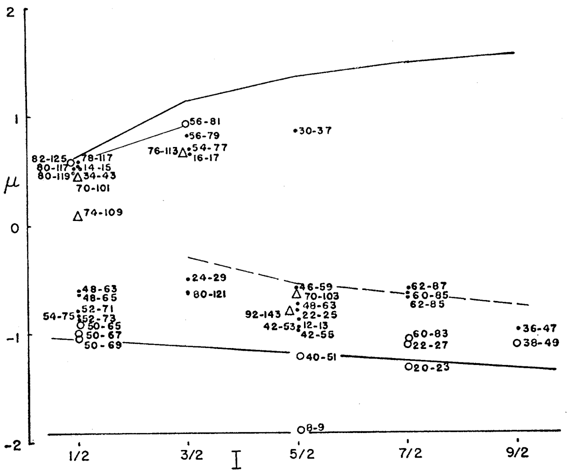

FIG. 6. Magnetic moments of odd-neutron nuclei with $A > 16$. Notation, significance of lines, and sources of data same as in Fig. 5.

Stoyle Perks[33] effect (shift due to $l+\frac{1}{2}$, $l-\frac{1}{2}$ particle mixing), then nuclei in groups I and II should fall into separate bands in the Schmidt diagram. Figures 5 and 6 show the odd proton and odd neutron Schmidt diagrams with nuclei in these groups especially marked. The groups have been defined as follows: I, *highly deformed*, nuclei with neutron numbers $90 \leq N \leq 113$, plus $_{92}U^{238}$ (the only nucleus beyond Bi^{209} with known magnetic moment) and the isotopes of $_{49}In$, because of their large measured quadrupole moments; II, *spherical*, nuclei with closed shell in protons or in neutrons, or closed shell plus or minus one in neutrons. The last group are included because of the generally more marked effect of closing a neutron shell than a proton shell. In addition, we include in the spherical group the isotopes of $_{81}Tl$ because they are in the near vicinity of a double closed shell, and the nucleus $_{41}Nb^{93}$, because of its very small measured quadrupole moment.

One sees in Fig. 5 that the $l+\frac{1}{2}$ moments lying closest to the Schmidt line at each spin value belong to "spherical" nuclei, and moreover that these moments lie rather closely along a single straight line (exception, $_9F^{19}$, whose moment, however, has been explained in terms of directly induced configuration mixing[13]). If one defines the net particle moment, Jg_J, by this wholly empirical line, and supposes that it does not vary much among different nuclei with the same spin and parity (there is no very strong argument for this assumption), then one may calculate by means of Eq. (42) the extra shift to be expected for strongly deformed nuclei. This leads to a revised "Bohr line," shown by the dashed line in Fig. 5. Four of the five known strongly deformed nuclei of the odd proton, $l+\frac{1}{2}$ type, lie very close to this line. The fifth, $_{63}Eu^{153}$, $I=5/2$, is displaced to the neighborhood of the opposite Schmidt line. All but two of the remaining moments lie between the weak-coupling and strong-coupling lines. For the $l-\frac{1}{2}$ moments in Fig. 5, a similar procedure is carried out. An empirical weak-coupling line is drawn through the moments of the spherical nuclei (solid line), and a strong-coupling line is then constructed (dashed line) using the weak-coupling line to define g_J. The "$d_{\frac{3}{2}}$" moments are then more widely scattered than one would expect on this picture. At spin 7/2, however, one obtains agreement with experiment in that the spherical nuclei have the largest moments, and the highly deformed nucleus, $_{73}Ta^{181}$, has the smallest moment (i.e., is displaced *outward*), while other nuclei lie between.

