RESISTANCE AND MAGNETIZATION.

BY C. W. HEAPS.

IN its present condition the theory of free electrons in metals explains qualitatively the general aspects of most of the observed phenomena, but fails to account quantitatively for some of the more special experiments. In particular, the Hall effect and the other galvanomagnetic phenomena exhibit peculiarities in different metals which are difficult to bring into line with the theory. The difficulty appears to arise because of complex and little understood molecular actions inside the metal which the theory does not take into account. For this reason the above mentioned class of phenomena deserves special consideration. Furthermore, the explaining of the motion of free electrons in magnetized metals involves the explanation of magnetostriction and of magnetism. The results of some previous experiments[1] made by the writer have indicated very clearly that the resistance change induced in iron and nickel by magnetization is very intimately associated with the structural changes producing magnetostriction. In order to bring out this relationship more clearly it seemed important to make experiments on the influence of structure on the resistance change of metallic conductors when placed in magnetic fields. The present paper describes these experiments.

In interpreting the results it will be necessary to consider the theoretical side of the question. On the usual theory of free electrons in a metal it is found that when a magnetic field acts at right angles to an electric current in the metal the expression for the change of resistance involves the term $H^2(e/m)^2T^2$ multiplied by a numerical factor. Here H is the magnetic field and T the free period of an electron. The numerical factor by which the above expression is multiplied has been determined as $\frac{1}{4}$[2] and as $\frac{1}{12}$[3], and there has been some disagreement as to whether or not the theoretical resistance change is an increase or a decrease.

In order to clarify our ideas it will be of advantage to consider the separate factors which enter to change the magnitude of the electric current in a wire when this wire is magnetized. It is ordinarily assumed that free electrons in a metal move about with high velocity very much

[1] PHYS. REV., VI., p. 34, 1915.

[2] E. P. Adams, PHYS. REV., XXIV., p. 428, 1907.

[3] O. W. Richardson, "The Electron Theory of Matter."

like gas molecules but collide with molecules of metal instead of with each other. When an electric field acts on these electrons a drift motion is superposed on the ordinary velocity of agitation of the electrons and it is this drift motion which constitutes the electric current. It has been shown[1] that the velocity with which electrons drift under these conditions is given by the equation

$$U_0 = X\frac{e}{m}\frac{\lambda_0}{V}, \tag{1}$$

where U_0 is the drift velocity in the direction of the electric force X, λ_0 is the mean free path of the electrons, and V is the velocity of agitation of the electrons.

We have now to consider the effect of a magnetic field on this drift velocity in two special cases as follows: (1) when H is applied parallel to the drift velocity (the longitudinal effect), and (2) when H is applied perpendicular to the drift velocity (the transverse effect). In case (1) it is clear that the only way in which H can change U_0 is by affecting the mean free path of an electron. Such an effect of H on λ_0 is to be expected, for the existence of magnetostriction implies an alteration of intermolecular distances and a consequent change in λ_0. It has also been pointed out[2] that the paths made by electrons between collisions would on the average be curved in the case where a magnetic field acts, and hence would be changed in length for this reason also. If λ_h is the mean free path in the longitudinal magnetic field we may write $\lambda_h - \lambda_0 = \delta\lambda$

$$\frac{\delta U}{U_0} = \frac{\delta\lambda}{\lambda_0}, \tag{2}$$

where δU is the increase in U_0 produced by the increase $\delta\lambda$ in λ_0. The electric current, I, is proportional to U_0 in case the magnetic field does not affect the number of electrons per unit volume, so that if δI represents the increase of current produced by $\delta\lambda$ we have

$$\frac{\delta I}{I} = \frac{\delta\lambda}{\lambda_0}.$$

For the increase of resistance we may write

$$\frac{\delta R}{R} = -\frac{\delta\lambda}{\lambda_0}. \tag{3}$$

According to this equation a longitudinal magnetic field will produce an increase of resistance when the mean free path of the electrons is decreased by H.

[1] Langevin, Ann. de Chim. et de Phys., 28, p. 336, 1903.