Similar data and analysis are shown for the odd-neutron nuclei in Fig. 6. Again the moments of spherical nuclei are closest to the $(l+\frac{1}{2})$ Schmidt line at every spin value and are used to define a weak coupling line (solid line). A strong-coupling line is defined as in Fig. 5 (dashed line) and the two lines enclose most of the moments. At spin 5/2, the highly deformed nucleus $_{70}Yb^{173}$ lies close to the strong-coupling line. The moment of $_{92}U^{238}$ is not close but is not known with any accuracy. The moment of $_8O^{17}$ lies very near the Schmidt line; this has been explained by the fact that for this particular nucleus, the kind of configuration mixing which gives a first order contribution to the moment is absent.[33] Note that the same should be true of $_{20}Ca^{41}$, whose moment has not yet been measured. A corresponding analysis of the $l-\frac{1}{2}$ moments in Fig. 6 is not possible. One notes, however, that at $I=\frac{3}{2}$, the spherical moment is high, while the highly deformed moment is low. This agrees with the fact that Eq. (42) predicts a small *inward* shift due to strong coupling.

As emphasized by Bohr and Mottelson,[4] one should expect the moment of the extra-particle structure and the effect of configuration mixing to vary considerably among nuclei of the same spin and parity, and especially so for the low spin values and $j=l+\frac{1}{2}$. Therefore one more in the many efforts to give an over-all qualitative explanation of nuclear magnetic moments perhaps needs further justification. Our main points are these: (1) There is good evidence against an appreciable quenching of the nuclear magnetic moment in nuclear matter. (2) Collective effects alone cannot account for nuclear magnetic moments, but can account for most of the *variations* of moments at each spin and parity. (3) In the absence of surface coupling, nuclei still show a considerable inward deviation from the Schmidt line, as pointed out by Blin-Stoyle and Perks[33] (except for special cases such as $_8O^{17}$ which are understood). We feel that this description brings new order into the over-all picture of magnetic moments. It should be emphasized, however, that we have only brought together the ideas of Bohr and of Blin-Stoyle and Perks, together with the observation that weak surface coupling has only a second order effect on magnetic moments.

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[33] R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954).

B. Quadrupole Moments

For weak surface coupling, quadrupole effects may still be relatively great, since the surface coupling interaction energy is itself of quadrupole form. The collective quadrupole operator,

$$Q_s = (3/5)(5/\pi)^{\frac{1}{2}}ZeR^2\alpha_0 = (3/5)\gamma ZeR^2(b_0+b_0^*), \quad (44)$$

contributes in first order, giving in the weak coupling limit, a quadrupole moment,

$$Q=(6/5)ZeR^2\gamma \times[I(2I-1)/(I+1)(2I+3)]^{\frac{1}{2}}\bar{A}_{\alpha I00}A_{\alpha I12}, \quad (45)$$

where γ is defined by (3), and $A_{\alpha JNR}$ is the amplitude of the state with particle configuration αJ coupled to N phonons of spin R to give the total spin I. Thus in lowest order, admixed particle states do not contribute.[4] Typical collective quadrupole moments predicted for Ca^{43} by this formula are shown in Table IV. The amplitudes in (45) are given in the weak coupling limit by $\bar{A}_{\alpha I00}=1$, and

$$A_{\alpha I12}=(k\gamma/\hbar\omega)5^{\frac{1}{2}}(\alpha I\|\textstyle\sum_i \mathcal{Y}_2(i)\|\alpha I). \quad (46)$$

Some of the reduced particle matrix elements required in (46) are given in Sec. II. The qualitative picture is that collective quadrupole moments may be comparable to or greater than the direct particle moments of extra-shell protons for a strength of coupling so weak that energies and magnetic moments are only very slightly affected.

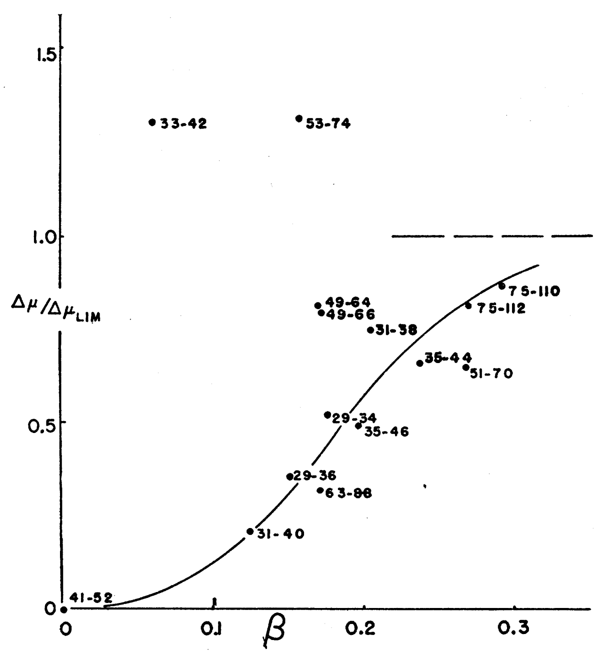

FIG. 7. Deviations of magnetic moments of odd-proton nuclei of $l+\frac{1}{2}$ type from the "Blin-Stoyle Perks line" (see Fig. 5) in units of predicted deviation for strong surface coupling *vs* the nuclear deformation β as deduced from the observed quadrupole moment (using the strong-coupling projection factor).

Various authors[34] have noted a correlation between the magnitude of magnetic moment shifts and the magnitudes of quadrupole moments. For weak surface coupling, $\Delta\mu_c$ should vary as Q^2, while for strong coupling, $\Delta\mu_c$ should be independent of Q, where $\Delta\mu_c$ is the magnetic moment shift due to collective nuclear motion. For the odd proton moments of $l+\frac{1}{2}$ type, we define $\Delta\mu_c$ as the difference between the observed moment and the moment of a "spherical" nucleus at the same spin value (see Fig. 5), and $\Delta\mu_{\text{lim}}$ as the predicted value of $\Delta\mu_c$ in the strong coupling limit. For the nuclei in this group, we plot in Fig. 7: $\Delta\mu_c/\Delta\mu_{\text{lim}}$ *vs* the nuclear deformation β calculated from the observed quadrupole moment *as if strong coupling were valid in every case.* If it were true that the g factor of the extra shell particle structure were a constant for given spin and parity, then the points in Fig. 7 should be along a curve which starts parabolically away from the origin at small β and approaches unity at large β. Indeed the best fit to the points is of this form, although an appreciable scatterer enters due to experimental errors in the quadrupole moments and to fluctuations in the particle g factors. In particular, two points fall far off the curve—those of ${}_{33}As^{75}$ and ${}_{53}I^{127}$. In both cases, the anomalously great magnetic moment deviation can be understood in terms of an unusually great mixing of nearly degenerate particle states—$f_{5/2}$–$p_{3/2}$ at $Z=33$ and $d_{5/2}$–$g_{7/2}$ at $Z=53$. A third anomaly is ${}_{33}Eu^{153}$, whose magnetic moment deviation appears to be far larger than that of any other nucleus, and is unexplained.

C. $E2$ Transition Rates

For weak surface coupling, the collective $E2$ transition rate may compete favorably with the direct particle rate for extra-shell protons, and may be entirely dominant for extra-shell neutrons. The rate is governed by the reduced probability,

$$B_e(2)=(2I'+1)^{-1}\sum_{MM'm}|(f|Q_{2m}|i)|^2, \quad (47)$$

where I' and I are spins of initial and final states. The collective quadrupole operator is,

$$Q_{2m}=q\alpha_m, \quad (48a)$$

where

$$q=(3/4\pi)ZeR^2, \quad (48b)$$

and

$$\alpha_m=(\hbar\omega/2C)^{\frac{1}{2}}[b_m+(-1)^m b_{-m}^*]. \quad (48c)$$

This leads, in lowest order to

$$B_e(2)=(\pi/5)\gamma^2q^2 \times[\bar{A}_{0II}A_{1II'}+(2I+1/2I'+1)^{\frac{1}{2}}\bar{A}_{1I'I}A_{0I'I'}]^2, \quad (49)$$

where A_{NJI} is the amplitude of the state with N

[34] H. Kopfermann, Naturwiss. **38**, 29 (1951); H. Miyazawa, Progr. Theoret. Phys. (Japan) **6**, 801 (1951).

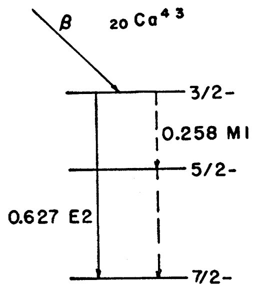

FIG. 8. Lowest states of Ca[43] (reference 35).

phonons coupled to particle configuration J to give total spin I.