[2] Richardson, "Electron Theory of Matter."

In case (2) we have an added effect to consider. Townsend has shown[1] that if λ_0 is unchanged by H the drift velocity in a transverse magnetic field is given by

$$U_h = \frac{U_0}{1 + \omega^2 T^2}, \tag{4}$$

where $\omega = H(e/m)$, and T, the mean value of the time between collisions, is equal to λ_0/V. We have seen, however, that a magnetic field may be expected to change λ_0. If we let $\lambda_h - \lambda_0 = d\lambda$ be this change produced in λ_0 by a transverse magnetic field we must use λ_h instead of λ_0 in equation (4). We thus have, approximately,

$$U_h(1 + \omega^2 T^2) = U_0 \frac{\lambda_h}{\lambda_0},$$

$$\frac{dU}{U_0} = \frac{U_h - U_0}{U_0} = \frac{d\lambda}{\lambda_0} - \omega^2 T^2. \tag{5}$$

The change of electric current is proportional to the change in U_0 so that this expression gives the increase of current produced by a transverse magnetic field. For the increase of resistance we may write

$$\frac{dR}{R} = \omega^2 T^2 - \frac{d\lambda}{\lambda_0}. \tag{6}$$

According to this equation a transverse magnetic field produces an increase of resistance unless $d\lambda$ is an increase in free path sufficiently large to make $d\lambda/\lambda_0$ greater than $\omega^2 T^2$.

If we subtract equation (3) from equation (6) we get

$$\frac{dR}{R} - \frac{\delta R}{R} = H^2 \left(\frac{e}{m}\right)^2 \frac{\lambda_0^2}{V^2} - \frac{d\lambda}{\lambda_0} + \frac{\delta\lambda}{\lambda_0}. \tag{7}$$

In the general case we may not set the last two terms of this equation equal to each other because of the crystalline nature of the specimens under examination. It is usual to experiment with wires which have been pulled through draw-plates, and in specimens of this kind it is possible for the crystal structure to differ along different directions in the metal. A magnetic field perpendicular to the wire, therefore, might produce an effect on molecular arrangement which is different from the effect of a field parallel to the wire. In case we have a substance which is magnetically isotropic the last two terms of the equation above become equal and we get

$$\frac{dR}{R} - \frac{\delta R}{R} = H^2 \left(\frac{e}{m}\right)^2 \frac{\lambda_0^2}{V^2}. \tag{8}$$

This equation has the advantage of being free from terms involving

[1] "Electricity in Gases."

unknown molecular changes and hence may be used for determining T. If we take the electrical conductivity of a metal to be given by[1]

$$\sigma = \frac{2}{3} n \frac{e^2}{m} \frac{\lambda_0}{V},$$

where n is the number of electrons per unit volume, we get

$$\frac{dR}{R} - \frac{\delta R}{R} = \frac{9}{4} H^2 \frac{\sigma^2}{n^2 e^2}. \tag{9}$$

According to this equation a transverse magnetic field should always produce a greater increase of resistance than a longitudinal field.

Experiments.

It has been observed[2] that some forms of graphite suffer a large resistance change while other forms are apparently unaffected by a magnetic field. No systematic study seems to have been made of the reasons for these variations, and inasmuch as the variations are quite large it seemed as if a knowledge of the causes of these variations would prove illuminating from the standpoint of the theory. Furthermore, powdered graphite, composed of small crystals, may be compressed into bars, and these bars are similar to most metals in that they are composed of crystal agglomerations.

The apparatus which was used in measuring the resistance of the specimens was a Wheatstone bridge with balancing shunts so arranged that changing one of these shunt resistances by a large amount compensated for a very small change in the resistance of the specimen. A Leeds and Northrup high sensibility galvanometer of resistance 13 ohms was used, and the apparatus was sufficiently sensitive in most cases to measure a value of dR/R as small as 3×10^{-5}. The specimen under examination was suspended by means of a wooden clamp or an ebonite rod between the poles of a Weiss electromagnet in such a way that a simple rotation of the magnet sufficed to change a transverse into a longitudinal field without disturbing the specimen. This arrangement was found to be of distinct advantage, since the moving or jarring of the specimen was found under some conditions to alter its resistance. The experiments were performed at room temperature (about 26° C.) and in order to shield the specimen from air currents thick pads of hair-felt were set up around it between the magnet poles, which were 2.3 cm. apart. The bridge current was allowed to flow for a sufficient length of time before taking

[1] Swann, Phil. Mag., March, p. 441, 1914.