As an example we consider the lowest levels of Ca^{43}, $7/2-$, $5/2-$, $3/2-$, as shown in Fig. 8. According to the work of Lindqvist and Mitchell[35] the $3/2-$ state decays predominantly by crossover rather than cascade. (a) Assume the particle states are $f_{7/2}$, $f_{5/2}$, $p_{3/2}$ (all neutrons). Then in the absence of collective effects, the cascade should be more probable than the crossover by about 10^6. Even for protons instead of neutrons, the cascade should be favored by at least 10^2. The observed favoring of the crossover would require a strong surface coupling of unreasonably large magnitude. (b) Assume that the particle states are all $(f_{7/2})^3J$. Then for pure states, the $M1$ cascade is forbidden, and the $E2$ crossover is greatly inhibited (going only by recoil of the core). The $M1$ cascade will be greatly speeded up by small configuration mixing (although still small compared to the one-particle cascade of (a) above), and the $E2$ crossover will be greatly speeded up by weak surface coupling. We calculate crudely that the $M1$ rate, including particle mixing, is,

$$T_{M1}(3/2 \rightarrow 5/2) \sim 3 \times 10^8 \text{ sec}^{-1}, \quad (50)$$

while the $E2$ rate is,

$$T_{E2}(3/2 \rightarrow 7/2) \cong 0.9 \times 10^{11} (k\gamma^2/\hbar\omega)^2 \text{ sec}^{-1}. \quad (51)$$

If we choose the "standard" values for the interaction constant k and the mass constant B of the collective model in order to reduce the number of parameters to one, the phonon energy (or the effective surface tension), we obtain

$$T_{E2}(\text{crossover})/T_{M1}(\text{cascade}) \sim (17 \text{ Mev}/\hbar\omega)^4, \quad (52)$$

i.e., the crossover will dominate for phonon energy less than about 17 Mev. This value corresponds to a very weak surface coupling. These arguments are crude, but provide the following information: (1) The observed dominance of the crossover transition is evidence for $(f_{7/2})^3$ configurations rather than one-particle configurations; (2) only a very weak surface coupling is sufficient to account for the dominance of the crossover; (3) the transition rates are perhaps slow enough to be measured—in particular, the 5/2 state might have a measurable lifetime.[36]

[35] T. Lindqvist and A. C. G. Mitchell, Phys. Rev. **95**, 444 (1954) and Phys. Rev. **95**, 1535 (1954).

VI. CONCLUSION

In claiming success for any nuclear model, one must be wary of those results which are model-independent, or at least given by more than one model. For example, almost any alteration of the pure jj coupling shell model improves the calculated ft values of unfavored beta transitions, since the pure jj coupling ft values are nearly minimal, and almost any change of coupling scheme will increase the predicted ft value. Similarly, in assessing the evidence for collective phenomena in low-energy nuclear properties, it is important to note that many of the effects of surface coupling can be duplicated by direct interactions among extra shell nucleons moving in the field of a rigid core. For example, energy level orders as shown in Fig. 1 are qualitatively close to those predicted for direct short range attractive forces; and deviations of nuclear magnetic moments from the pure state value are in the same direction, due to surface coupling or due to direct interaction. Large quadrupole moments and the striking regularities of the properties of the strongly deformed nuclei receive at least much simpler and more direct explanation from the collective model—but the important large anomaly in the nuclear moments of inertia leaves uncertain the explanation of even this class of phenomena.

For the nearly spherical nuclei, quadrupole effects afford the best empirical test of the strength of surface coupling. This evidence favors a coupling strength near closed shells considerably less than the "hydrodynamic" value, but still large enough to give important quadrupole effects, as discussed, for example, for the $E2$–$M1$ competition in Ca^{43}.

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[36] *Note added in proof.*—R. van Lieshout has kindly pointed out to us that the experimental results[35] do not in fact give unambiguous information on the relative speed of the crossover and cascade transitions discussed above, because of accidental energy degeneracy between different gamma rays. Equations (50) to (52) remain valid, but the actual relative rates remain to be determined.