[2] D. E. Roberts, Konink. Akad. Wetensch. Amsterdam, Proc. 15, p. 148, 1912. G. E. Washburn, Ann. d. Phys., 48, 2, p. 236, 1915.

measurements to let the specimen come into temperature equilibrium with its surroundings. The resistance of the wires leading to the specimen was carefully measured and allowed for in the computations. These leads, however, were of copper and they had a low resistance, so that the effect of the magnetic field on the resistance of these leads was negligible.

Several forms of graphite were used in preparing the specimens for examination. The first group of experiments was made on the graphite of ordinary pencils, electric light carbons, and rods constructed for lubricating purposes. In these materials the pure graphite is mixed with "binder" of some sort—usually clay in the case of pencils—so that the resulting mixture is quite hard and brittle. In all these substances the effect of a magnetic field was quite small. Fig. 1 shows the relation between H and the resistance change for a rod of soft graphite made by the Dixon Crucible Co. for lubricating purposes. These curves are typical of other curves obtained for other materials of this class. It will be observed that the transverse effect is greater than the longitudinal effect for a given value of H. The experiments seemed to indicate that in soft specimens, presumably containing a small amount of binder, the magnetic field produces a greater effect than in the hard specimens.

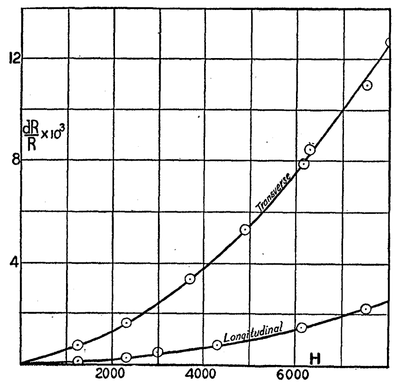

Fig. 1.

The next group of experiments was made on graphite powder compressed into sticks and bars. The following methods of compressing the powder were used:

(*a*) Hand compression between brass electrodes in small bore glass tubes. The electrodes were held by a screw arrangement in tight contact with the graphite and the graphite was not removed from the tube during the experiments.

(*b*) Compression of the powder by means of a hydraulic press into grooves in an ebonite block. The groove for each specimen consisted of a single straight shallow trough connecting two deep holes in the face of the block. Copper wires coiled in these holes were led out to serve as terminals. This block was placed face upwards at the bottom of a hollow iron cylinder, graphite poured into the cylinder, and pressure applied. When the block was removed and the excess graphite scraped off, the groove was found to contain a bar of graphite formed by pressure applied in a direction perpendicular to its length.

(*c*) Compression into the form of thin plates by means of the hydraulic press. The same method was used as in (*b*) except that no grooves were cut in the ebonite block. The thin brittle plates which were formed were cut into bars with a razor blade, the ends copper plated, and copper leads were soldered to the specimen.

Two different grades of powder were used—the visible difference between the two grades being that one consisted of larger crystals than the other. It was evident upon inspection of the specimens prepared in the three ways described above that the result was not even approximately an isotropic material. The bars exhibited cleavage planes perpendicular to the direction in which pressure had been applied. We should, however, expect a result of this nature. The crystals of the powder were like very small thin plates, hence pressure applied to such an aggregation might be expected to produce an anisotropic effect by forcing the crystals to turn their large plane faces perpendicular to the direction in which pressure had been applied while the specimen was being made up. When two test specimens were made up according to methods (*b*) and (*c*) respectively, it was found that a magnetic field produced the greater effect on the sample made by method (*c*). Since this method produces a much greater compression of the graphite powder than the other two methods it appears that high compression tends to increase the resistance change in a magnetic field. However, it was possible to change by hand the compression of samples made by method (*a*) until the resistance was halved, without producing an appreciable increase in the effect of a magnetic field. We must conclude, therefore, that a high degree of compression is necessary in order to produce much of an effect.

In Fig. 2 the results obtained with the fine power are expressed in graphical form; in Fig. 3 some of the corresponding results for the coarse powder are plotted. The letter on each curve specifies the method (described above) used in preparing the specimen. Circles are used to specify all results obtained when the magnetic field was perpendicular to the direction of the current; small crosses are used when H was parallel to the current. When the magnetic field was parallel to the direction in which pressure had been applied during the process of making the specimen the symbol $||$ is written on the curve; when H was perpendicular to this direction the symbol $\perp$ is used. The immediate conclusions which we may draw from these curves are as follows:

1. Increasing the size of the graphite particles increases the effect of the magnetic field on the resistance; for the curves of Fig. 3 are all higher than the corresponding curves of Fig. 2.

2. The direction of the magnetic field with respect to the crystal structure in graphite is of greater importance than its direction with respect to the electric current. For in (*b*) of Fig. 2 the effect of a transverse field is only a small amount greater than the effect of a longitudinal field, provided the field is kept perpendicular to the direction of compres-

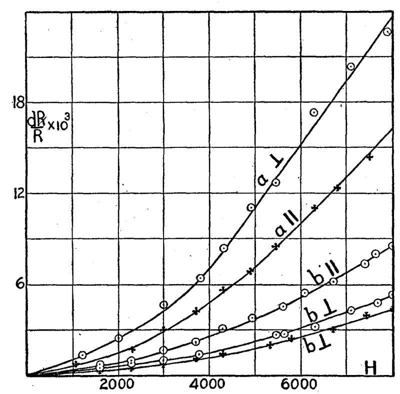

Fig. 2.

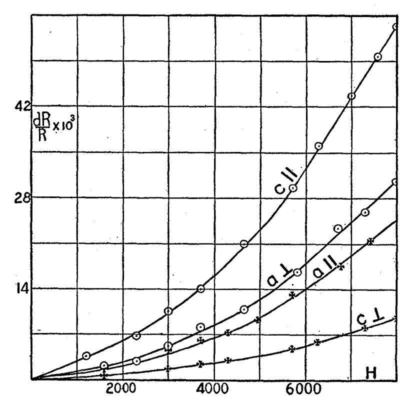

Fig. 3.

sion. Turning the field parallel to the direction of compression increases the effect of the transverse field.

3. If the effect of crystal structure in the graphite is eliminated we secure an effect in a transverse field which is greater than in a longitudinal field. This conclusion follows if we assume magnetic isotropy in all directions perpendicular to the direction of compression. We are then able to compare the two lowest curves of Fig. 2 with each other on the assumption that the influence of crystal structure is the same for each curve. In Fig. 2 the transverse field of 8,000 gausses gives a value of dR/R, which is greater by about 8×10^{-4} than that of the longitudinal field.

In order to test these conclusions more completely an experiment was performed on some fairly large crystals of graphite. These crystals consisted of laminated fragments embedded in a calcareous material which could be broken with comparative ease into small pieces. After some difficulty a satisfactory specimen of graphite was secured. The laminations of this specimen were parallel to its length, were very numerous and closely pressed together, and appeared to be very free from impurities. It was cut into the form of a bar, the ends copper plated, and copper terminals were soldered to these ends. The dimensions of the specimen were roughly $0.3 \times 0.4 \times 5.0$ mm., and its resistance was 0.08 ohm. The experimental results obtained with this specimen are plotted in Fig. 4. Here the symbols $\perp$ and $||$ mean that H is respectively perpendicular and parallel to the normal to the laminæ of the specimen. These curves are similar to those of Figs. 2 and 3 except for the relative magnitudes of the effects. Where a single crystal or a single group of crystals is used it is evident that the magnitude of the resistance change is greatly increased. In short, we may consider that the experiments with the large crystal group corroborate conclusions (1) and (2) above. Regarding conclusion (3) it is to be observed that of the two lowest curves of Fig. 3 that for the transverse field is the higher. Another sample of crystalline graphite tested out very carefully in this respect gave similar results to those plotted in Fig. 3. If we make the assumption that the crystal is magnetically isotropic for all directions in the plane of the laminations then we may assert that a transverse magnetic field produces a greater effect than a longitudinal field when the structural changes produced by H are the same in both cases.

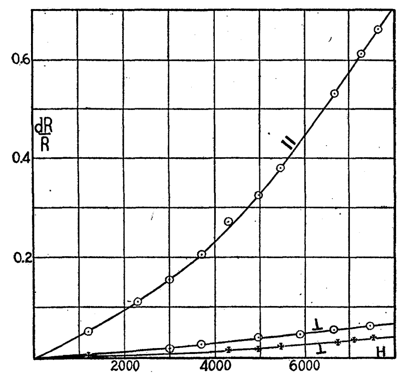

Fig. 4.

The general conclusions, however, which are to be drawn from the above study of a group of small crystals indicate very clearly that we must inquire carefully into the crystalline nature of metals before attempting to interpret experiments which are made on metals. The point of immediate interest is to decide whether or not a wire of given material is magnetically isotropic as regards directions parallel and perpendicular to its length. Ordinary magnetic measurements are difficult to make with sufficient accuracy in this case, and furthermore, we cannot be sure that isotropy as regards magnetic permeability, for example, will guarantee isotropy as regards resistance change in a magnetic field. A way in which we may gain information, however, is as follows. If a thin section of metal is cut from a cylindrical bar of cast metal (the cut being perpendicular to the length of the bar) and then hammered out into a thin plate we may suppose from principles of symmetry that for all directions in the plane of the plate we will have magnetic isotropy.

A thin rectangular plate of cadmium was prepared in this way and a series of parallel cuts made across it. These cuts did not completely traverse the plate, but were so made that four very thin strips of metal were secured, each about two centimeters long and half a millimeter wide, lying side by side and joined in series. When experiments were made on this specimen it was found that a transverse magnetic field of 7,550 gausses produced the same increase of resistance whether parallel or perpendicular to the surface of the strips. A difference in dR/R as small as 3×10^{-5} could have been detected in the two cases. Keeping the magnetic field in the plane of the strips but rotating it so as to compare the longitudinal and transverse effects, it was found that the transverse field produced the greater increase of resistance. The average of two trials gave

$$\frac{dR}{R} - \frac{\delta R}{R} = 9 \times 10^{-5}$$

with $H = 7{,}550$. This result agrees as well as could be expected with previous experiments[1] made on a different sample of cadmium in the form of wire. We may conclude then, that the cadmium strips were magnetically isotropic with respect to resistance change. If strips of metal hammered out in this fashion are magnetically isotropic it is probably safe to assume that cadmium wires are similarly isotropic as regards resistance change.

It was difficult to test other non-ferromagnetic metals than cadmium in this way for magnetic isotropy because those metals which are easily worked with mechanically have a small resistance change in a magnetic field, and hence require a more sensitive apparatus than the one available. In a previous paper[2] the writer has measured both the transverse and longitudinal effects in wires made of different metals. In all cases the transverse effect was greater than the longitudinal as we should expect from equation (9). If we assume that the other metals are like cadmium in being magnetically isotropic we may apply equations (8) and (9) to these substances and calculate the mean free period of the electrons and the number of electrons per unit volume of metal. The following table gives the results of the calculations, data being taken from the previous paper except in the case of bismuth and graphite. In the calculations it is assumed that $e = 1.6 \times 10^{-20}$ e.m.u., $e/m = 1.7 \times 10^7$, and $H = 8{,}000$ gausses.

Patterson[3] obtained values of n somewhat different from those given above but in his equation no account was taken of the longitudinal effect or of the effect of H on molecular structure. Other methods of determining n are based upon Drude's theory of the optical properties of

[1] Phil. Mag., Dec., p. 900, 1911.
[2] Phil. Mag., Dec., p. 900, 1911; Dec., p. 813, 1912,
[3] Phil. Mag., 3, p. 643, 1902.

	σ(e.m.u.).	$\frac{dR}{R} - \frac{\delta R}{R}$.	T.	n.
Te[4]	[2]4.7×10^{-5}	4.9×10^{-3}	5.1×10^{-13}	5.0×10^{20}
Bi[5]	[2]0.84	0.144	39.2	11.8×10^{18}
PbS[6]	[3]0.042	1.15×10^{-4}	0.8	2.9×10^{19}
Cd	[2]13.0	6.5×10^{-5}	0.59	1.2×10^{22}
Zn	[2]16.0	2.9×10^{-5}	0.4	2.2×10^{22}
Au	[2]41.0	0.3×10^{-5}	0.12	18.8×10^{22}
Graphite[7]	[3]0.036	0.024	11.4	1.74×10^{18}

[2] From Kaye and Laby.

[3] Baedeker, "Elektrischen Erscheinungungen in Metallischen Leitern."

[4] Cast in cylindrical form.

[5] The tests on bismuth were made using a Hartmann and Braun spiral. The magnetic field used in this case was 5,700 gausses.

[6] Natural crystal. Tests showed it to be magnetically isotropic.

[7] Large natural crystals of Fig. 4.

metals and upon the theory of thermionic emission. Experiments made by Spence[1] on the refractive index of metals lead him to give as probable values of n for gold the number 3.3×10^{22}, for silver 3.7×10^{22}, and for platinum a number less than 8.1×10^{22}. From measurements on the thermionic emission of hot metals H. A. Wilson[2] deduces for platinum a value of $n = 1.5 \times 10^{22}$. These results are of the same order of magnitude as those tabulated above.

It is of interest to consider how equations (3) and (6) may be applied to the experiments. In the case of all the substances listed in the above table we must consider the term $\delta\lambda/\lambda_0$ to be very important, for in all these materials the change of resistance in a longitudinal magnetic field is comparable with the change in a transverse field. We are thus led to believe that in equation (6) the term $d\lambda/\lambda_0$ must account for a large part of the resistance change. Furthermore $\delta\lambda$ must represent a decrease in free path, otherwise a longitudinal magnetic field would decrease the resistance of the conductor—a result contrary to experiment. It is to be hoped that a more complete understanding of this term, $\delta\lambda$, will be reached by further experiment.

Conclusions.

1. The theory of the effect of magnetic fields on resistance has been developed to cover the cases of longitudinal and transverse magnetic fields. The theory leads us to expect a greater transverse than longitudinal effect. All experiments on non-ferromagnetic metals appear to be in agreement with this conclusion. In order to make definite tests,

[1] Phys. Rev., 28, p. 337, 1909.

[2] "Electrical Properties of Flames."

however, we are obliged to consider carefully the crystalline structure of the metals examined.

2. With this fact in mind experiments have been made on crystalline graphite in different forms. In compressed graphite powder the effect of a magnetic field on the resistance is greater in coarse powder than in fine. This fact implies that in a substance composed of large crystals the effect is greater than when it is composed of small crystals. It is probable that this effect of crystalline structure explains why bismuth, which in ordinary form suffers a large increase of resistance in a magnetic field, is very little affected when in the form of fine powder produced by chemical reduction of a bismuth salt.

There is a greater effect in large crystals of graphite than has so far been discovered in any substance which is metallically conducting. From the study of graphite it is to be concluded that crystal structure is an important factor to consider when experimenting on the conductivity of any substance in a magnetic field. By making up specimens in the proper way it is possible to compare the longitudinal and transverse effects experimentally without any trouble arising from differences in crystalline structure.

3. From experiments on a specially prepared sample of cadmium it is to be concluded that comparisons of the longitudinal and transverse effects may be made legitimately using an ordinary wire as the specimen. Assuming that wires of other metals are like cadmium in this respect calculations have been made for several metals of the number of electrons per unit volume and of the free period of the electrons.

4. The theory affords a satisfactory explanation of the behavior of all para- and diamagnetic substances which have so far been examined.

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